Fundamental Symmetries – III
Muons

R. Tribble
Texas A&M University
All about muons

Topics:

• Lifetime – MuLAN
• Normal decay – TWIST
• Exotic decays – MEGA, MEG, SINDRUM
• Anomalous Moment – (g-2)
Muon Lifetime

• Determines $G_F$ by (two loop QED and SM)

$$
\tau_\mu^{-1} = \frac{G_F^2 m_\mu^5}{192\pi^3} F \left( \frac{m_e^2}{m_\mu^2} \right) \left( 1 + \frac{3m_\mu^2}{5m_W^2} \right) \left[ 1 + \frac{\alpha(m_\mu)}{2\pi} \left( \frac{25}{4} - \pi^2 \right) \right]
$$

where

$$
F(x) = 1 - 8x + 8x^3 - x^4 - 12x^2 \ln x
$$

and

$$
\frac{1}{\alpha(m_\mu)} = \frac{1}{\alpha} - \frac{2}{3\pi} \ln \left( \frac{m_\mu}{m_e} \right) + \frac{1}{6\pi} \approx 136
$$

• MuLAN collaboration recently completed and published new result for lifetime
The experimental concept...

- **Kicker On**
- **Fill Period**
- **Measurement Period**

170 Inner/Outer tile pairs

Real data

Counts per 42 ns

Accumulation Period, $T_A$

Measurement Period, $T_M$

Kicker Transition

Background Level

- **450 MHz WaveForm Digitization** (2006/07)
- **MHTDC** (2004)

Slide from D. Hertzog
MuLan collected two datasets, each containing $10^{12}$ muon decays

- Two (very different) data sets
  - Different blinded clock frequencies used
  - Revealed only after all analyses of both data sets completed
  - Most systematic errors are common
## Final Errors and Numbers

### ppm units

| Effect                      | 2006  | 2007  | Comment                                               |
|-----------------------------|-------|-------|                                                      |
| Kicker extinction stability | 0.20  | 0.07  | Voltage measurements of plates                       |
| Upstream muon stops         | 0.10  | 0.10  | Upper limit from measurements                        |
| Overall gain stability:     | 0.25  | 0.25  | MPV vs time in fill; includes:                       |
| Short time; after a pulse   |       |       | MPVs in next fill & laser studies                    |
| Long time; during full fill |       |       | Different by PMT type                                |
| Electronic ped fluctuation  |       |       | Bench-test supported                                |
| Unseen small pulses         |       |       | Uncorrected pileup effect \(\rightarrow\) gain       |
| Timing stability            | 0.12  | 0.12  | Laser with external reference ctr.                   |
| Pileup correction           | 0.20  | 0.20  | Extrapolation to zero ADT                            |
| Residual polarization       | 0.10  | 0.20  | Long relax; quartz spin cancelation                   |
| Clock stability             | 0.03  | 0.03  | Calibration and measurement                          |
| **Total Systematic**        | **0.42** | **0.42** | Highly correlated for 2006/2007                     |
| **Total Statistical**       | **1.14** | **1.68** |                                                   |

\[ \tau(R06) = 2\ 196\ 979.9 \pm 2.5 \pm 0.9 \text{ ps} \]

\[ \tau(R07) = 2\ 196\ 981.2 \pm 3.7 \pm 0.9 \text{ ps} \]
Lifetime “history”

The most precise particle or nuclear or (we believe) atomic lifetime ever measured

New $G_F$

$G_F(\text{MuLan}) = 1.166\,378\,8(7) \times 10^{-5} \text{ GeV}^{-2} \ (0.6 \text{ ppm})$

Slide from D. Hertzog
Muon decay spectrum

The energy and angle distributions of positrons following polarized muon decay obey the spectrum:

\[
\frac{d^2\Gamma}{x^2 dx d(cos \theta)} \propto (3 - 3x) + \frac{2}{3} \rho (4x - 3) + 3\eta \frac{x_0}{x} (1 - x) \\
+ P_\mu \xi \cos \theta \left[(1 - x) + \frac{2}{3} \delta(4x - 3)\right]
\]

where \( x = \frac{E_e}{E_{e,\text{max}}} \)

[Radiative corrections not included]
Muons decay matrix element

- Most general local, derivative-free, lepton-number conserving muon decay matrix element:

\[
M = \frac{4G_F}{\sqrt{2}} \sum_{\gamma=S,V,T} \sum_{\varepsilon,\mu=R,L} g^{\varepsilon\mu}_{\gamma} \langle \bar{e}_\varepsilon | \Gamma_{\gamma} | (\nu_\varepsilon)_n \rangle \langle (\bar{\nu}_\mu)_m | \Gamma_{\gamma} | \mu_\mu \rangle
\]

- In the Standard Model, \( g^{V}_{LL} = 1 \), all others are zero
- Pre-TWIST global fit results (all 90% c.l.):

<table>
<thead>
<tr>
<th>( g^{S}_{RR} )</th>
<th>( g^{V}_{RR} )</th>
<th>( g^{T}_{RR} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt; 0.066</td>
<td>&lt; 0.033</td>
<td>( \equiv 0 )</td>
</tr>
<tr>
<td>( g^{S}_{LR} )</td>
<td>( g^{V}_{LR} )</td>
<td>( g^{T}_{LR} )</td>
</tr>
<tr>
<td>&lt; 0.125</td>
<td>&lt; 0.060</td>
<td>&lt; 0.036</td>
</tr>
<tr>
<td>( g^{S}_{RL} )</td>
<td>( g^{V}_{RL} )</td>
<td>( g^{T}_{RL} )</td>
</tr>
<tr>
<td>&lt; 0.424</td>
<td>&lt; 0.110</td>
<td>&lt; 0.122</td>
</tr>
<tr>
<td>( g^{S}_{LL} )</td>
<td>( g^{V}_{LL} )</td>
<td>( g^{T}_{LL} )</td>
</tr>
<tr>
<td>&lt; 0.550</td>
<td>&gt; 0.960</td>
<td>( \equiv 0 )</td>
</tr>
</tbody>
</table>
Muon decay parameters and coupling constants

\[ \rho = \frac{3}{4} - \frac{3}{4} \left[ |g_{RL}^V|^2 + |g_{LR}^V|^2 + 2 |g_{RL}^T|^2 + 2 |g_{LR}^T|^2 \right] + \Re \left( g_{RL}^S g_{RL}^{T*} + g_{LR}^S g_{LR}^{T*} \right) \]

\[ \eta = \frac{1}{2} \Re \left[ g_{RR}^V g_{LL}^S + g_{LL}^V g_{RR}^S + g_{RL}^V (g_{LR}^S + 6g_{LR}^T) + g_{LR}^V (g_{RL}^S + 6g_{RL}^T) \right] \]

\[ \xi = 1 - \frac{1}{2} |g_{LR}^S|^2 - \frac{1}{2} |g_{RR}^S|^2 - 4 |g_{RL}^V|^2 + 2 |g_{LR}^V|^2 - 2 |g_{RR}^V|^2 + 2 |g_{LR}^T|^2 - 8 |g_{RL}^T|^2 + 4 \Re \left( g_{LR}^S g_{LR}^{T*} - g_{RL}^S g_{RL}^{T*} \right) \]

\[ \xi \delta = \frac{3}{4} - \frac{3}{8} |g_{RR}^S|^2 - \frac{3}{8} |g_{LR}^S|^2 - \frac{3}{2} |g_{RR}^V|^2 - \frac{3}{4} |g_{RL}^V|^2 - \frac{3}{4} |g_{LR}^V|^2 - \frac{3}{2} |g_{RL}^T|^2 - 3 |g_{LR}^T|^2 + \frac{3}{4} \Re \left( g_{LR}^S g_{LR}^{T*} - g_{RL}^S g_{RL}^{T*} \right) \]

Prior to \textbf{TWIST}

\[ \rho = 0.7518 \pm 0.0026 \quad 3/4 \]
\[ \eta = -0.007 \pm 0.013 \quad 0 \]
\[ P_\mu \xi = 1.0027 \pm 0.0079 \pm 0.0030 \quad 1 \]
\[ \delta = 0.7486 \pm 0.0026 \pm 0.0028 \quad 3/4 \]
\[ P_\mu (\xi \delta/\rho) > 0.99682 \ (90\% \ c.l.) \quad 1 \]
Goal of *TWIST*

- Search for new physics that can be revealed by **order-of-magnitude improvements** in our knowledge of $\rho$, $\delta$, and $P_\mu \xi$

Two examples

- **Model-independent limit on muon handedness**

$$Q^\mu_R = \frac{1}{2} \left[ 1 + \frac{1}{3} \xi - \frac{16}{9} \xi \delta \right]$$

- **Left-right symmetric models**

$$\frac{3}{4} - \rho = \frac{3}{2} \zeta^2 \quad 1 - P_\mu \xi = 4 \left( \zeta^2 + \zeta \left( \frac{M_L}{M_R} \right)^2 + \left( \frac{M_L}{M_R} \right)^4 \right)$$

- ....
Must:

- Determine spectrum shape
  -- All three parameters
- Understand sources of muon depolarization
  -- $P_\mu$ and $\xi$ come as a product
- Measure forward-backward asymmetry
  -- For $P_\mu \xi$ and $\delta$

To within a few parts in $10^4$
Analysis method

• **Extract energy and angle distributions for data:**
  – Apply (unbiased) cuts on muon variables.
  – Reject fast decays and backgrounds.
  – Calibrate $e^+$ energy to kinematic end point at 52.83 MeV.

• **Fit to identically derived distributions from simulation:**
  – GEANT3 geometry contains virtually all detector components.
  – Simulate chamber response in detail.
  – Realistic, measured beam profile and divergence.
  – Extra muon and beam positron contamination included.
  – Output in digitized format, identical to real data.
2-d momentum-angle spectrum

Acceptance of the TWIST spectrometer
Fitting the data distributions

- Decay distribution is linear in $\rho$, $\eta$, $P_{\mu \xi}$, and $P_{\mu \xi \delta}$, so a fit to first order expansion is exact.

- Fit data to simulated (MC) base distribution with hidden assumed parameters, $\lambda_{MC} = (\rho, \eta, P_{\mu \xi}, P_{\mu \xi \delta})$ plus MC-generated distributions from analytic derivatives, times fitting parameters ($\Delta \lambda$) representing deviations from base MC. ($\eta$ is now fixed to global analysis value)

(graphic thanks to Blair Jamieson)
Results from first two data sets

- From Fall, 2002 run:
  - $\rho = 0.75080 \pm 0.00032 \text{ (stat)} \pm 0.00097 \text{ (syst)} \pm 0.00023 \text{ (}\eta\text{)}$
  - $\delta = 0.74964 \pm 0.00066 \text{ (stat)} \pm 0.00112 \text{ (syst)}$

- From Fall, 2004 run:
  - $\rho = 0.75014 \pm 0.00017 \text{ (stat)} \pm 0.00044 \text{ (syst)} \pm 0.00011 \text{ (}\eta\text{)}$
  - $\delta = 0.74964 \pm 0.00030 \text{ (stat)} \pm 0.00067 \text{ (syst)}$

R. McDonald et al., PRD 78, 032010
Global Analysis

Use general form of interaction:

\[
M = \frac{4G_F}{\sqrt{2}} \sum_{\gamma=S,V,T} \sum_{\varepsilon,\mu=R,L} g_{\varepsilon\mu}^\gamma \langle \bar{e}_\varepsilon | \Gamma_\gamma | (\nu_e)_n \rangle \langle (\bar{\nu}_\mu)_m | \Gamma_\gamma | \mu_\mu \rangle
\]

- Follow Fetscher, Gerber, Johnson formulation (Phys. Lett. 173B, 102 (1986))
Global Analysis

\[ Q_{RR} = \frac{1}{4} |g_{RR}^S|^2 + |g_{RR}^V|^2, \]
\[ Q_{LR} = \frac{1}{4} |g_{LR}^S|^2 + |g_{LR}^V|^2 + 3|g_{LR}^T|^2, \]
\[ Q_{RL} = \frac{1}{4} |g_{RL}^S|^2 + |g_{RL}^V|^2 + 3|g_{RL}^T|^2, \]
\[ Q_{LL} = \frac{1}{4} |g_{LL}^S|^2 + |g_{LL}^V|^2, \]
\[ B_{LR} = \frac{1}{16} |g_{LR}^S + 6g_{LR}^T|^2 + |g_{LR}^V|^2, \]
\[ B_{RL} = \frac{1}{16} |g_{RL}^S + 6g_{RL}^T|^2 + |g_{RL}^V|^2, \]
\[ I_\alpha = \frac{1}{4} [g_{LR}^V (g_{RL}^S + 6g_{RL}^T)^* + (g_{RL}^V)^* (g_{LR}^S + 6g_{LR}^T)] \]
\[ = (\alpha + i\alpha')/2A, \]
\[ I_\beta = \frac{1}{2} [g_{LL}^V (g_{RR}^S)^* + (g_{RR}^V)^* g_{LL}^S] = -2(\beta + i\beta')/A \]

Constraints:

\[ 0 \leq Q_{\epsilon \mu} \leq 1, \quad \text{where } \epsilon, \mu = R, L, \]
\[ 0 \leq B_{\epsilon \mu} \leq Q_{\epsilon \mu}, \quad \text{where } \epsilon \mu = RL, LR, \]
\[ |I_\alpha|^2 \leq B_{LR} B_{RL}, \quad |I_\beta|^2 \leq Q_{LL} Q_{RR}, \]

Normalization:

\[ Q_{RR} + Q_{LR} + Q_{RL} + Q_{LL} = 1 \]

Note that \( Q_{LL} \approx 1 \)

(from Phys. Lett. 173B)
Global Analysis

Relation to muon decay observables:

\[
\begin{align*}
\rho &= \frac{3}{4} + \frac{1}{4} (Q_{LR} + Q_{RL}) - (B_{LR} + B_{RL}), \\
\xi &= 1 - 2 Q_{RR} - \frac{10}{3} Q_{LR} + \frac{4}{3} Q_{RL} + \frac{16}{3} (B_{LR} - B_{RL}), \\
\xi \delta &= \frac{3}{4} - \frac{3}{2} Q_{RR} - \frac{7}{4} Q_{LR} + \frac{1}{4} Q_{RL} + (B_{LR} - B_{RL}), \\
\xi' &= 1 - 2 Q_{RR} - 2 Q_{RL}, \\
\xi'' &= 1 - \frac{10}{3} (Q_{LR} + Q_{RL}) + \frac{16}{3} (B_{LR} + B_{RL}), \\
\text{rad. decay} \{ \bar{\eta} = \frac{1}{3} (Q_{LR} + Q_{RL}) + \frac{2}{3} (B_{LR} + B_{RL}), \\
\text{e}^+_L \{ \eta = (\alpha - 2 \beta)/A, \quad \eta'' = (3 \alpha + 2 \beta)/A. \\
\text{e}^+_T \{ 
\end{align*}
\]
## Global Analysis

### 2005 Input:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho$</td>
<td>$0.7518 \pm 0.0026$</td>
</tr>
<tr>
<td></td>
<td>$0.75,080 \pm 0.00,105^a$</td>
</tr>
<tr>
<td>$\delta$</td>
<td>$0.7486 \pm 0.0038$</td>
</tr>
<tr>
<td></td>
<td>$0.74,964 \pm 0.00,130$</td>
</tr>
<tr>
<td>$P_\mu \xi$</td>
<td>$1.0027 \pm 0.0085^b$</td>
</tr>
<tr>
<td>$P_\mu \xi \delta / \rho$</td>
<td>$0.99,787 \pm 0.00,082^b$</td>
</tr>
<tr>
<td>$\xi'$</td>
<td>$1.00 \pm 0.04$</td>
</tr>
<tr>
<td>$\xi''$</td>
<td>$0.65 \pm 0.36$</td>
</tr>
<tr>
<td>$\bar{\eta}$</td>
<td>$0.02 \pm 0.08$</td>
</tr>
<tr>
<td>$\alpha / A$</td>
<td>$0.015 \pm 0.052^c$</td>
</tr>
<tr>
<td>$\beta / A$</td>
<td>$0.002 \pm 0.018^c$</td>
</tr>
<tr>
<td>$\eta$</td>
<td>$0.071 \pm 0.037^d$</td>
</tr>
<tr>
<td>$\eta''$</td>
<td>$0.105 \pm 0.052^d$</td>
</tr>
<tr>
<td>$\alpha' / A$</td>
<td>$-0.047 \pm 0.052^e$</td>
</tr>
<tr>
<td></td>
<td>$-0.0034 \pm 0.0219^f$</td>
</tr>
<tr>
<td>$\beta' / A$</td>
<td>$0.017 \pm 0.018^e$</td>
</tr>
<tr>
<td></td>
<td>$-0.0005 \pm 0.0080^f$</td>
</tr>
</tbody>
</table>

### 2005 Output:

#### Fit Result ($\times 10^3$)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Fit Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q_{RR}$</td>
<td>&lt;1.14(0.60 ± 0.38)</td>
</tr>
<tr>
<td>$Q_{LR}$</td>
<td>&lt;1.94(1.22 ± 0.53)</td>
</tr>
<tr>
<td>$B_{LR}$</td>
<td>&lt;1.27(0.72 ± 0.40)</td>
</tr>
<tr>
<td>$Q_{RL}$</td>
<td>&lt;44(26 ± 13)</td>
</tr>
<tr>
<td>$B_{RL}$</td>
<td>&lt;10.9(6.4 ± 3.3)</td>
</tr>
<tr>
<td>$Q_{LL}$</td>
<td>&gt;955(973 ± 13)</td>
</tr>
<tr>
<td>$\alpha / A$</td>
<td>0.3 ± 2.1</td>
</tr>
<tr>
<td>$\beta / A$</td>
<td>2.0 ± 3.1</td>
</tr>
<tr>
<td>$\alpha' / A$</td>
<td>−0.1 ± 2.2</td>
</tr>
<tr>
<td>$\beta' / A$</td>
<td>−0.8 ± 3.2</td>
</tr>
</tbody>
</table>

**PRD 72, 073002**
Final TWIST Results

\[ \rho = 0.74977 \pm 0.00012 \text{ (stat)} \pm 0.00023 \text{ (syst)} \]
\(<1 \sigma \text{ from SM})

\[ \delta = 0.75049 \pm 0.00021 \text{ (stat)} \pm 0.00027 \text{ (syst)} \]
\(+1.4 \sigma \text{ from SM})

\[ \mathcal{P}_{\mu} \pi \xi = 1.00084 \pm 0.00029 \text{ (stat)} \pm 0.00063 \text{ (syst)} \]
\(+0.00165 \text{ (syst)} \)
\(-0.00063 \text{ (syst)} \)
\(+1.2 \sigma \text{ from SM})

\[ \mathcal{P}_{\mu} \pi \xi \delta \rho > 0.99909 \text{ (90\%CL)} \]
from global analysis
Final Global Analysis Results

- Include new results with other muon decay observables to restrict coupling constants
  - summary of all terms (pre-\textbf{TWIST} in parentheses)
    \[
    \begin{align*}
    |g^S_{RR}| &< 0.035 (0.066) & |g^V_{RR}| &< 0.017 (0.033) & |g^T_{RR}| &\equiv 0 \\
    |g^S_{LR}| &< 0.050 (0.125) & |g^V_{LR}| &< 0.023 (0.060) & |g^T_{LR}| &< 0.015 (0.036) \\
    |g^S_{RL}| &< 0.420 (0.424) & |g^V_{RL}| &< 0.105 (0.110) & |g^T_{RL}| &< 0.105 (0.122) \\
    |g^S_{LL}| &< 0.550 (0.550) & |g^V_{LL}| &> 0.960 (0.960) & |g^T_{LL}| &\equiv 0
    \end{align*}
    \]

- influences mostly right-handed muon terms
  \[
  Q^\mu_R = \frac{1}{4}|g^S_{LR}|^2 + \frac{1}{4}|g^S_{RR}|^2 + |g^V_{LR}|^2 + |g^V_{RR}|^2 + 3|g^T_{LR}|^2 \\
  = \frac{1}{2}[1 + \frac{1}{3}\xi - \frac{16}{9}\xi \delta] \\
  < 8.2 \times 10^{-4} \text{ (90\%C.L.)}
  \]
Neutrino-less Muon Decays

• Three lepton-flavor violating muon decays are possible:
  – $\mu \rightarrow e + \gamma$
  – $\mu^+ \rightarrow e^+e^+e^-$
  – $\mu \rightarrow e$ conversion

• These decay modes are not allowed with massless neutrinos

• Highly suppressed in SM with known neutrino masses

• SM extensions affect the decay rates differently
$\mu \rightarrow e + \gamma$ Decay

• History of searches for this decay at LAMPF following preliminary work at TRIUMF and PSI
• Very high flux of muons at LAMPF
• MEGA collaboration most recent $\mu \rightarrow e + \gamma$ experiment (1985-1995)

High energy $\gamma$ background:
• $\mu \rightarrow e\nu\bar{\nu}\gamma$
• Annihilation in flight
• External bremsstrahlung
Status of $\mu \rightarrow e + \gamma$

- Last results reported in 1999

<table>
<thead>
<tr>
<th>Place</th>
<th>Year</th>
<th>$\Delta E_e$</th>
<th>$\Delta E_\gamma$</th>
<th>$\Delta t_{e\gamma}$</th>
<th>$\Delta \theta_{e\gamma}$</th>
<th>Upper limit</th>
<th>References</th>
</tr>
</thead>
<tbody>
<tr>
<td>TRIUMF</td>
<td>1977</td>
<td>10%</td>
<td>8.7%</td>
<td>6.7 ns</td>
<td></td>
<td>$&lt;3.6 \times 10^{-9}$</td>
<td>Dcpommier et al. (1977)</td>
</tr>
<tr>
<td>SIN</td>
<td>1980</td>
<td>8.7%</td>
<td>9.3%</td>
<td>1.4 ns</td>
<td></td>
<td>$&lt;1.0 \times 10^{-9}$</td>
<td>Van der Schaaf et al. (1980)</td>
</tr>
<tr>
<td>LANL</td>
<td>1982</td>
<td>8.8%</td>
<td>8%</td>
<td>1.9 ns</td>
<td>37 mrad</td>
<td>$&lt;1.7 \times 10^{-10}$</td>
<td>Kinnison et al. (1982)</td>
</tr>
<tr>
<td>LANL</td>
<td>1988</td>
<td>8%</td>
<td>8%</td>
<td>1.8 ns</td>
<td>87 mrad</td>
<td>$&lt;4.9 \times 10^{-11}$</td>
<td>Bolton et al. (1988)</td>
</tr>
<tr>
<td>LANL</td>
<td>1999</td>
<td>1.2%$^a$</td>
<td>4.5%$^a$</td>
<td>1.6 ns</td>
<td>15 mrad</td>
<td>$&lt;1.2 \times 10^{-11}$</td>
<td>Brooks et al. (1999)</td>
</tr>
</tbody>
</table>

- New experiment underway at PSI – MEG
- Liquid xenon calorimeter for $\gamma$’s, solenoid for positrons
- Goal is factor of $\approx 100$ below MEGA
- Analysis of results underway – invited talk with new results is scheduled for DNP fall meeting!
Status of $\mu^+ \rightarrow e^+e^+e^-$

- Last results reported in 1991
- SINDRUM at PSI has best limit – solenoid tracking chamber

<table>
<thead>
<tr>
<th>Place</th>
<th>Year</th>
<th>90%-C.L. upper limit</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>JINR</td>
<td>1976</td>
<td>$&lt;1.9 \times 10^{-9}$</td>
<td>Korenchenko et al. (1976)</td>
</tr>
<tr>
<td>LANL</td>
<td>1984</td>
<td>$&lt;1.3 \times 10^{-10}$</td>
<td>Bolton et al. (1984)</td>
</tr>
<tr>
<td>SII</td>
<td>1984</td>
<td>$&lt;1.6 \times 10^{-10}$</td>
<td>Bertl et al. (1984)</td>
</tr>
<tr>
<td>SII</td>
<td>1985</td>
<td>$&lt;2.4 \times 10^{-12}$</td>
<td>Bertl et al. (1985)</td>
</tr>
<tr>
<td>LANL</td>
<td>1988</td>
<td>$&lt;3.5 \times 10^{-11}$</td>
<td>Bolton et al. (1988)</td>
</tr>
<tr>
<td>SII</td>
<td>1988</td>
<td>$&lt;1.0 \times 10^{-11}$</td>
<td>Bellgardt et al. (1988)</td>
</tr>
<tr>
<td>JINR</td>
<td>1991</td>
<td>$&lt;3.6 \times 10^{-11}$</td>
<td>Baranov et al. (1991)</td>
</tr>
</tbody>
</table>

- No new experiments planned that I know
- A non-zero result for $\mu \rightarrow e + \gamma$ would likely change that
\( \mu^- \rightarrow e^- \) Conversion – I

- Searches for this conversion process carried out in several different nuclei (Cu, S, Ti, Pb)
- Process involves \( \mu \) capture by atom and then a cascade to 1s atomic orbital
- After cascade, \( \mu \) orbit overlaps nucleus then have normal muon decay or
  - \( \mu^- + (A,Z) \rightarrow \nu_\mu + (A,Z-1) \) (allowed)
  - \( \mu^- + (A,Z) \rightarrow e^- + (A,Z) \) (not allowed)
- Ratio \( \Rightarrow \) branching ratio for conversion
$\mu^- \rightarrow e^-$ Conversion – II

- Signal $\Rightarrow$ mono-energetic $e^-$ at end point energy
- Backgrounds: $\mu$ decay in orbit, $\pi$ capture, radiative $\mu$ capture with very asymmetric pair creation
- Titanium has high end point so attractive

Predicted signal and background level for $\mu - e$ conversion on Ti
Status of $\mu^- \rightarrow e^-$ Conversion

- Last results reported in 1998

<table>
<thead>
<tr>
<th>Process</th>
<th>90%-C.L. upper limit</th>
<th>Place</th>
<th>Year</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu^- + Cu \rightarrow e^- + Cu$</td>
<td>$&lt;1.6 \times 10^{-8}$</td>
<td>SREL</td>
<td>1972</td>
<td>Bryman et al. (1972)</td>
</tr>
<tr>
<td>$\mu^- + ^{32}S \rightarrow e^- + ^{32}S$</td>
<td>$&lt;7 \times 10^{-11}$</td>
<td>SIN</td>
<td>1982</td>
<td>Badertscher et al. (1982)</td>
</tr>
<tr>
<td>$\mu^- + Ti \rightarrow e^- + Ti$</td>
<td>$&lt;1.6 \times 10^{-11}$</td>
<td>TRIUMF</td>
<td>1985</td>
<td>Bryman et al. (1985)</td>
</tr>
<tr>
<td>$\mu^- + Ti \rightarrow e^- + Ti$</td>
<td>$&lt;4.6 \times 10^{-12}$</td>
<td>TRIUMF</td>
<td>1988</td>
<td>Ahmad et al. (1988)</td>
</tr>
<tr>
<td>$\mu^- + Pb \rightarrow e^- + Pb$</td>
<td>$&lt;4.9 \times 10^{-10}$</td>
<td>TRIUMF</td>
<td>1988</td>
<td>Ahmad et al. (1988)</td>
</tr>
<tr>
<td>$\mu^- + Ti \rightarrow e^- + Ti$</td>
<td>$&lt;4.3 \times 10^{-12}$</td>
<td>PSI</td>
<td>1993</td>
<td>Dohmen et al. (1993)</td>
</tr>
<tr>
<td>$\mu^- + Pb \rightarrow e^- + Pb$</td>
<td>$&lt;4.6 \times 10^{-11}$</td>
<td>PSI</td>
<td>1996</td>
<td>Honecker et al. (1996)</td>
</tr>
<tr>
<td>$\mu^- + Ti \rightarrow e^- + Ti$</td>
<td>$&lt;6.1 \times 10^{-13}$</td>
<td>PSI</td>
<td>1998</td>
<td>Wintz (1998)</td>
</tr>
</tbody>
</table>

- New experiment proposed in U.S. – Mu2e to run at FNAL with accelerator upgrade
- Estimates of background suggest $10^{-16}$ possible
Measuring the **Muon (g-2) Factor**

- Like other precision measurements, the determination of \( g-2 \) for the muon has a long history.
- Most recent results from BNL E821.
- Store a polarized muon beam in ring and measure precession frequency as a function of time.
- AGS provides muons for ring.
- Requires precise knowledge of magnetic.
- The SM prediction for a non-zero \( g-2 \) includes several correction factors – higher order loop diagrams.
The Storage Ring for E821
An “event” is an isolated positron above a threshold.
$a_\mu = (g - 2)/2$ is non-zero because of virtual loops, which can be calculated very precisely.

**Contributions:**

<table>
<thead>
<tr>
<th>Contribution</th>
<th>Result ($\times 10^{-11}$) Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>QED (leptons)</td>
<td>$116\ 584\ 718.09 \pm 0.14 \pm 0.04\alpha$</td>
</tr>
<tr>
<td>HVP (lo)</td>
<td>$6\ 914 \pm 42_{\text{exp}} \pm 14_{\text{rad}} \pm 7_{\rho\text{QCD}}$</td>
</tr>
<tr>
<td>HVP (ho)</td>
<td>$-98 \pm 1_{\text{exp}} \pm 0.3_{\text{rad}}$</td>
</tr>
<tr>
<td>HLxL</td>
<td>$105 \pm 26$</td>
</tr>
<tr>
<td>EW</td>
<td>$152 \pm 2 \pm 1$</td>
</tr>
<tr>
<td>Total SM</td>
<td>$116\ 591\ 793 \pm 51$</td>
</tr>
</tbody>
</table>

The **“g-2 test”**: Compare experiment to theory. Is SM complete?

$$\delta a_\mu^{\text{NewPhysics}} = a_\mu^{\text{Expt.}} - a_\mu^{\text{Theory}}$$

Slide from D. Hertzog
Historical Evolution

+/-a_μ uncertainty
abs(a_μ) contribution

a_μ in units of 10^{-11}

Slide from D. Hertzog
HVP is determined from data

\[ a_\mu(\text{had}) = \left( \frac{\alpha m_\mu}{3\pi} \right)^2 \int_{4m_\pi^2}^{\infty} \frac{ds}{s^2} K(s) \left( \frac{\sigma(e^+e^- \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)} \right) \]
A world-wide effort exists to measure over full range

\[
a_\mu(\text{had}) = \left( \frac{\alpha m_\mu}{3\pi} \right)^2 \int_0^\infty \frac{ds}{s} K(s) \left( \frac{\sigma(e^+e^- \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)} \right)
\]
HVP evaluations by 2 groups, updated Tau’10

- Hagiwara, Liao, Martin, Nomura, Teubner (HLMNT)

\[ a_\mu^\text{exp} - a_\mu^\text{SM} = (296 \pm 81) \times 10^{-11} \rightarrow 3.2 \sigma \]

- M. Davier, A. Hoecker, B. Malaescu, Z. Zhang (DHMZ)
  - (BaBar team with access to preliminary data)

Biggest difference is from high multiplicity states now measured at BaBar; > 1 GeV region
  \( a_\mu^\text{exp} - a_\mu^\text{SM} = (259 \pm 81) \times 10^{-11} \rightarrow 3.2 \sigma \)

Slide from D. Hertzog
The new HVP evaluations also affect $\alpha_{\text{QED}}$ running ... and enter the global electroweak fits ...

Slide from D. Hertzog
The values & the new experimental goal

Theory uncertainty = $51 \times 10^{-11}$ (0.44 ppm)
Experimental uncertainty = $63 \times 10^{-11}$ (0.54 ppm)

- 0.46 ppm statistical ← limit was counts
- 0.21 ppm precession systematic
- 0.17 ppm field systematic

The values & the new experimental goal

Leads to $\Delta a_\mu(\text{Expt} - \text{Thy}) = 297 \pm 81 \times 10^{-11}$ 3.6 $\sigma$

Experimental goal: 63 → $16 \times 10^{-11}$
Theory uncertainty expect: 51 → $30 \times 10^{-11}$

Leads to $\Delta a_\mu(\text{Expt} - \text{Thy}) = XXX \pm 34 \times 10^{-11}$

If central value remained, $\Delta a_\mu$ would exceed 8$\sigma$

Slide from D. Hertzog
The Storage Ring exists. It will be moved to FNAL
- Transport coils to and from barge via Sikorsky aircrane
- Ship through St Lawrence -> Great Lakes -> Calumet SAG
- Subsystems can be transported overland, but probably more cost effective to ship steel on barge as well.
Fundamental Symmetries

- Many experimental avenues to explore
- Much to understand about neutrino’s
- Possible signatures for SM deviations from the LHC
- Low-energy SM tests

- A bright future for the field!