On $^{21}\text{Na}$ States between $E_x=5.5$ and $6.2$ MeV

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Abstract. States in $^{21}\text{Na}$ between 5.5 and 6.2 MeV have been investigated by means of the $^{20}\text{Ne}(p, p')^{20}\text{Ne}$ and $^{20}\text{Ne}(p, \gamma)^{20}\text{Ne}$ reactions. The gas target is Ne in naturally occurring abundances or enriched to 99.9\% in $^{20}\text{Ne}$. No new levels were seen but a new value of the spin of the 5.814 MeV state is given: $J^e=7/2^-$. The $J^e=(5/2, 7/2)^-$ assignment of the 6.076 MeV state is confirmed.

Introduction

Many experimental studies have been performed on the $^{21}\text{Na}$ nucleus [1] and have led to definite spin and parity assignments for most of the states with $E_x<5.5$ MeV [2–10]. Between 5.5 and 6.2 MeV, the situation is not so clear. Cox et al. [11] studied the $^{20}\text{Ne}(p, p)$ and $^{20}\text{Ne}(p, \gamma)$ reactions and gave the energy levels of the excited states of $^{21}\text{Na}$. Haeberli [12] and Bloch et al. [13] analyzed their $^{20}\text{Ne}(p, p\gamma)$ cross-section data and gave a set of possible parity and spin values. In particular, spin and parity assignments for the 5.814 MeV and the 6.076 MeV states are given by Bloch in his preliminary report [13] to be $(1/2, 3/2)^-$ and $(5/2, 7/2)^-$, respectively. Nevertheless, these two levels are not reported in the definitive paper [8] which follows. This is probably due to difficulties arising from the use of a natural target containing 10\% of $^{22}\text{Ne}$. Bloch assumed a $^{22}\text{Ne}$ non-resonant scattering cross-section; this can be considered as a good approximation for proton energies < 2 MeV but is not convenient for higher proton energies due to the presence of many important resonances in $^{23}\text{Na}$ [14]. Thus, using a highly enriched differentially pumped gas target, it was felt that $J^e$ determination for these two levels at 5.814 and 6.076 MeV could be given, combining the studies of elastic and inelastic proton scattering on $^{20}\text{Ne}$ with measurements of the gamma rays resulting from the inelastically scattered protons. In addition, recent experiments, through the $^{23}\text{Na}(p, t)^{21}\text{Na}$ reaction [15] and the $^{20}\text{Ne}(^3\text{He}, 2n)^{21}\text{Mg}(\beta^+)^{21}\text{Na}$ reaction [16], revealed many new levels. The possible presence of these new levels is also studied in the present experiment in the energy region between 5.5 and 6.2 MeV.

Experimental Set Up

The elastic and inelastic proton scattering experiments were performed at the 4 MeV Van de Graaff accelerator of the Institut de Physique Nucléaire, Lyon. The incident proton beam was directed into a scattering chamber containing a differentially pumped gas cell. The beam was monitored by means of a Faraday cup under high vacuum and a current integrator. The target was neon enriched to 99.95\% in $^{20}\text{Ne}$. The peaks arising from inelastic proton scattering from the neon isotopes are well-separated and allowed use of a natural neon (90.5\% $^{20}\text{Ne}$) target, permitting higher counting times and resulting in reduced statistical errors. The pressure of the gas in the cell was approximatively 2 Torr. The scattered protons were detected simultaneously by eight surface barrier detectors placed at angles between $\pm 40^0$ and $\pm 160^0$ in the laboratory system. In the $^{20}\text{Ne}(p, \gamma)$ angular distribution measurements, the gas chamber was a tantalum cylinder 2 cm high with inside diameter 1 cm and wall thickness 2 mm.
Fig. 1. Excitation functions of elastic scattering $^{20}$Ne($p,p$) for the $E_p = 3.552$ MeV resonance for: $J^* = 7/2^-, \Gamma = 0.4$ keV, $I_p = 0.5 \Gamma$ (a); $J^* = 5/2^-, \Gamma = 0.4$ keV, $I_p = 0.5 \Gamma$ (b); $J^* = 1/2^-, \Gamma = 0.4$ keV, $I_p = 0.7 \Gamma$ (c); $J^* = 3/2^-, \Gamma = 0.4$ keV, $I_p = 0.83 \Gamma$ (d)
The incident proton beam entered this cell through an isolated diaphragm and was stopped in the cell. The pressure of the gas was approximately 5 Torr. The 1.63 MeV gamma radiation from the de-excitation of the first state in $^{20}$Ne was detected by a 92 cm$^3$ Ge(Li) detector.

**Experimental Results and Procedure of Analysis**

Parts of the excitation curves of the $^{20}$Ne($p$, $p$) (see Figs. 1 and 2) and $^{20}$Ne($p$, $p'$) reactions are given for incident proton energy between 3.5 and 3.9 MeV. The absolute differential cross sections were determined by comparison with the Kr($p$, $p$) reaction at 2.5 MeV. The relative uncertainties in the data were estimated from the method of analysis and from the usual statistical errors; they were about 5% as shown by the error bars in the figures. The systematic error in the absolute normalization of the cross-sections is about 10%. At the proton energies considered here, the $^{20}$Ne($p$, $p$) and $^{20}$Ne($p$, $p'$) reactions proceed by the compound nucleus formation as described by the Breit-Wigner theory. Accordingly, the theoretical excitation functions were calculated within the formulation of [17]. The potential scattering was described by the hard-sphere phase-shifts adjusted to fit the experimental data in the regions unperturbed by the resonances. For each angle, the energy dispersion, due to the beam and to the target thickness, was taken into account by using a normal distribution as the resolution function (the width at half maximum was estimated to be $\approx 2.5$ keV).

The measured angular distribution of the inelastically scattered proton and of the gamma ray were fitted to the expression

$$W(\theta) = \sum_{n=0}^{n_{\text{max}}} A_n P_n(\cos \theta)$$

by means of a least squares program, to determine the coefficients $A_n$. The presence of a background has led to study the behaviour of the $A_n$ coefficients both off and on resonance as a function of proton bombarding energy (Figs. 3 and 4). For an isolated resonance the behaviour of the $A_n$ coefficients over the resonance gives a lower limit for the spin $J$ of the excited state of $^{21}$Na. There is a relationship between angular distribution complexity ($n_{\text{max}}$) and the spin ($J$) of an isolated resonant state (Table 1). More generally, in the presence of two interfering resonances, the Legendre polynomial expansion (1) could be regarded as:

$$W(\theta) = \sum_{n_{1}=0}^{n_{\text{max}}} A_{n_{1}1} P_{n_{1}1}(\cos \theta) + \sum_{n_{2}=0}^{n_{\text{max}}} A_{n_{2}2} P_{n_{2}2}(\cos \theta) + \sum_{n_{1,2}=0}^{n_{\text{max}}} A_{n_{1,2}} P_{n_{1,2}}(\cos \theta)$$

where the superscripts correspond successively to the first resonance (1), the second resonance (2) and the interference between both (12). The limitations for the polynomial degree are given by:

$$|l_{\text{in}} - l_{\text{out}}| \leq n_{\text{max}} \leq |l_{\text{in}} + l_{\text{out}}|$$

$$|l_{\text{in}} - l_{\text{out}}| \leq n_{\text{max}} \leq |l_{\text{in}} + l_{\text{out}}|$$

Table 1. Correspondence between angular distribution complexity ($n_{\text{max}}$) and the spin ($J$) of an isolated resonant state

<table>
<thead>
<tr>
<th>$n_{\text{max}}$</th>
<th>$J$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$\frac{1}{2}$</td>
</tr>
<tr>
<td>2</td>
<td>$\geq \frac{3}{2}$</td>
</tr>
<tr>
<td>4</td>
<td>$\geq \frac{5}{2}$</td>
</tr>
</tbody>
</table>

Fig. 2: Excitation functions of elastic scattering $^{20}$Ne($p$, $p$) for the $E_p = 3.80$ MeV resonance, for $T = 6$ keV, $T_p = 0.05 T$, $J^* = 7/2$ and $5/2$ (both curves are identical)
|\(J^i - J^j| \leq n^i_{\text{max}} \leq J^i + J^j| \)  
\( \pi^i \cdot \pi^j = (-1)^{n^i} \)  

with \(i, j = 1, 2\) have the meaning defined above and where \(l^i_{\text{in}}\) and \(l^i_{\text{out}}\) are the orbital angular momentum of the incident particle and of the exit particle, respectively. The unprimed quantities refer to values which minimize the left hand side of relations (3), (4) and the primed quantities to angular momentum giving the highest values to the right hand sides of these relations. \(J^i\) and \(J^j\) are the spins of the excited states in \(^{21}\)Na corresponding to the \(i^{th}\) and \(j^{th}\) resonance respectively. For inelastically scattered proton angular distributions, conditions (3), (4), (5), (6) occur, whereas for the gamma angular distribution, only (3), (5), (6) must be taken into account with the supplementary limitations: \(n^i_{\text{max}} \leq L + L', \) where \(L\) and \(L'\) are the values of the multipolarity of the emitted gamma (here \(L = L' = 2\)), and \(n^i_{\text{max}} \leq 2c\) where \(c\) is the spin of the excited level of \(^{20}\)Ne (here \(c = 2\)).

**Discussion**

In the investigated energy range no evidence of the new levels seen in [14, 15] was found. For the
The angular distribution coefficients $A_n$ for the $^{20}$Ne($p, p'\gamma$) reaction obtained from least-square fits to $W(\theta) = \sum A_n P_n(\cos \theta)$ for the $E_p = 3.828$ MeV resonance ($E_x = 6.076$ MeV), which has been considered as isolated, the existence of a strong resonating $A_4$ term (see Fig. 4) indicates a spin $J \geq 5/2$ (see Table 1). Thus, we calculated the excitation curves for the elastic scattering with $J^\pi = (5/2, 7/2, 9/2)^+$. The only satisfactory fits are obtained for $J^\pi = (5/2^-, 7/2^-)$ (Fig. 2). This confirms the conclusions of Haeberli [15] but it is not possible to choose between these two determinations. To study the $E_p = 3.552$ MeV resonance ($E_x = 5.814$ MeV), it is necessary to consider the interference with the resonance at $E_p = 3.566$ MeV ($E_x = 5.827$ MeV, $J^\pi = 3/2^-$). Table 2 indicates the maximum degree of the adjusted $A_n^\pi$ coefficients for the $^{20}$Ne($p, p'$) and $^{21}$Ne($p, p'\gamma$) angular distributions for different spin values of the 5.814 MeV state. These coefficients determined over the 5.814 MeV resonance are shown in Figs. 3 and 4. The $A_4$ coefficient resonates for ($p, p'\gamma$) reaction suggesting a spin of 5/2 or greater (see Table 2).

Table 2. Correspondence between the maximum degree of the Legendre polynomial development for the $^{20}$Ne($p, p'$) and the $^{21}$Ne($p, p'\gamma$) angular distributions and the spins of the two interfering states at $E_p = 5.814$ and 5.827 MeV. (Notations are given in the text)

<table>
<thead>
<tr>
<th>$E_p$ (MeV)</th>
<th>5.814</th>
<th>5.827</th>
<th>$pp'$</th>
<th>$pp'\gamma$</th>
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<tbody>
<tr>
<td>$J^\pi$</td>
<td></td>
<td></td>
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<td>7/2</td>
<td>3/2</td>
<td>6</td>
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<td>1/2</td>
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<td>2</td>
<td>1</td>
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The elastic scattering excitation functions calculated at different angles for $J^\pi = (1/2^-, 7/2^-)$ are displayed in Fig. 1. Other spins (9/2) and parity hypotheses do not lead to satisfactory shapes (we note that the...
values of $A_1/A_0$ and $A_3/A_0$ terms at the resonance energy (Fig. 3) are weak and thus are not inconsistent with the assumption of two interfering negative parity states at 5.814 MeV and 5.827 MeV.) The theoretical curves are not in total agreement with the experimental data especially at certain angles. This is probably due to the non resonant background or to the unseen resonances which may result in interference and which are not included in the calculations. Nevertheless comparing the amplitude and the shape of the anomaly (especially at 142°, 116° and 88°), it appears that $J^p = 5/2^-$ must be ruled out. The resonance is then analyzed with $J^p = 7/2^-$, $\Gamma = 0.4$ keV and $I_p/\Gamma = 0.5$. This result is in contradiction with the $J^p=(1/2, 3/2)$ determination of Bloch et al. [13].

Further informations were expected from the interpretation of the inelastically scattered protons and gamma rays angular distributions. In the $(p, p')$ and $(p, p' \gamma)$ yield curves, a background exists outside the resonances, which might possibly be due to unseen levels. The widths of such levels must be large and thus would explain the resonance of the $A_1$ coefficient (Fig. 3) by interference effects with the studied states. In particular, two levels at 5.758 and 5.867 MeV were seen by Sextro et al. [15] with assigned $J^p=(3/2 - 7/2)^+$ (see also Agard [14]). But, in the absence of further knowledge of the unseen levels, we substracted the background and took into account the interference between the levels at $E_x=5.814$ and 5.827 MeV with no assumptions about the possible partial waves in the decay of the inelastic proton. This procedure fails to correctly fit the $A_n$ coefficients at the resonance energy.

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References