Cross section vs. experimental yield

\[
\sigma \equiv \frac{\text{number of interactions per time}}{\text{number of incident particles per area per time} \cdot \text{number of target nuclei within the beam}} = \frac{N_r}{N_0 N_t}
\]

\[
Y \equiv \frac{N_r}{N_0} = \alpha N_t \quad \text{[for special case of “thin” target]}
\]

Example: \(^1\text{H} + ^1\text{H} \rightarrow ^2\text{H} + \text{e}^+ + \nu\) (first step of pp chain; solar Gamow peak: 6 keV)

- cross section: \(\sigma_{\text{theo}} = 8 \cdot 10^{-48} \text{ cm}^2 = 8 \cdot 10^{-24} \text{ barn} \) at \(E_{\text{lab}} = 1 \text{ MeV} \) [\(E_{\text{cm}} = 0.5 \text{ MeV}\)]
- proton beam: total charge \((Q)\), current \((I)\), elementary charge
  \[N_0/t = (Q/t)/q = 1 \text{ Ampere}/1.6 \cdot 10^{-19} \text{ C} = 6 \cdot 10^{18} \text{ incident protons/s}\]
- proton target: dense hydrogen gas \([N_t = 10^{20} \text{ protons/cm}^2]\)

\[N_r = (8 \cdot 10^{-48} \text{ cm}^2)(10^{20} \text{ protons/cm}^2)(6 \cdot 10^{18} \text{ protons/s}) = 5 \cdot 10^{-9} / \text{s} \] [1 reaction in 6 years!]

We need very high beam currents of very low energies!
We can parametrize the $\gamma$-ray partial width for the $E1$ ground state transition as

$$\Gamma_{\gamma,E1}^0(E) = \Gamma_{\gamma,E1}^0(E_r) \left( \frac{E + Q}{E_r + Q} \right)^{2L+1} = (9.4 \cdot 10^{-6} \text{ MeV}) \left( \frac{E + 7.550 \text{ MeV}}{0.518 \text{ MeV} + 7.550 \text{ MeV}} \right)^3$$

and the proton partial width as

$$\Gamma_p^0(E) = \Gamma_p^0(E_r) \frac{P_{l=0}(E)}{P_{l=0}(E_r)} = (0.037 \text{ MeV}) \frac{e^{-5.721/\sqrt{E}}}{e^{-5.721/\sqrt{0.518}}}$$

since

$$P_{l=0}(E) \approx e^{-2\kappa l} = e^{-0.989 \cdot 1.6 \sqrt{\frac{1.13}{1 + 13 \sqrt{E}}} \frac{1}{\sqrt{E}}} = e^{-\frac{5.721}{\sqrt{E}}}$$

It follows that $\Gamma_{\gamma}^0 \approx \Gamma_p^0$ near $E=122$ keV. Note that in this case the exact penetration factor, evaluated by using Coulomb wave functions, agrees with the Gamow factor within a factor of 2. See also Brune and Kavanagh, Phys. Rev. C 44, 1665 (1991).