TENSOR POLARIZED DEUTERON CAPTURE BY $^3\text{He}$ AT AND ABOVE THE FUSION RESONANCE REGION

by

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Date: February 10, 1992

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Dissertation submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy in the Department of Physics in the Graduate School of Duke University

1992
ABSTRACT

(Physics-Nuclear)

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Abstract

Tensor-Polarized Deuteron Capture by $^3$He at and above the Fusion-Resonance Region

The absolute differential cross section and vector and tensor analyzing powers of the $^3$He($^3$He,$\gamma$)$^5$Li reaction have been measured at three energies: $E_d$(lab) = 8.6 MeV, $E_d$(lab) = 0.8 MeV where the beam was stopped in the target, and $E_d$(lab) = 0.6 MeV where the beam lost 0.3 MeV in the target. Additionally, absolute cross sections were measured at $E_d$(lab) = 0.8 MeV where the beam lost 0.4 MeV in the target, and $E_d$(lab) = 0.4 MeV where the beam was stopped in the target.

Transition matrix element (TME) analyses of the observables at or below $E_d$(lab) = 0.8 MeV concluded that the reaction proceeds predominantly through s-wave (E1) capture, with small admixtures of M1 or E2 radiation. While multiple TME solutions exist, multichannel Resonating Group Model calculations performed at the peak of the fusion resonance at $E_d$(lab) = 0.45 MeV unambiguously select the solution with >89% $^4s_{3/2}$(E1) TME strength as the physically preferred one. In the RGM picture, the strong capture cross section in the fusion resonance region is a consequence of the tensor force, which couples the $[d+^3\text{He}]^{S=3/2}_{J=3/2}$ channel to the $[p+^4\text{He}]^{S=1/2}_{J=3/2}$ channel in the scattering wave function, enabling the capture reaction to proceed via the $\Delta S=0$ (E1) transition to the ground state.

At $E_d$=8.6 MeV, a transition matrix element fit indicates that the capture cross-section strength is dominated by s-wave E1 capture with an admixture of ~20% d-wave E1 capture. In an RGM calculation at this energy, a non-zero tensor force is necessary to get agreement between data and theory for the relative strengths of the TME's. As with the $E_d$(lab) = 0.45 MeV calculation, the strong $^4s_{3/2}$ (E1) capture strength is due to the coupling of the $[d+^3\text{He}]^{S=3/2}_{J=3/2}$ scattering channel to the $[p+^4\text{He}]^{S=1/2}_{J=3/2}$ scattering channel by the tensor force.
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Chapter 1: Introduction

Light nuclei are important for understanding the two-body nuclear force which includes spin-orbit and tensor components. With small, uncomplicated systems the fundamental physical effects become easier to untangle from each other. Capture reactions are particularly well suited for probing these small systems since the photon only weakly perturbs the nuclear medium. Also, the electromagnetic interaction is well understood from Maxwell's equations allowing a strong theoretical foundation with which to interpret empirical results. In this dissertation, the results of experiments to study polarized-deuteron capture by $^3\text{He}$ are discussed. These experiments have been conducted in two energy regions: at $E_{5_{\text{Li}}}^*=21.5$ MeV and at $E_{5_{\text{Li}}}^*=16.7$ MeV which corresponds to the $J^\pi=3/2^+$ resonance at $E_d^{(\text{lab})}=0.45$ MeV above the $d+^3\text{He}$ threshold.

From a practical point of view, the $d+^3\text{He}$ reaction is important because of its potential as a fuel for an energy-producing fusion reactor. Current fusion technology relies on the $d+^3\text{H}$ reaction because of the lower ignition temperature necessary for a sustained burn. In both reactions, the energy yield is enhanced by the existence of a $J^\pi=3/2^+$ resonance (called the "fusion" resonance) which exists only a few hundred keV above fusion-reactor energies. The advantage that the $d+^3\text{He}$ reaction offers over the $d+^3\text{H}$ reaction is a source of cleaner energy. Tritium ($^3\text{H}$) itself is radioactive and tends to contaminate the work environment. The process of fusing it with deuterium creates a large source of neutrons from the $^3\text{H}(d,n)^4\text{He}$ reaction which are a health hazard both by themselves and by creating unstable isotopes through neutron-induced nuclear reactions. Using the $d+^3\text{He}$ reaction eliminates the neutrons, instead producing protons from the $^3\text{He}(d,p)^4\text{He}$ reaction which are easily stopped in matter. Unfortunately, current technology is unable to reach the temperatures necessary to ignite the $d+^3\text{He}$ plasma, but
the more knowledge which is available increases the probability of someday building just such a device.

Even in the \( d + ^3\text{He} \) reactors which are being designed and studied today, the \( ^3\text{He}(d,\gamma)^5\text{Li} \) reaction is being considered as a possible diagnostic for the plasma since \( \gamma \)-rays interact relatively weakly with matter and are therefore more easily detected outside of the reactor than other particles. The reaction comes about from the deuterium fusing with \( ^3\text{He} \) which is created in the \( d(d,n)^3\text{He} \) reaction. The \( \gamma \)-rays from this reaction, along with the previously measured \( ^3\text{He}(d,\gamma)^5\text{Li} \)-to-\( ^3\text{He}(d,p)^4\text{He} \) branching ratio gives a direct measure of the fusion energy production rate of the reactor [Cec85a].

The properties of the fusion resonance were originally determined primarily by studies of the \( ^3\text{He}(d,p)^4\text{He} \) reaction [Bon52, Freier54, and Yar53]. The isotropic cross section indicates that the resonance is formed via s-wave capture indicating that the resonance has positive parity. Since the spins of the deuteron and \( ^3\text{He} \) can only couple to \( S=1/2 \) or \( S=3/2 \), the total spin \( (L+S) \) of the fusion resonance can only be \( J^{\pi}=1/2^+ \) or \( J^{\pi}=3/2^+ \). If a one-level approximation is made, then the total cross section has an upper bound of

\[
\sigma \leq \pi(2J+1)/6k^2. \tag{1.1}
\]

where \( k \) is the wave number of the deuteron in the center-of-mass. The measurements of the absolute cross section falls between the upper limits for \( J=1/2 \) and \( J=3/2 \). Since Eqn. 1.1 is an upper bound the resonance must have \( J^{\pi}=3/2^+ \). Subsequent \( ^4\text{He}(p,p)^4\text{He} \) phase shift analyses [Hou78, Pla72, and Sch71] have confirmed this initial spin and parity assignment. A phase shift analysis of the \( ^3\text{He}(d,d)^3\text{He} \) reaction by [Jen80] found a solution where the \( ^2s_{1/2} \) phase shift is approximately 5° while the \( ^4s_{3/2} \) phase shift is
resonant. This indicates that any L=0, S=1/2 background is on the order of two magnitudes less than the L=0, S=3/2 resonance.

Shell-model calculations of $^5\text{Li}$ predict both a 3/2$^+$ state and a 1/2$^+$ state at excitation energies above the 1/2$^-$ first excited state [Van83]. Coupled-channel calculations disperse the strength of the 1/2$^+$ state because it couples strongly to the $\alpha$+p channel [Bur83 and HofPri]. The 3/2$^+$ state couples to the $\alpha$+p channel only through the tensor and spin-orbit forces and therefore has a relatively narrow width. Resonating Group Model calculations in the fusion-resonance region also require the presence of a tensor force to predict the strength of the $^3\text{He}(d,p){}^4\text{He}$ low-energy cross section [Blu90].

For our investigation of this $J^\pi=3/2^+$ "fusion" resonance region of $^5\text{Li}$, capture reactions with tensor-polarized deuterons are well suited for probing tensor-force effects because of the constraints imposed by the $\Delta S=0$ selection rule for E1 radiation. As stated above, this resonance is populated predominantly by deuterons with L=0 and S=3/2. The $J^\pi=3/2^-$ ground state of $^5\text{Li}$, on the other hand, is predominantly L=1, S=1/2 (as seen in the single-particle model, where the $^5\text{Li}$ ground state consists predominantly of a proton in the $p_{3/2}$ orbital above a closed $s_{1/2}{}^4\text{He}$ core). Therefore, in keeping with the $\Delta S=0$ selection rule which says that the dominant term of the E1 operator does not affect channel spin, there are at least two likely mechanisms for an E1 $\gamma$-ray transition from the predominantly S=3/2 resonance to the predominantly S=1/2 ground state. First, the S=3/2 resonance can decay directly to the S=3/2 component of the ground state which may be large due to tensor force mixing in the ground state. Second, the tensor force can mix the S=3/2 and S=1/2 scattering states, allowing the transition to the S=1/2 component of the ground state.

The cross section for the $^3\text{He}(d,\gamma)^5\text{Li}$ reaction was first measured by Blair, Hintz, and Van Patter [Bla54]. An excitation-function measurement showed the fusion
resonance at $E_{\text{d(lab)}} = 0.45$ MeV with an angle-integrated total cross section of $50 \pm 10 \mu b$. An angular distribution taken at $E_{\text{d}} = 0.58$ MeV was consistent with isotropy to within 10%. [Bus68] also measured an isotropic angular distribution (at $E_{\text{d}} = 0.48$ MeV) but claim a lower total cross section of $21 \pm 4 \mu b$. [Kra68] measured an excitation function at 90° with a total cross section of 38 $\mu b$ at resonance.

Analyzing powers of the $^3\text{He}(d,\gamma)^5\text{Li}$ reaction have not been measured previously. However, a study has been made by [Ril89] of the $^3\text{H}(d,\gamma)^5\text{He}$ reaction in the region of the $^5\text{He}$ fusion resonance. An analysis of the data concluded that the isotropic cross section was dominated by $S=3/2$ (E1) capture, which contributed 95% of the cross section strength. Admixtures of 2.6% $S=1/2$ (E1) and 2.0% $S=3/2$ (M1) capture strengths were required to fit the data of that experiment, especially the tensor analyzing power $A_{\gamma\gamma}(\theta)$. From charge-symmetry considerations, similar results are expected for the $^3\text{He}(d,\gamma)^5\text{Li}$ reaction.

Above the fusion resonance, the $^3\text{He}(d,\gamma)^5\text{Li}$ reaction has been studied from 2-26 MeV by [Tom67], [Del68], [Sch70], [Kin72], and others (see [Ajz88]). [Tom67] suggests the existence of a broad state at 20 MeV with a possible spin assignment of $3/2^+$ or $5/2^+$, whereas [Sch70] also considers a spin assignment of $1/2^+$. [Del68] found no clear evidence for the existence of any state at this energy. [Kin72] measured an excitation function from $17 \text{ MeV} \leq E^*_{5\text{Li}} \leq 26 \text{ MeV}$, and several angular distributions within this energy range. The $\gamma_0$ and $\gamma_1$ transition intensities were extracted, the analysis of which was consistent with a $1/2^+$ state around 18 MeV and a $5/2^+$ state around 20 MeV. If such states do exist, then the current experiment at $E^*_{5\text{Li}} = 21.5 \text{ MeV}$ lies on the high-energy side of the 20 MeV resonance. Angular distributions by [Kin72] indicate a non-isotropic cross section around these energies. Analyzing powers of the $^3\text{He}(d,\gamma)^5\text{Li}$ reaction have not previously been measured.
The following chapters present the experimental techniques, methods of analysis, and results of our recently concluded study of the $^3$He($^3$H,$\gamma$)$^5$Li reaction. Cross sections and analyzing powers have been measured in the fusion resonance region ($E_{5\text{Li}}^*=16.7$ MeV) and at $E_{5\text{Li}}^*=21.5$ MeV. The results of Legendre and transition matrix element fits to the data are presented along with interpretations of these results. The results of a Direct Capture calculation at the fusion-resonance energy is reported, along with the more sophisticated Resonating Group Model calculation which was performed in both energy regions. Finally, conclusions about the reaction and the role of the tensor force will be discussed.
Chapter 2: Experimental Techniques

The following is a discussion of the experimental techniques used for beam creation and acceleration, and photon and particle detection. Deuteron beams of 8.85 MeV (which resulted in an 8.6 MeV beam at the center of the target) and approximately 3.3 MeV (subsequently degraded to provide 0.8 MeV, 0.6 MeV, and 0.4 MeV beams on target) were created in three sources and accelerated by the TUNL FN tandem Van de Graaff to one of two gaseous $^3$He targets. The signals from two NaI spectrometers were processed by fast CAMAC electronics and sorted by the VAX 11/780 computer using the XSYS data acquisition system.

2.1 Sources

The simplest of the three sources that will be discussed is the Direct Extraction Negative Ion Source (DENIS). In this source, hydrogen or deuterium is bombarded by electrons supplied by a hot filament. The negative ions are then accelerated by an electric field away from the neutral gas. The entire source is biased to -50 kilovolts with respect to the lab which further accelerates the beam out of the source. The beam travels through a parallel-plate chopper followed by an auxiliary square-wave chopper. It is then deflected by a magnetic field and passes through a double-drift bunching system [Mil79 and Wen80]. This allows the creation of a pulsed beam with 2 to 4 ns long bursts every 200 ns.

The Lamb Shift Polarized Ion Source (LSPIS) [Cle74] has been decommissioned since the completion of this experiment. In that source, low-energy positive ions from a duoplasmatron were passed through a cesium vapor in a charge-exchange canal. Approximately 1/3 of these particles exited the canal as neutral particles in the metastable $2S_{1/2}$ state. Negatively charged particles (which resulted from the exchange of two electrons) were deflected out of the beam by an electric field. The beam then traveled through a spin filter [McK68] which consisted of static and radio-frequency
electric fields which selectively quenched all but one hyperfine state of the metastable atoms. In order to select out the polarized $2S_{1/2}$ atoms from the ground-state atoms and also to allow for subsequent acceleration of the beam, the atoms were passed through an argon charge-exchange canal. The probability for the polarized metastable atoms to pick up an extra electron is much greater than for the unpolarized ground state atoms. After exiting the argon region, the beam typically had a 10% to 25% unpolarized background. The beam was then focused and accelerated to 50 keV in preparation for injection into the tandem accelerator. If a pulsed beam was needed, the anode signal of the duoplasmatron could be ramped [How82] to prebunch the beam before passing through the double-drift bunching system.

The LSPIS was replaced by an intense Atomic Beam Polarized Ion Source (ABPIS) [CleTBP]. In this new source, deuterium molecules are dissociated by an applied radio-frequency field and drift into the region of a sextupole magnet which splits the six hyperfine levels of the deuterium atoms and defocuses the three lower energy states out of the beam. Weak and strong field rf cavities selectively induce transitions between the three higher-energy populated states and the three lower-energy unpopulated states thus polarizing the atoms. The neutral atoms are then passed through a plasma created by ionized nitrogen to create positively charged ions. They are then accelerated through gaseous Cs in order to pick up two electrons. Finally, the source itself is at -50 kilovolts with respect to the lab, which accelerates the negative ions out of the source. This source is followed by a Wien Filter (described below) and a set of chopper plates which are identical to those following the DENIS source.

The quantization axis of the polarized particles emitted by the above polarized-ion sources points in the direction of the beam momentum axis. In order to be able to measure the full range of analyzing powers, the quantization axis must be rotated into other orientations. This is accomplished with a Wien Filter, which previously was
located after the LSPIS and now follows the ABP\textls{12}S. The Wien Filter consists of crossed
electric and magnetic fields which point in the plane normal to the beam momentum. The
magnetic field, which is read by a calibrated Hall probe, precesses the particle spins in the
plane normal to the direction of the magnetic field. The entire assembly can be rotated
about the beam-momentum axis to allow for spin precession to any desired orientation.
The electric field is then tuned so that there is no net deflection of the deuterons.

2.2 Acceleration and Transport

The negatively-charged deuterium ions are accelerated by means of anFN tandem
Van de Graaff accelerator. The negative ions are attracted to the positively charged
terminal located at the center of the accelerator tank. A thin (~10\(\mu\)g)\({}^{12}\)C foil strips the
two electrons from the negative ions at the terminal and the now-positive ions are
accelerated back to ground potential at the high-energy end of the tank.

The accelerator is followed by two high-resolution 90° bending magnets. Their
fields are set by means of a calibrated NMR probe in the first 90° magnet such that
particles of the correct charge-to-mass ratio pass through a pair of control slits located
between the magnets. Beam-energy stabilization of the Tandem accelerator is achieved by
means of an optically-coupled, fast-feedback circuit which applies a voltage, derived from
the slit-current difference, to the accelerator terminal stripper. Thus the calibrated magnet
is used to control and regulate the beam energy.

The beam is transported to target by means of several sets of electromagnetic
steerers, magnetic-quadrupole focusing devices, and a switching magnet which deflects
the beam into the desired beam-line. After passing through the target, the beam is
collected in a Faraday cup. The total charge is read by a beam-current integrator,
digitized, and counted by a scaler whose output is stored in the computer.
2.3 NaI Spectrometers

Capture $\gamma$ rays were detected in two anticoincidence-shielded 25.4 cm x 25.4 cm NaI(Tl) spectrometers [Wel81]. Each spectrometer was surrounded by 10 cm of Pb shielding as well as 20 cm of lithium-carbonated paraffin to moderate neutrons. Cadmium or boron sheets in front of the spectrometers also absorbed thermal neutrons. A tapered collimator in the front Pb shield defined a solid angle which completely illuminated the back face of the spectrometer.

The NaI crystals, which were manufactured by the Bicron Corporation, are viewed from the rear by six RCA 8575 photomultiplier tubes. The tubes have been gain matched for increased resolution and the output signals have been passively added. LED pulsers in the crystals provide light pulses for gain stabilization. All but the back face of the NaI crystals are surrounded by five inches of NE 110 plastic scintillator. This plastic shield is viewed by 8 XP 1031 photomultiplier tubes, 2 of which are mounted on the side to view the front volume of the shield. The remaining six tubes are equally spaced on a circle in the rear of the shield to view the remaining volume.

A coincidence condition between the shield and the NaI spectrometer rejects signals which occur within 60 ns of each other. This allows the rejection of $\sim$99% of the cosmic ray events in the NaI crystal. As well as rejecting cosmic ray event, the shield rejects events in which some radiation has leaked out of the crystal. An example of this (although relatively unimportant at the energies of this experiment) would be a $\gamma$ ray which enters the crystal and is Compton scattered into the shield. The shield gain was run in high-rejection mode which also rejects the 511 keV and 1022 keV escape peaks, due to escaping radiation from $e^+\cdot e^-$ annihilation in the NaI crystal. This decreases the efficiency of the detector system but increases its resolution to a total of $\sim$3% of the $\gamma$-ray energy for 15-25 MeV $\gamma$ rays.
There is a finite probability that a good $\gamma$-ray event will be rejected due to an accidental coincidence with a shield event. The probability of this occurring depends on the rate of signals from the shield, and the maximum length of time between shield and NaI signals which is considered coincident. The accidental rate can be measured approximately by setting up a second coincidence circuit where the NaI signals are now delayed by about 300 ns. Most coincidences occurring in this circuit are from uncorrelated events, giving an estimate of the number of accidental coincidences in the undelayed coincidence circuit.

At 8.85 MeV, the beam was pulsed at 5 MHz. A Time to Amplitude Converter (TAC) with the NaI signal as the start and the signal from a pickoff capacitor (located upstream from the target) as the stop, was used to create a time profile of the events in the NaI crystals. Good separation between $\gamma$-ray events and neutron events allowed rejection of the neutron events from the energy spectra. Occasionally, a stop signal will not be generated and a crystal event with the proper timing will be rejected. By storing the number of pickoff capacitor signals in a scaler counter and knowing the frequency of the beam bursts, the spectra can be corrected for this effect. Fig. 3.4 shows a typical $\gamma$-ray TAC spectrum. The experiments in the fusion resonance region used an unpulsed beam and therefore no time profile of events could be generated.

2.4 Solid-State Detectors

Solid state detectors have been used for three different purposes during this experiment. In the 8.6 MeV configuration, a solid state detector was used to detect elastically scattered deuterons in order to monitor the stability of the beam and target thickness. In the fusion-resonance experiments, a calibrated solid-state detector was used to set the beam energy and gas pressure needed, again by detecting elastically scattered deuterons. Two transmission-mount solid-state detectors are also installed in the
polarimeter in order to detect α particles which are needed to measure the beam polarization out of the ABPIS.

Despite the different purposes of these detectors they have the same electronics configuration. The detector is connected to a preamplifier which has inputs for the voltage bias and pulser test signal and outputs for an energy signal and a fast-timing signal. The energy signal is fed into the control room and the signal is amplified by a Spectroscopy Amplifier. The unipolar output of the Spec. Amp. is used as the linear input of a Linear Gated Stretcher operating in the gated, coincidence mode. The bipolar output of the Spec. Amp. is fed into a Single Channel Analyzer (SCA), which creates the gate for the Linear Gated Stretcher. In this way, the SCA can be used to set energy thresholds.

2.5 Target Configurations

Two distinctly different target geometries were used for the two energy regions studied in this experiment. Fig. 2.1 shows the target geometry for the experiment at $E_d = 8.6$ MeV. The gaseous $^3$He target (Research Grade 99.95% pure) was operated at a pressure of 69.0 kPa with a 1.27 μm Nickel beam-entrance foil and a 2.54 μm Havar beam-exit foil. To create the 8.6 MeV beam at the center of the target, an 8.85 MeV beam from the accelerator was used. According to an energy-loss calculation, this beam lost 60 keV in the Ni entrance foil and 150 keV in the four inches of gas between the Ni entrance foil and the center of the target. Another 150 keV was lost in the remaining $^3$He gas, and 115 keV was lost in the Havar foil. Lead and tungsten shadow bars were used to prevent the NaI spectrometers from viewing these foils and allowed the spectrometers to see a region of gas which was 3.58 cm long when the spectrometer was at 90°, corresponding to a target thickness of 0.301 mg/cm$^2$ and a solid angle of 23 msr. As an additional monitor of the experiment, an Ortec transmission mount, 500 μm thick, solid-state detector was mounted inside the target chamber at 30° and was doubly collimated to view
the center of the target. A spectrum with no gas in the target was taken in this configuration to determine the $\gamma$-ray background from the foils. No background in the energy range of interest was detected. Fig. 2.2 shows a typical $d+^{3}\text{He}$ $\gamma$-ray spectrum. Fig. 2.3 shows the corresponding solid-state detector spectrum.
Fig. 2.1 The target geometry for the experiment at $E_d = 8.6\text{MeV}$.
Fig. 2.2 A typical γ-ray energy spectrum from the $^3\text{He}(d,\gamma)^5\text{Li}$ reaction at $E_d=8.6$ MeV.

Top Panel: The spectrum of all events in the NaI crystal. Bottom Panel: The same spectrum after the shield anticoincidence condition and TAC condition have been applied. The small peak marked $\gamma_1$ may be due to capture to the first excited state of $^5\text{Li}$. 
Fig. 2.3 A solid-state detector spectrum from the $^3\text{He}(\text{d},\text{d})^3\text{He}$ reaction at $E_d=8.6\text{ MeV}$.

The high peak is from the elastically scattered deuterons. The background is due to high-energy protons from the $^3\text{He}(\text{d},\text{p})^4\text{He}$ reaction which are not stopped in the detector.
In the fusion resonance energy region, deuteron beams of several different energies were passed through 6.35 μm Havar foils which degraded the beam energy to the desired value. A 2.54 cm long cylindrical gas cell (see Fig. 2.4) was filled with $^3\text{He}$ gas (Research Grade 99.95% pure). Table 2.1 lists the conditions for each experiment. Accurate beam-current integration was achieved by electrically isolating the gas cell and applying a suppression voltage to prevent the escape of low energy electrons. (The gas cell could not be used for the 8.6 MeV experiment due to the high-energy background in the γ-ray spectra which was generated due to the NaI detectors viewing the beam stop.) The back faces of the NaI spectrometers were placed 106 cm from the target center, defining a solid angle of 45 msr. Spectra from a $^3\text{He}$ target and a $^4\text{He}$ background target along with the resultant background-subtracted spectrum are shown in Fig. 2.5.

Table 2.1  Target conditions for measurements made in the fusion-resonance region. $E_{\text{beam}}$ is the beam energy before entering the Havar degrading foils, $E_i$ is the energy immediately after the foils, and $E_f$ is the beam energy when exiting the target. The uncertainties $\Delta E = \frac{1}{2} \Gamma$, where $\Gamma$ is the full width at half maximum (FWHM) of the deuteron peaks as seen in the solid-state detector spectrum.

<table>
<thead>
<tr>
<th>$E_{\text{beam}}$ (MeV)</th>
<th># of Ha foils (6.35 μm)</th>
<th>$^3\text{He}$ Pressure (kPa)</th>
<th>$E_i\pm\Delta E_i$ (MeV)</th>
<th>$E_f\pm\Delta E_f$ (MeV)</th>
<th>Observables</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.43</td>
<td>4</td>
<td>414</td>
<td>0.80±0.09</td>
<td>0.0</td>
<td>$\sigma, A_y, A_{yy}, T_{20}$</td>
</tr>
<tr>
<td>3.43</td>
<td>4</td>
<td>124</td>
<td>0.80 ± 0.09</td>
<td>0.41±0.12</td>
<td>$\sigma$</td>
</tr>
<tr>
<td>3.33</td>
<td>4</td>
<td>93.8</td>
<td>0.60±0.08</td>
<td>0.30±0.14</td>
<td>$\sigma, A_y, A_{yy}$</td>
</tr>
<tr>
<td>3.28</td>
<td>4</td>
<td>89.7</td>
<td>0.4 ± 0.12</td>
<td>0.0</td>
<td>$\sigma$</td>
</tr>
</tbody>
</table>
Fig. 2.4 The 2.54 cm long cylindrical gas cell used in the $^3\text{He}(d,\gamma)^{5}\text{Li}$ reaction experiments in the fusion-resonance energy region.
Fig. 2.5  Top Panel: The upper energy spectrum is from the $^3\text{He}(d,\gamma)^5\text{Li}$ reaction at $E_d=0.8$ MeV where the beam is stopped in the target. The lower energy spectrum is from the same target configuration with the $^3\text{He}$ replaced by $^4\text{He}$. Contaminant peaks from 3.43 MeV deuteron capture by $^{12}\text{C}$ and $^{16}\text{O}$ are also visible. Bottom Panel: The resultant background-subtracted spectrum.
2.6 Beam-energy calibration

For the experiment at $E_d = 8.6$ MeV, the beam energy was set using the calibration for the 90°-90° high-resolution magnet system in the high-transmission mode. From experience, it has been found that the beam has less than a 50 keV uncertainty and less than an 8 keV dispersion [Wes84].

In order to set the beam energy and gas pressure for the fusion-resonance experiments, the deuteron beam was tuned into the 60° beam-leg scattering chamber and scattered from a thin (~10 μg) $^{12}$C foil into a calibrated solid-state detector located at $\theta_{ss}=30°$. The procedure for creating an 0.8 MeV beam began by rotating four Havar degrading foils into position between the target and the detector. The accelerator voltage was then adjusted to give an 0.8 MeV deuteron peak in the solid-state detector spectrum. The foils were rotated out of position, giving an undegraded scattered-deuteron peak with an energy of 3.43 MeV. Therefore the beam energy before scattering needs to be set to 3.43 MeV since no scattering takes place in the capture experiments. The beam energy was adjusted such that a 3.275 MeV scattered-deuteron peak was seen in the solid-state detector spectrum, corresponding to a 3.43 MeV deuteron beam. Having accounted for the kinematic shift due to the scattering, these same Havar degrading foils were transferred back to the capture set-up to ensure uniformity in foil thickness. The above procedure was also used to produce 0.6 MeV and 0.4 MeV deuteron beams.

In order to determine the gas pressure needed, the solid-state detector was placed inside a small air-tight chamber with a Havar entrance foil and the whole assembly was placed inside the scattering chamber. The scattered-deuteron energy was set by evacuating the chamber and measuring the energy deposited in the calibrated solid-state detector. The chamber was then pressurized with $^3$He until the deuterons reaching the detector had the desired energy. (See Table 2.2) As their energy loss became greater, the deuterons had more energy spreading. Therefore, the gas pressure was increased in small
increments in order to determine the peak energy in the spectrum more accurately. The distance from the foils to the detector is 3.81 cm. The pressure used in the 2.54 cm long gas cell was one and a half times the pressure in the solid state detector chamber so that the $^3$He target thicknesses measured in atoms/cm$^2$ were equal. Figs. 2.6 and 2.7 show four solid-state detector spectra corresponding to scattered deuterons with no degrading foil, scattered deuterons whose energy has been degraded from $E_d=3.43$ MeV to 0.80 MeV, scattered and degraded deuterons after losing 0.40 MeV through the $^3$He gas, and scattered and degraded deuterons after losing all of their energy through the $^3$He gas.
Table 2.2 Energy loss of deuterons in $^3$He gas. $E_{\text{initial}}$ is the energy of the deuterons entering the gas. $E_{\text{final}}$ is the energy of the deuterons after passing through 3.81 cm of gas. Also listed is the gas pressure and the FWHM of the deuterons (as measured in the solid-state detector) after passing through the gas.

<table>
<thead>
<tr>
<th>$E_{\text{initial}}$ (keV)</th>
<th>Pressure (kPa)</th>
<th>$E_{\text{final}}$ (keV)</th>
<th>FWHM (keV)</th>
</tr>
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<tbody>
<tr>
<td>798</td>
<td>0.0</td>
<td>804</td>
<td>174</td>
</tr>
<tr>
<td>798</td>
<td>55.2</td>
<td>570</td>
<td>225</td>
</tr>
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<td>798</td>
<td>82.7</td>
<td>414</td>
<td>237</td>
</tr>
<tr>
<td>798</td>
<td>110.3</td>
<td>235</td>
<td>350</td>
</tr>
<tr>
<td>798</td>
<td>172.4</td>
<td>Almost all particles are stopped.</td>
<td></td>
</tr>
<tr>
<td>798</td>
<td>241.3</td>
<td>All particles are stopped.</td>
<td></td>
</tr>
<tr>
<td>595</td>
<td>0.0</td>
<td>595</td>
<td>155</td>
</tr>
<tr>
<td>595</td>
<td>34.5</td>
<td>455</td>
<td>211</td>
</tr>
<tr>
<td>595</td>
<td>48.3</td>
<td>380</td>
<td>223</td>
</tr>
<tr>
<td>595</td>
<td>57.9</td>
<td>324</td>
<td>265</td>
</tr>
<tr>
<td>595</td>
<td>62.1</td>
<td>295</td>
<td>281</td>
</tr>
</tbody>
</table>
Fig. 2.6 Top Panel: A solid-state detector spectrum from the $^{12}\text{C}(d,d)^{12}\text{C}$ reaction. The scattered-deuteron peak is at 3.43 MeV. Three peaks from the $^{12}\text{C}(d,p_{1,2,3})^{13}\text{C}$ reaction are also visible. Bottom Panel: Spectrum after the deuterons have been degraded by 4 Havar foils (placed between the $^{12}\text{C}$ target and the solid-state detector) to 0.8 MeV.
Fig. 2.7  Top Panel: Spectrum from the $^{12}\text{C}(d,d)^{12}\text{C}$ reaction after the deuterons have been degraded by 4 Havar foils to 0.8 MeV and lost an additional 0.4 MeV in $^3\text{He}$ gas. Bottom Panel: Spectrum from the same reaction after the deuterons have been degraded by 4 Havar foils to 0.8 MeV and stopped in the $^3\text{He}$ gas. The peak in the spectrum at 0.9 MeV is caused by protons from the $^{12}\text{C}(d,p)^{13}\text{C}$ reaction.
2.7 Polarimetry

The polarization of the beam from the LSPIS was measured inside the source itself using the quench ratio method [Ohl71]. By quenching out the polarized particles, the beam which remained was completely unpolarized. The ratio of beam current from the quenched and unquenched beams gave a direct measure of the unpolarized background. The major disadvantage to this method is that, although the fraction of the beam which is polarized is determined, the specific state of the polarization at the target is unknown.

In order to allow greater knowledge of the beam polarization, a polarimeter was constructed which measures vector and tensor polarized deuterons from $E_d = 5-12$ MeV using the $^3\text{He}(d,p)^4\text{He}$ and $^3\text{He}(d,\alpha)^1\text{H}$ reactions. Based on the Stephenson polarimeter at Wisconsin, the polarimeter consists of a $^3\text{He}$ gas-target cell, two CsI detectors at $27.5^\circ$ and one CsI detector at $0^\circ$, all for proton detection, and two transmission-mount solid-state detectors in front of the $27.5^\circ$ CsI's for detecting recoil $\alpha$ particles. For an extensive discussion of the polarimeter see [Ste80].

For the detectors on the left side of the polarimeter, the yield in each detector, $N_L(\theta)$, can be expressed as:

$$N_L(\theta) = K_L \sigma_0(\theta)[1 + 2iT_{11}(\theta)\text{Re}(it_{11}) + T_{20}(\theta)t_{20} + 2T_{21}(\theta)\text{Re}(t_{21}) + 2T_{22}(\theta)\text{Re}(t_{22})],$$  \[2.1\]

where $K_L$ contains the target thickness and detector efficiency, the $T_{ij}$'s are the reaction's tensor observables, and the $t_{ij}$'s are the tensor moments of the beam. Similarly, for a detector on the right side:

$$N_R(\theta) = K_R \sigma_0(\theta)[1 - 2iT_{11}(\theta)\text{Re}(it_{11}) + T_{20}(\theta)t_{20} - 2T_{21}(\theta)\text{Re}(t_{21}) + 2T_{22}(\theta)\text{Re}(t_{22})].$$  \[2.2\]

The analyzing powers for the $^3\text{He}(d,p)^4\text{He}$ and $^3\text{He}(d,\alpha)^1\text{H}$ reactions have been measured in [Ste80]. Therefore, it is only necessary to solve five equations, one for each detector, for four unknowns to get the beam polarization. Using all five detectors gives a
redundancy check on the accuracy of the measurement. A Fortran code (see Appendix B) reads the yields in each detector directly from gates set in the spectra on the computer. It then performs the MINUIT $\chi^2$ minimization procedure to determine the polarization moments of the beam and their associated errors. The accuracy of the analyzing-power measurements used to compute the beam polarization limits the polarimeter to no better than 2% accuracy.
Chapter 3: Absolute Cross Sections

3.1 Introduction

An angular distribution measurement of a differential cross section is conceptually simple to perform. For a given number of particles passing through a target of known thickness, the events in a detector with a known efficiency and set at a given angle are counted. More specifically, the differential cross section is:

$$\frac{d\sigma(\theta)}{d\Omega} = \frac{Y(\theta)}{N \epsilon d\Omega} \quad [3.1]$$

where

- $Y(\theta)$ = the number of events coming from the reaction of interest in the detector at the angle $\theta$.
- $\theta$ = the angle between the beam-momentum vector and the vector pointing from the target to the detector.
- $N$ = the total number of particles which passed through the target.
- $\epsilon$ = the target thickness in units of particles/cm².
- $d\Omega$ = the solid angle subtended by the detector.

In practice, this becomes a very difficult measurement because complete knowledge of the beam, target, and detector geometry is needed. In addition, estimates must be made of how many counts were lost due to instrumental and/or computer effects.

In a perfect spectrum containing only one peak which comes from the reaction of interest, it would be a straightforward procedure to sum the total number of counts in the spectrum to get $Y(\theta)$. However, background radiation from beam-induced reactions and natural radiation from the Earth complicate matters by obscuring the low-energy portion of the $\gamma$-ray peak. It is therefore only possible to sum a portion of the $\gamma$-ray peak and estimate the fraction of the total counts which were summed. To do this, the peak of
interest must be fitted with a physically reasonable response function which extends to zero energy. The area of the entire response function is then compared to the area of the region which was summed. For a monoenergetic γ ray this functional form is simply the line-shape response of the NaI detector system. For a transition to an unbound state such as the ground state of $^5$Li, however, the functional form must be a convolution of the NaI line-shape with the energy distribution of γ-rays coming from the reaction.

The measurements of all the various contributions needed for a determination of the absolute cross section are discussed below. Errors for each contribution are estimated and then tabulated in Table 3.3.

3.2 Beam Current Integration

Deuteron beam currents on target ranged from 50 to 250 nA. For the experiment at $E_d = 8.6$ MeV, the beam was deposited in a tantalum-lined Faraday cup located about $4\frac{3}{4}$ meters beyond the target. For the fusion resonance experiments, the beam was stopped either in the target or at the back end of the gas cell. The cell was electrically isolated from the beam-line and a suppressor ring biased to -300 Volts prevented recoiling electrons from escaping the gas cell. The total charge was integrated by a Model 1000 Brookhaven Instruments Corporation Current Integrator (BCI). The procedure to manually zero the current scale and input balance introduces less than a 1% uncertainty in the total charge accumulated.

3.3 Target Thickness

The target geometries used in the experiments are described in Experimental Techniques. The density of $^3$He gas at 310.93 K is given in [Mat80] as 25,537.6 cm$^3$/mole. By applying the Ideal Gas Laws to correct for room temperature and pressure conditions and by multiplying by the length of gas viewed by the detectors, the target thickness in atoms/cm$^2$ can be calculated. The gas pressure of the target was measured by a Penwalt gauge which could be read to better than 0.2 PSIA. The pressure tended to
increase over time by 0.2 to 0.4 PSIA over the original fill pressure due to heating of the gas by the beam. For consistency, and since there was no evidence of a gas leak, the original fill pressure was used for the absolute cross section calculations. The absolute temperature of the $^3$He gas is not known just after filling the chamber since the rapid expansion of the gas may tend to cool it. However, even temperature variations of 5° C introduce less than 2% uncertainty in the target density.

For the 8.6 MeV geometry, the target thickness was $6.03 \times 10^{19}$ atoms/cm$^2$ at $\theta_{\text{lab}}=90°$. A Monte Carlo calculation was performed by S. E. King [Kin83] to find the variation in target thickness as a function of detector angle. The correction factor was within 2% of the first-order correction of $\frac{1}{\sin(\theta)}$. Therefore, only the first-order correction has been used for this data.

For the cross section measurements in the fusion-resonance region, the target thickness remained constant with angle since the entire 2.54 cm long gas cell was viewed by the NaI spectrometers. However, for the two cross section measurements where the beam was stopped in the target ($E_d=0.8$ MeV and $E_d=0.4$ MeV), the actual target length is known only to about 5%. This comes about due to the discrepancy in the range calculation for deuterons traversing $^3$He as computed by the energy-loss code BABEL and the extrapolated target thickness from the measured energy losses of Table 2.2. In all fusion-resonance measurements, the $^3$He was replaced by an equal pressure of $^4$He in order to obtain a background spectrum while maintaining the energy loss of the deuterons through the target. Table 3.1 lists the target thicknesses used.
Table 3.1 Target thickness for measurements made in the fusion resonance energy region. \( E_i \) is the beam energy entering the target and \( E_f \) is the beam energy exiting the target. The uncertainties are given by \( \Delta E = \frac{1}{2} \Gamma \), where \( \Gamma \) is the full width at half maximum (FWHM) of the deuteron peaks as seen in the solid-state detector spectrum. The errors for the target thickness are computed from the uncertainties in target length, \(^3\text{He} \) temperature, and \(^3\text{He} \) pressure.

<table>
<thead>
<tr>
<th>( E_i \pm \Delta E_i ) (MeV)</th>
<th>( E_f \pm \Delta E_f ) (MeV)</th>
<th>target thickness (atoms/cm(^2))</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.80( \pm 0.09 )</td>
<td>0.0</td>
<td>((1.54 \pm 0.14) \times 10^{20})</td>
</tr>
<tr>
<td>0.80( \pm 0.09 )</td>
<td>0.41( \pm 0.12 )</td>
<td>((7.69 \pm 0.18) \times 10^{19})</td>
</tr>
<tr>
<td>0.43( \pm 0.22 )</td>
<td>0.0</td>
<td>((5.56 \pm 0.31) \times 10^{19})</td>
</tr>
<tr>
<td>0.60( \pm 0.08 )</td>
<td>0.30( \pm 0.14 )</td>
<td>((5.82 \pm 0.15) \times 10^{19})</td>
</tr>
</tbody>
</table>

3.4 Efficiency

The measurement of the two NaI-spectrometer efficiencies is crucial for the calculation of the absolute cross section. The absolute \( \gamma \)-ray efficiency was measured at \( E_\gamma = 15.1 \) MeV and \( \theta_\gamma = 125^\circ \) by populating a resonance in \(^{13}\text{N} \) via the \(^{12}\text{C} (p,\gamma) ^{13}\text{N} \) reaction using the thick-target technique of [Mar77]. The product of the detector efficiency (\( \varepsilon \)) and solid angle (\( d\Omega \)) is extracted by comparing to the absolute yield given by [Mar77]. For a typical region of the spectrum which was summed, \( \varepsilon d\Omega = 6.6 \) msr and 5.7 msr for the left and right NaI spectrometers, respectively.

The efficiencies of the NaI spectrometers were confirmed by a second, independent method which utilized the \(^3\text{H}(p,\gamma)^4\text{He} \) reaction at \( E_\gamma = 26 \) MeV. A spectrum was measured with no paraffin in front of the NaI. The full line-shape was summed and corrected for lost counts due to the attenuation through the plastic shield [HayPri].
spectrum was then measured and summed for a fully shielded detector. The ratio of the shielded to unshielded yields is the detector efficiency at 26 MeV, assuming that the efficiency of the unshielded detector is nearly 100% for the full response function [Pau74]. Using the known energy dependence [McB82 and Wel81], the efficiency was calculated for 15.1 MeV γ-rays and found to agree with the first measurement to within 5% of the value.

3.5 Line-shape measurement

The NaI detector line-shape response function was measured using the $^3$H(p,γ)$^4$He reaction at $E_p = 2.5$ MeV, yielding γ rays with $E_γ = 21.6$ MeV. The response of the detector to these incident γ rays yields a spectrum with finite resolution (~3%) and a long low-energy tail (see Fig. 3.1). In order to use the line-shape to fit other NaI γ-ray peaks, it must be parameterized in a convenient functional form. It has been found that a composite function of two exponentials with quartic arguments can describe the line-shape quite well. In fact, until the recent addition of a new NaI with improved resolution, only cubic arguments had been needed. The functional form for the line-shape parameterization is:

$$F(E) = \begin{cases} 
0 & E \geq E_{\text{high}} \\
C_5 + C_6E + C_7E^2 + C_8E^3 + C_9E^4 & E_j \leq E < E_{\text{high}} \\
C_0 + C_1E + C_2E^2 + C_3E^3 + C_4E^4 & E_{\text{low}} \leq E \leq E_j \\
\frac{F(E_{\text{low}})}{E_{\text{low}}} & E < E_{\text{low}}
\end{cases}$$

[3.2]

where the two exponentials are required to match smoothly at the joint $E_j$, which usually is positioned high on the low-energy side of the peak (see Fig. 3.1). $E_{\text{low}}$ is the low-energy cut-off point below which the line-shape is extrapolated linearly to zero energy.
$E_{\text{high}}$ is the high-energy cut-off point above which the line-shape is zero. A $\chi^2$ minimization code (see Appendix C) using the routine MINUIT has been written in order to fit $F(E)$ to $^3\text{H}+\text{p}$ $\gamma$-ray spectra accumulated with the left and right NaI spectrometers. The results of these fits are listed in Table 3.2. Fig. 3.1 shows a typical fit to the $\gamma$-ray spectrum.

Table 3.2 The results from a search of $\chi^2$ space for solutions of Eqn. 3.2 for the left and right NaI spectrometers. Parameters are given to extreme precision due to the large cancellation effects in $F(E)$.

<table>
<thead>
<tr>
<th></th>
<th>Left NaI</th>
<th>Right NaI</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_0$</td>
<td>505.941760</td>
<td>1446.01464</td>
</tr>
<tr>
<td>$C_1$</td>
<td>-120.991939</td>
<td>-325.455574</td>
</tr>
<tr>
<td>$C_2$</td>
<td>10.8054336</td>
<td>27.4027008</td>
</tr>
<tr>
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<td>-0.426110365</td>
<td>-1.02228202</td>
</tr>
<tr>
<td>$C_4$</td>
<td>0.00627399981</td>
<td>0.0142771522</td>
</tr>
<tr>
<td>$C_5$</td>
<td>177032.628</td>
<td>438922.787</td>
</tr>
<tr>
<td>$C_6$</td>
<td>29665.4010</td>
<td>-80895.7846</td>
</tr>
<tr>
<td>$C_7$</td>
<td>-1856.87722</td>
<td>5586.461089</td>
</tr>
<tr>
<td>$C_8$</td>
<td>51.4420715</td>
<td>-171.313332</td>
</tr>
<tr>
<td>$C_9$</td>
<td>-0.532014012</td>
<td>1.96830020</td>
</tr>
<tr>
<td>$E_J$</td>
<td>21.701</td>
<td>21.060</td>
</tr>
<tr>
<td>$E_{\text{low}}$</td>
<td>14.000</td>
<td>15.000</td>
</tr>
<tr>
<td>$E_{\text{high}}$</td>
<td>23.500</td>
<td>23.000</td>
</tr>
<tr>
<td>Centroid</td>
<td>21.820</td>
<td>21.512</td>
</tr>
</tbody>
</table>
The energy of the \( \gamma \) ray entering the crystal in a given experiment is not always the same as a \( \gamma \)-ray energy for which a line-shape has been measured. It has been assumed in the present experiments that the percent resolution, \( \Delta E/E_{\gamma} \), is constant in the region of interest for the current experiment [Wel81]. It is then a simple procedure to transform the line-shape response function at 21.6 MeV into a response function for any \( \gamma \)-ray energy. This is accomplished by linearly transforming the line-shape energy such that the centroid corresponds to the desired \( \gamma \)-ray energy. For example, to create a line-shape response function at \( E_{\gamma}=10.8 \) MeV, \( F(E) \) is mapped to \( E/2 \), which maps the centroid at \( F(E=21.6 \) MeV\) to \( E=10.8 \) MeV. It is therefore necessary to specify the \( \gamma \)-ray energy \( E_{\gamma} \) when discussing the the response function, which from now on shall be denoted as \( F(E,E_{\gamma}) \).
Fig. 3.1 Spectrum of the NaI detector response function as measured in the $^3\text{He}(p,\gamma)^4\text{He}$ reaction at $E_p = 2.5$ MeV. The error bars represent the statistical uncertainties in the measurement. The solid curve represents the best fit to the data. $E_J$ (see Eqn. 3.2) is the energy at which the composite function $F(E)$ is joined.
3.6 NaI Spectrum fitting with a convolution function

In order to get consistent summing regions with different spectra, it is necessary to fit the spectra with a physically reasonable functional form. For γ-ray spectra coming from reactions with transitions to bound states, this form is simply the line-shape response of the NaI crystal as described in Section 3.5. Complications arise when the final state is unbound as in the case of $^5\text{Li}$.

According to the theory of resonance reactions [Lan58], the energy distribution of γ rays which are produced in an electric-dipole transition to an isolated resonance with a $p+α$ breakup channel can be described in terms of a Breit-Wigner form:

$$\text{BW}(E_\gamma) \propto \frac{1}{k^2} \frac{E_\gamma^3 \Gamma_p(E_p)}{\left[ \left( E_\gamma - E_{\text{res}} + \Delta(E_p) \right)^2 + \left( \frac{\Gamma_p}{2} \right)^2 \right]} \tag{3.3}$$

where $\Gamma_p$ is the width of the $p+α$ channel, $E_p$ is the nucleon emission energy, and $E_\gamma^3$ is the energy-dependent term of the γ-ray width $\Gamma_\gamma$. It is assumed that $\Gamma_\gamma$ is much less than $\Gamma_p$ so that the total width $\Gamma = \Gamma_p + \Gamma_\gamma = \Gamma_p$. This formalism is discussed further in Appendix A.1.

In the $E_q = 8.6$ MeV experiment, the γ-ray spectra from the NaI spectrometers were fit using the minimization code MINUIT with a convolution of the measured energy-dependent response function of the NaI spectrometer and the Breit-Wigner resonance shape described above. The functional form used is

$$F'(E) \propto \int F(E,E_\gamma) \text{BW}(E_\gamma) \, dE_\gamma \tag{3.4}$$

where the overall height of $F'(E)$ was varied, and the entire convoluted form was shifted up or down in energy to allow for spectrum calibration errors. The value for the width of the $^5\text{Li}$ ground state, extracted in a previous study [Bal91], was fixed at
\( \Gamma_p(E_{\text{res}}) = 2.44 \text{ MeV} \). Fig. 3.2 shows the best fit to the \(^5\text{Li}\) spectrum at 8.6 MeV, as well as the NaI response function and the BW\((E_\gamma)\) function used in the convolution.

A further complication arose when fitting the spectra from experiments in the fusion energy region. In these experiments, the energy loss of the beam in the target was comparable to the width of the ground-state resonance. This additional spreading was taken into account by a second convolution with a Gaussian function. Although the Gaussian width was treated as a free parameter, the full width at half maximum was always consistent with the energy loss of the beam in the target. A typical fit is shown in Fig. 3.3. Appendix A.2 contains the Fortran code for the fitting routine.
Fig. 3.2 A typical $\gamma$-ray spectrum for the $^3\text{He}(d,\gamma)^5\text{Li}$ reaction at $E_d=8.6$ MeV and $\theta_{\text{lab}}=90^\circ$. The convolution of the NaI response function and the resonance shape was fitted in the range $19.0$ MeV $\leq E_\gamma \leq 24.0$ MeV. The spectrum is not plotted below 16 MeV.
Fig. 3.3 A typical background-subtracted $\gamma$-ray spectrum from the $^3\text{He}(d,\gamma)^5\text{Li}$ reaction for an 0.8 MeV beam which was stopped in the target. The dashed curve represents a convolution fit to the data, where the vertical lines denote the fitted region. This is also the region of the peak that was summed to calculate the observables. The dip in the spectrum at 12.5 MeV is caused by background subtraction of a peak from the $^{12}\text{C}(d,\gamma)^{14}\text{N}$ reaction.
3.7 Dead-Time Correction

Dead time may arise from the fact that the computer cannot process an event because it is still busy processing a previous event or other information. Higher count rates and multiple active processes increase this effect. Fortunately, it is easy to measure this dead time and correct the spectra for the lost counts. A high frequency ("deadtime-less") scaler counter keeps track of the total number of gate signals going into the computer. Each gate corresponds to one event. Summing the full energy spectrum gives the total number of events processed by the computer. To correct for dead time, yields are multiplied by the factor:

$$\text{Dead Time Correction (DTC)} = \frac{\# \text{ of gates for energy signals}}{\# \text{ of counts in full energy spectrum}} \quad [3.5]$$

This correction is used for the NaI spectrometers where the complete NaI spectrum is summed before any coincidence or timing conditions are applied. The same method is used to correct for dead time effects in the solid state detector spectra. Typically, dead times are kept below 10% and were usually less than 4% for the current experiments.

Other sources of dead time are the electronic modules used to process the signals before storing them in the computer. However, these effects are negligible compared to the computer dead time. For example, for 10 $\mu$s after an event in the NaI crystal no other signals are accepted by the electronics. This is to prevent the pile up of multiple signals. This 10 $\mu$s is dead time. A typical run lasts about one hour with less than 100,000 NaI signals reaching the computer. Then the amount of time that the electronics are idle is $\sim 10^5$ counts $\times 10\times10^{-6}$ s $= 1$ s. The dead time correction is then:

$$\text{Electronics DTC} \sim \frac{3600 \text{ s}}{3600 \text{ s} - 1 \text{ s}} = 1.0003$$

which is a small correction compared to the computer dead time.
3.8 Accidental-Coincidence Correction

A small fraction of good events in the NaI crystal are rejected due to accidental coincidences with shield events. The number of good events which have been accidentally rejected are estimated electronically in a delayed-coincidence circuit which is described in Section 2.3. Knowing the total number of coincidences (COIN) and accidental coincidences (ACC), the multiplicative correction for this effect is:

\[
\text{Accidental Correction (AC)} = \frac{1}{1 - \frac{\text{ACC}}{\text{COIN}}} \tag{3.6}
\]

where the true yield is equal to the accidental correction times the measured yield. The accidental corrections for this experiment were typically 1 to 4% in the current experiments.

This correction factor is only an estimate of the true accidental correction due to correlated events which can take place. For example, a cosmic-ray shower may produce a secondary event 300 ns after the initial event is detected. This coincides with the delay in the accidental-coincidence circuit and therefore gives rise to an accidental-coincidence count. Assuming that 10% of the accidentals are not true accidentals, then the yields should actually be corrected by:

\[
\text{Actual Accidental Correction (AAC)} = \frac{1}{1 - \frac{0.9\text{ACC}}{\text{COIN}}} \tag{3.7}
\]

In a typical run, the number of coincidences is about 30,000 and the number of accidentals is perhaps 500. Then AC = 1.017 and AAC = 1.015. In other words, there is less than a 0.2% discrepancy.

3.9 Pickoff Correction

For the experiment at 8.6 MeV, the beam was pulsed at 5 MHz with the beam compressed into 2 to 3 ns long bursts. A timing profile (TAC spectrum) for the events in the NaI was generated (see Fig. 3.4) by measuring the time between the beam burst
passing through a capacitive pickoff, located before the target, and an event in the crystal. Since neutrons travel slower than γ rays, counts from the two events are separated in the time spectrum. The neutrons can then be gated out of the energy spectrum by allowing only events which arrive at the detector at the right time to be sorted into the energy spectrum. This sorting process is done by software in the computer. The TAC window size was set visually during on-line sorting. Therefore, it is possible for some good events to be outside this window. By summing the TAC spectrum using a wider window it has been determined that at most 5% of the good counts are outside the original TAC window.

Occasionally, a pickoff signal from a beam burst is not strong enough to get through the discriminator. Any events from this burst are then rejected by the timing condition. To correct for this, the number of signals (S) from the pickoff capacitor are counted. The frequency (f) of the beam pulsing and the length of time (t) of each run are known. Then the multiplicative pickoff correction is given by:

$$\text{Pickoff Correction} = \frac{ft}{S}$$  \hspace{1cm} [3.7]

where the true yield of γ rays is equal to the pickoff correction times the measured yield. Pickoff corrections varied from 0.1% to 7% and were typically under 2%. With no beam on target, background noise produced in the pickoff circuit is typically less than 0.1%.
Fig. 3.4 A TAC spectrum from the $^3\text{He}(d,\gamma)^5\text{Li}$ reaction at $E_d = 8.6$ MeV. The vertical lines denote the gate from which a TAC-gated energy spectrum was constructed (as in Fig. 3.2).
3.10 Results

Independent measurements of the angular distributions of the absolute differential cross sections were made in the left and right NaI spectrometers. These two angular distributions were then normalized to each other by an overall averaging factor and then the data at each angle were combined in an weighted averaging procedure. The overall averaging factor was typically about 0.95 for the right NaI data and 1.05 for the left NaI data. This falls well within the systematic errors of the experiment.

The absolute differential cross section (obtained from the weighted averaging procedure described above) for $E_d = 8.6 \text{ MeV}$, $E_d = 0.8-0.0 \text{ MeV}$, $E_d = 0.8-0.4 \text{ MeV}$, $E_d = 0.4-0.0 \text{ MeV}$, and $E_d = 0.6-0.3 \text{ MeV}$ are listed in Table 3.3. Figs. 3.5-3.9 plot these cross sections, respectively, along with the cross sections obtained individually from the left and right NaI spectrometers. Table 3.4 lists the various sources of errors of these measurements as discussed in Sections 3.2-3.9.

Previous studies of the $^3\text{He}^*(d,\gamma)^5\text{Li}$ reaction at the peak of the fusion resonance measured absolute cross sections which range from 1.7 $\mu$b/sr [Bus68] to 4 $\mu$b/sr [Bla54]. The current measurements energy-integrate over four regions of the resonance and are therefore underestimates of the on-resonance cross section. However, they do fall within the range of the previously measured values, indicating that the measurement of [Bus68] may be too low. Present measurements of angular distributions in the fusion resonance region are nearly isotropic, which is also consistent with these previous experiments.

Measurements of the differential cross section at $\theta=90^\circ$ near $E_d = 8.6 \text{ MeV}$ indicate that the cross section is between 3 and 4 mb/sr [Del68 and Kin72]. This is significantly higher than the current measurement of 2.4 $\mu$b/sr, however the angular distributions are similar in shape (see Section 5.3.4)
Table 3.3  The absolute differential cross section of the $^3\text{He}(d,\gamma)^5\text{Li}$ reaction in the center-of-mass system as measured in the present experiment. The errors represent the statistical uncertainty of the data.

<table>
<thead>
<tr>
<th>$\theta_{\text{c.m.}}$ (deg)</th>
<th>8.6 MeV ($\mu$b/sr)</th>
<th>$\theta_{\text{c.m.}}$ (deg)</th>
<th>0.8-0.0 MeV ($\mu$b/sr)</th>
<th>$\theta_{\text{c.m.}}$ (deg)</th>
<th>0.8-0.4 MeV ($\mu$b/sr)</th>
</tr>
</thead>
<tbody>
<tr>
<td>36.280</td>
<td>1.694 ± 0.018</td>
<td>30.252</td>
<td>1.747 ± 0.072</td>
<td>30.252</td>
<td>1.984 ± 0.057</td>
</tr>
<tr>
<td>51.703</td>
<td>1.945 ± 0.025</td>
<td>50.386</td>
<td>1.802 ± 0.027</td>
<td>50.386</td>
<td>2.037 ± 0.027</td>
</tr>
<tr>
<td>72.077</td>
<td>2.225 ± 0.022</td>
<td>70.473</td>
<td>1.868 ± 0.015</td>
<td>70.473</td>
<td>2.111 ± 0.037</td>
</tr>
<tr>
<td>92.196</td>
<td>2.396 ± 0.055</td>
<td>90.502</td>
<td>1.917 ± 0.015</td>
<td>90.502</td>
<td>2.105 ± 0.059</td>
</tr>
<tr>
<td>112.050</td>
<td>2.361 ± 0.019</td>
<td>110.471</td>
<td>1.907 ± 0.033</td>
<td>110.471</td>
<td>2.099 ± 0.072</td>
</tr>
<tr>
<td>131.662</td>
<td>2.121 ± 0.169</td>
<td>130.384</td>
<td>1.890 ± 0.067</td>
<td>130.384</td>
<td>2.151 ± 0.025</td>
</tr>
<tr>
<td>151.080</td>
<td>2.150 ± 0.052</td>
<td>150.250</td>
<td>1.838 ± 0.038</td>
<td>150.250</td>
<td>2.122 ± 0.062</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$\theta_{\text{c.m.}}$ (deg)</th>
<th>0.4-0.0 MeV ($\mu$b/sr)</th>
<th>$\theta_{\text{c.m.}}$ (deg)</th>
<th>0.6-0.3 MeV ($\mu$b/sr)</th>
</tr>
</thead>
<tbody>
<tr>
<td>30.238</td>
<td>1.549 ± 0.075</td>
<td>35.289</td>
<td>1.957 ± 0.074</td>
</tr>
<tr>
<td>50.364</td>
<td>1.507 ± 0.070</td>
<td>50.386</td>
<td>1.854 ± 0.069</td>
</tr>
<tr>
<td>70.446</td>
<td>1.530 ± 0.024</td>
<td>70.473</td>
<td>1.877 ± 0.028</td>
</tr>
<tr>
<td>90.474</td>
<td>1.558 ± 0.022</td>
<td>90.502</td>
<td>1.822 ± 0.134</td>
</tr>
<tr>
<td>110.445</td>
<td>1.652 ± 0.058</td>
<td>110.471</td>
<td>1.935 ± 0.011</td>
</tr>
<tr>
<td>130.362</td>
<td>1.574 ± 0.029</td>
<td>130.384</td>
<td>1.938 ± 0.029</td>
</tr>
<tr>
<td>150.236</td>
<td>1.739 ± 0.108</td>
<td>145.287</td>
<td>1.836 ± 0.091</td>
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</table>
Table 3.4 Sources of error for the absolute cross section measurements listed in Table 3.1. These errors are discussed in Sections 3.2 through 3.9.

<table>
<thead>
<tr>
<th>Sources of Error</th>
<th>8.6</th>
<th>0.8-0.0</th>
<th>0.8-0.4</th>
<th>0.4-0.0</th>
<th>0.6-0.3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>MeV</td>
<td>MeV</td>
<td>MeV</td>
<td>MeV</td>
<td>MeV</td>
</tr>
<tr>
<td>Beam-current integration</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Gas Pressure</td>
<td>2.0</td>
<td>0.3</td>
<td>1.1</td>
<td>1.5</td>
<td>1.5</td>
</tr>
<tr>
<td>Gas Temperature</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Target Length</td>
<td>2</td>
<td>5</td>
<td>0</td>
<td>5</td>
<td>0</td>
</tr>
<tr>
<td>Efficiency Measurement</td>
<td>10</td>
<td>10</td>
<td>10</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>Dead Time Correction</td>
<td>0.03</td>
<td>0.03</td>
<td>0.03</td>
<td>0.03</td>
<td>0.03</td>
</tr>
<tr>
<td>Accidental Correction</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
</tr>
<tr>
<td>TAC Gate Error</td>
<td>5</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Pickoff Correction</td>
<td>0.1</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Statistical Uncertainty</td>
<td>2.3</td>
<td>2.0</td>
<td>2.3</td>
<td>3.6</td>
<td>2.4</td>
</tr>
<tr>
<td>TOTAL</td>
<td>12</td>
<td>12</td>
<td>11</td>
<td>12</td>
<td>11</td>
</tr>
</tbody>
</table>
Fig. 3.5 The absolute differential cross section for the $^3\text{He}(d,\gamma)^5\text{Li}$ reaction at $E_d(\text{lab}) = 8.6$ MeV as measured in the left and right NaI spectrometers. The combined result is derived from a normalization and weighted-averaging procedure as described in the text. The error bars represent the statistical uncertainty of the measurements.
Fig. 3.6 The absolute differential cross section for the \(^3\text{He}(d,\gamma)^5\text{Li}\) reaction at \(E_d(\text{lab}) = 0.8\) MeV, where the deuteron beam was stopped in the target, as measured in the left and right NaI spectrometers. The combined result is derived from a normalization and weighted-averaging procedure as described in the text. The error bars represent the statistical uncertainty of the measurements.
Fig. 3.7 The absolute differential cross section for the $^3\text{He}(d,\gamma)^5\text{Li}$ reaction at $E_d(\text{lab})=0.8\text{ MeV}$, where the deuteron beam exited the target with $E_d=0.4\text{ MeV}$, as measured in the left and right NaI spectrometers. The combined result is derived from a normalization and weighted-averaging procedure as described in the text. The error bars represent the statistical uncertainty of the measurements.
Fig. 3.8 The absolute differential cross section for the $^3\text{He}(d,\gamma)^5\text{Li}$ reaction at $E_d(\text{lab}) = 0.4$ MeV, where the deuteron beam was stopped in the target, as measured in the left and right NaI spectrometers. The combined result is derived from a normalization and weighted-averaging procedure as described in the text. The error bars represent the statistical uncertainty of the measurements.
Fig. 3.9 The absolute differential cross section for the $^3$He($^3$He,$\gamma$)$^5$Li reaction at $E_d$(lab) = 0.6 MeV, where the deuteron beam exited the target with $E_d = 0.3$ MeV, as measured in the left and right NaI spectrometers. The combined result is derived from a normalization and weighted-averaging procedure as described in the text. The error bars represent the statistical uncertainty of the measurements.
Chapter 4: Polarization Observables

4.1 Introduction

One of the advantages of conducting experiments at TUNL is being able to utilize the polarized proton and deuteron beams to measure the analyzing powers of reactions. This chapter will discuss the mathematical and physical properties which describe polarization phenomena for the case of the spin-1 deuteron as well as presenting the results of experimental measurements of the vector and tensor analyzing powers of the $^3\text{He}(^3\text{He},\gamma)^5\text{Li}$ reaction. All formalism is discussed under the guidelines of the Madison Convention [Bar71].

When an external magnetic field ($\mathbf{B}$-field) is applied to a deuterium atom, the degenerate energy levels of the electron split due to the hyperfine interaction. Fig. 4.1 shows a plot of the level splitting as a function of applied B-field strength. $B_c$ is the B-field applied to the electron due to it's motion through the electric field of the nucleus. At field strengths less than $B_c$ (weak fields), the proton and electron spins are strongly coupled so that the total spin of the atom $F$ and its associated magnetic quantum number $m_F$ are good quantum numbers. Here $F = s + I$, where $s$ is the spin of the electron, $I$ is the spin of the nucleus, and $F$ is the vector sum of $s$ and $I$. Bold-face type indicates these are vector quantities. At field strengths much greater than $B_c$ (strong fields), the electron and nuclear spin become decoupled such that $m_s$ and $m_I$ are good magnetic quantum numbers. Since all experiments described in this dissertation were conducted with $B > B_c$, only the strong field case shall be discussed further.

In a strong field, the non-degenerate energy levels can be labeled by their nuclear magnetic substate $m_I (+1, 0, \text{or} -1)$ and the electron magnetic substate $m_s (\pm 1/2)$. Alternatively, these states are often referred to as states 1, 2, and 3 for the three higher energy states with $m_I = +1, 0,$ and -1 respectively, and states 4, 5, and 6 for the three
lower energy states with $m_1 = -1, 0, \text{ and } +1$ respectively. The relative populations of these states determine the polarization of the deuteron beam.

![Deuterium](image)

**Fig. 4.1** Plot of the hyperfine splitting of deuterium as a function of applied B-field strength. $B_c$ is the B-field applied to the electron due to its motion through the electric field of the nucleus.

**4.2 The Madison Convention**

Fig. 4.2 illustrates the Madison-convention coordinate system. The z-axis points in the direction of the beam momentum $k_{\text{in}}$. The y-axis is perpendicular to the reaction plane in the direction of $k_{\text{in}} \times k_{\text{out}}$, where $k_{\text{out}}$ points from the target to the detector. $\beta$ is defined as the angle between the z-axis and the quantization axis $\zeta$. $\phi$ is defined as the angle due to a rotation of $\zeta$ about the z-axis out of the y-z plane.
Fig. 4.2 The Madison convention coordinate system. Left figure: The $z$ axis is along the direction of the incident beam momentum vector $\vec{k}_{in}$. The vector $\vec{k}_{out}$ points from the target to the detector. The $y$ axis is defined along $\vec{k}_{in} \times \vec{k}_{out}$. The $x$ axis is defined such that a right-handed coordinate system is formed. Right figure: The angle $\beta$ is defined by the $z$ axis and the quantization axis $\zeta$. The angle $\phi$ is defined by the $y$ axis and the projection of $\zeta$ on the $x$-$y$ plane.

The preparation of polarized beams is discussed in Section 2.1. For the purposes of the current discussion, which follows that of W. Haeberli [Hae75], it is assumed that a polarized deuteron beam with a known population of states has been prepared. For a beam which has been polarized along a quantization axis $\zeta$ given by the angles $\beta$ and $\phi$, the polarization can be represented by a tensor of rank 2. This tensor can either be expressed in cartesian coordinates with elements $p_i$ and $p_{ij}$, or in spherical coordinates with elements $t_{ij}$.

The cartesian spin moments are defined as the expectation values of combinations of the spin operators:

$$ p_i = \langle S_i \rangle \quad \text{vector polarization,} \quad [4.1] $$

and

$$ p_{ij} = \frac{3}{2} \langle S_i S_j + S_j S_i \rangle - 2 \delta_{ij} \quad \text{tensor polarization.} \quad [4.2] $$

Similarly, the spin tensor moments in spherical coordinates are defined below.
\[ t_{00} = 1 \]
\[ t_{10} = \sqrt{3/2} \langle S_z \rangle \]
\[ t_{11} = -\sqrt{3/2} \langle S_x + iS_y \rangle \]
\[ t_{20} = \sqrt{1/2} \langle 3S_z^2 - 2 \rangle \]
\[ t_{21} = -\sqrt{3/2} \langle (S_x + iS_y)S_z + S_z(S_x + iS_y) \rangle \]
\[ t_{22} = \sqrt{3/2} \langle (S_x + iS_y)^2 \rangle . \]

Beams prepared in a polarized-ion source have azimuthal symmetry such that \( \zeta \) points along the z-axis. It is then possible to express the polarization in terms of only two quantities, \( p_\zeta \) and \( p_{\zeta\zeta} \), in cartesian coordinates or \( \tau_{10} \) and \( \tau_{20} \) in spherical coordinates. If \( N_+ \) is the fraction of particles with spin \( m_1 = +1 \), \( N_- \) is the fraction of particles with spin \( m_1 = -1 \), and \( N_0 \) is the fraction of particles with spin \( m_1 = 0 \), then \( p_\zeta \) and \( p_{\zeta\zeta} \) can be written as:
\[ p_\zeta = (N_+ - N_-) \quad \text{and} \quad p_{\zeta\zeta} = (1 - 3N_0), \]
and \( \tau_{10} \) and \( \tau_{20} \) as:
\[ \tau_{10} = \sqrt{3/2} (N_+ - N_-) \quad \text{and} \quad \tau_{20} = \sqrt{1/2} (1 - 3N_0). \]

A Wien Filter with crossed E and B fields located downstream from the polarized-ion source allows \( \zeta \) to be precessed to any desired orientation. For any rotations \( \beta \) and \( \phi \) of the quantization axis the spin tensor moments are given in spherical coordinates by:
\[ t_{10} = \tau_{10} \cos(\beta) \]
\[ i t_{11} = \tau_{10} \sin(\beta) e^{i\phi}/\sqrt{2} \]
\[ t_{20} = (1/2) \tau_{20} (3\cos^2\beta - 1) \]
\[ t_{21} = -i\sqrt{3/2} \tau_{20} \sin(\beta)\cos(\beta) e^{i\phi} \]
\[ t_{22} = -\sqrt{3/8} \tau_{22} \sin^2(\beta) e^{2i\phi} . \]

The spin tensor moments in cartesian coordinates may be obtained from linear combinations of the above expressions as denoted in [Bar71, p. xxix].
4.3 Analyzing Powers

The cross section \( \sigma_p(\theta) \) for a reaction \( A(\vec{a},b)B \) with an incident polarized spin-1 particle can be expressed in terms of the unpolarized cross section \( \sigma_0(\theta) \) and the spin tensor moments [Hae75 and Dar71].

\[
\sigma_p(\theta) = \sigma_0(\theta)[1+2iT_{11}(\theta)\text{Re}(i\gamma_{11})+T_{20}(\theta)\gamma_{20} + 2T_{21}(\theta)\text{Re}(\gamma_{21}) + 2T_{22}(\theta)\text{Re}(\gamma_{22})] \tag{4.16}
\]

or

\[
\sigma_p(\theta) = \sigma_0(\theta)[1+\frac{3}{2}p_{\xi}A_{y}(\theta)\sin(\beta)\cos(\phi) + \frac{1}{4}p_{\xi\zeta}A_{zz}(\theta)(3\cos^2\beta - 1) \\
- p_{\zeta\xi}A_{xz}(\theta)\sin(\beta)\cos(\beta)\sin(\phi) \\
- \frac{1}{4}p_{\zeta\xi}(A_{xx}(\theta) - A_{yy}(\theta))\sin^2(\beta)\cos(2\phi)]. \tag{4.17}
\]

These expressions provide the foundation for measuring the analyzing powers of reactions. By judicious choices of beam polarizations each analyzing power becomes accessible to experimental measurements. For example, a polarized beam where the spin points in the direction of motion has only a \( t_{20} \) tensor spin moment. Then equation [4.16] reduces to:

\[
\sigma_p(\theta) = \sigma_0(\theta)[1+T_{20}(\theta)\gamma_{20}]. \tag{4.18}
\]

A cross-section measurement with a polarized beam followed by a measurement with an unpolarized beam leads to the expression:

\[
T_{20}(\theta) = \frac{1}{\gamma_{20}}\left[ \frac{\sigma_p(\theta)}{\sigma_0(\theta)} - 1 \right]. \tag{4.19}
\]

To measure \( A_y(\theta) \) and \( A_{yy}(\theta) \) simultaneously, the spin-quantization axis is aligned along the y axis (\( \beta=90^\circ, \phi=0^\circ \)). Equation [4.17] then reduces to:

\[
\sigma_p(\theta) = \sigma_0(\theta)[1+\frac{3}{2}p_{\xi}A_{y}(\theta) + \frac{1}{2}p_{\xi\zeta}A_{yy}(\theta)]. \tag{4.20}
\]

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With the TUNL Lamb Shift Polarized Ion Source (LSPIS), the beam was polarized into State 1, 2, or 3. For Q defined as the fraction of the beam which is polarized, i.e. there is a (1-Q) unpolarized background, then the polarization in State 1 is \( p_\zeta = Q \) and \( p_{\zeta\zeta} = Q \); for State 2, \( p_\zeta = 0 \) and \( p_{\zeta\zeta} = -2Q \); for State 3, \( p_\zeta = -Q \) and \( p_{\zeta\zeta} = Q \). Writing Equation [4.20] for each state and solving for \( \sigma_0(\theta) \), \( A_y(\theta) \), and \( A_{yy}(\theta) \) gives:

\[
\sigma_0(\theta) = \frac{1}{3}[\sigma_1(\theta) + \sigma_2(\theta) + \sigma_3(\theta)] \tag{4.21}
\]

\[
A_y(\theta) = \frac{1}{Q} \left[ \frac{\sigma_1(\theta) - \sigma_3(\theta)}{\sigma_1(\theta) + \sigma_2(\theta) + \sigma_3(\theta)} \right] \tag{4.22}
\]

\[
A_{yy}(\theta) = \frac{1}{Q} \left[ \frac{\sigma_1(\theta) + \sigma_3(\theta) - 2\sigma_2(\theta)}{\sigma_1(\theta) + \sigma_2(\theta) + \sigma_3(\theta)} \right] \tag{4.23}
\]

where \( \sigma_i(\theta) \) denotes the polarized cross section in state i (i=1,2,3).

To measure \( A_y(\theta) \) and \( A_{yy}(\theta) \) simultaneously using the Atomic Beam Polarized Ion Source (ABPIS), the beam was polarized using the Strong Field 2, 3\( \leftrightarrow \)5 transition. The theoretical maxima for \( p_\zeta \) and \( p_{\zeta\zeta} \) are \( \frac{1}{3} \) and -1, respectively. Typically, the actual polarizations are about 80% of these values. By measuring \( \sigma_p(\theta) \) and \( \sigma_0(\theta) \) for a left and right detector, the analyzing powers are given by:

\[
A_y(\theta) = \frac{1}{3p_\zeta} \left[ \frac{\sigma_{LP}(\theta)}{\sigma_{LO}(\theta)} - \frac{\sigma_{RP}(\theta)}{\sigma_{RO}(\theta)} \right] \tag{4.24}
\]

\[
A_{yy}(\theta) = \frac{1}{p_{\zeta\zeta}} \left[ \frac{\sigma_{LP}(\theta)}{\sigma_{LO}(\theta)} + \frac{\sigma_{RP}(\theta)}{\sigma_{RO}(\theta)} - 2 \right]. \tag{4.25}
\]
4.4 Statistical Errors of Analyzing Powers

To derive the statistical errors for the analyzing powers, it is necessary to start with the expression for the error of a function which contains contributions from several uncorrelated, independent measurements [Bev69].

\[ \Delta F(x_i) = \sqrt{\sum \left( \frac{\partial F}{\partial x_i} \right)^2 (\Delta x_i)^2} \]  \[ \text{[4.26]} \]

Using equations [4.19] and [4.26] the error in \( T_{20}(\theta) \) is given as:

\[ \Delta T_{20}(\theta) = \frac{1}{t_{20}} \sqrt{\left( \frac{1}{\sigma_0} \right)^2 (\Delta \sigma_p)^2 + \left( \frac{\sigma_p}{\sigma_0} \right)^2 (\Delta \sigma_0)^2}. \]  \[ \text{[4.27]} \]

\( \Delta \sigma_p \) and \( \Delta \sigma_0 \) are the errors in the cross section measurement defined as:

\[ \Delta \sigma_i = n_i \sqrt{Y_i} \] \[ \text{[4.28a]} \]

\[ n_i = \frac{\sigma_i}{Y_i}. \] \[ \text{[4.28b]} \]

The proportionality constant \( n_i \) depends on experimental parameters such as the target thickness and detector efficiency. (See Sections 3.3 and 3.4)

For \( A_y(\theta) \) and \( A_{yy}(\theta) \) in the LSPIS configuration, equations [4.22], [4.23], and [4.26] give:

\[ \Delta A_y(\theta) = \frac{1}{Q \Sigma^2} \sqrt{(\sigma_2+2\sigma_3)^2 (\Delta \sigma_1)^2 + (\sigma_1-\sigma_3)^2 (\Delta \sigma_2)^2 + (2\sigma_1+\sigma_2)^2 (\Delta \sigma_3)^2}, \] \[ \text{[4.29]} \]

and

\[ \Delta A_{yy}(\theta) = \frac{3}{Q \Sigma^2} \sqrt{(\sigma_2)^2 (\Delta \sigma_1)^2 + (\sigma_1+\sigma_3)^2 (\Delta \sigma_2)^2 + (\sigma_2)^2 (\Delta \sigma_3)^2}, \] \[ \text{[4.30]} \]

where \( \Sigma = \sigma_1(\theta) + \sigma_2(\theta) + \sigma_3(\theta). \)
For $A_y(\theta)$ and $A_{yy}(\theta)$ in the ABPIS configuration, equations [4.24], [4.25], and [4.26] give:

$$\Delta A_y(\theta) = \frac{1}{3p_{\zeta}} \sqrt{D}$$  \hspace{1cm} [4.31]

$$\Delta A_{yy}(\theta) = \frac{1}{p_{\zeta}} \sqrt{D}$$  \hspace{1cm} [4.32]

$$D = \left( \frac{\Delta \sigma_{LP}}{\sigma_{L0}} \right)^2 + \left( \frac{\sigma_{LP} \Delta \sigma_{L0}}{(\sigma_{L0})^2} \right)^2 + \left( \frac{\Delta \sigma_{RP}}{\sigma_{R0}} \right)^2 + \left( \frac{\sigma_{RP} \Delta \sigma_{R0}}{(\sigma_{R0})^2} \right)^2. \hspace{1cm} [4.33]$$

There are two basic effects in the error equations. First, the error goes inversely as the square root of the total number of counts. Second, the error goes inversely as the polarization. Therefore it is very important to have the maximum polarization possible. If the polarization drops by a factor of two, the error doubles. It then takes four times as many counts (and four times as long) to make up for the lost precision.

### 4.5 Results

The analyzing powers of the the $^3\text{He}(d,\gamma)^5\text{Li}$ reaction have been measured in three energy regions, using the methods discussed above. The analyzing powers $A_y(\theta)$ and $A_{yy}(\theta)$ were measured at $E_d = 8.6$ MeV and $E_d = 0.8$ MeV (where the beam was stopped in the target) using the LSPIS configuration. Independent measurements were made in this configuration for the left and right NaI spectrometers and the results were combined by a weighted averaging procedure. $T_{20}(\theta)$ was measured at $E_d = 0.8$ MeV using the ABPIS configuration. Finally, the ABPIS was used to measure the analyzing powers $A_y(\theta)$ and $A_{yy}(\theta)$ at $E_d = 0.6$ MeV where the deuteron beam lost 0.3 MeV in the target.

In Chapter 3, many sources of errors were discussed for the cross section measurement. One of the nice properties of analyzing powers is that they are expressed
as ratios of cross sections. This means that many systematic errors in the cross section do not affect the analyzing powers. Target thickness, detector efficiency, and the solid angle of the detector all cancel out in Eqn. 4.19 and Eqns. 4.21-4.25. The dominant systematic errors in analyzing power measurements are from the uncertainty in the beam polarization. This is usually less than 10% but no better than 2% (see Section 2.7). The results of the analyzing power measurements are given in Tables 4.1-4.3 and Figs 4.3-4.6. The quoted errors represent the statistical uncertainty of the data. The analysis and interpretation of the measured cross sections and analyzing powers are discussed in the next chapter.

Table 4.1  The results of an angular distribution measurement of the analyzing powers for the $^3\text{He}(\vec{d},\gamma)^5\text{Li}$ reaction at $E_d = 8.6 \text{ MeV}$. The errors represent the statistical uncertainty of the measurement.

<table>
<thead>
<tr>
<th>$\theta_{\text{c.m.}}$</th>
<th>$A_v(\theta)$</th>
<th>$A_{vv}(\theta)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>36.280</td>
<td>-0.026±0.013</td>
<td>-0.002±0.019</td>
</tr>
<tr>
<td>51.703</td>
<td>-0.040±0.017</td>
<td>0.056±0.023</td>
</tr>
<tr>
<td>72.077</td>
<td>-0.009±0.011</td>
<td>0.049±0.035</td>
</tr>
<tr>
<td>92.196</td>
<td>-0.023±0.013</td>
<td>0.044±0.019</td>
</tr>
<tr>
<td>112.050</td>
<td>-0.002±0.011</td>
<td>0.016±0.031</td>
</tr>
<tr>
<td>131.662</td>
<td>0.026±0.016</td>
<td>0.019±0.018</td>
</tr>
<tr>
<td>151.080</td>
<td>0.027±0.012</td>
<td>-0.009±0.017</td>
</tr>
</tbody>
</table>

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Fig. 4.3 Plot of the angular distribution of the analyzing powers for the $^3$He($d,\gamma$)$^5$Li reaction at $E_d = 8.6$ MeV. The errors represent the statistical uncertainty of the measurement.
Table 4.2 The results of an angular distribution measurement of the analyzing powers for the $^3\text{He}(^3\text{He},\gamma)^5\text{Li}$ reaction at $E_d = 0.8$ MeV where the deuteron beam was stopped in the target. The errors represent the statistical uncertainty of the measurement.

<table>
<thead>
<tr>
<th>$\theta_{\text{c.m.}}$</th>
<th>$A_V(\theta)$</th>
<th>$A_{VV}(\theta)$</th>
<th>$T_{20}(\theta)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>30.252</td>
<td>0.052±0.052</td>
<td>0.188±0.069</td>
<td>-0.19±0.02</td>
</tr>
<tr>
<td>50.386</td>
<td>-0.018±0.019</td>
<td>0.174±0.026</td>
<td>-0.05±0.02</td>
</tr>
<tr>
<td>70.473</td>
<td>0.064±0.026</td>
<td>0.201±0.035</td>
<td>0.04±0.03</td>
</tr>
<tr>
<td>90.502</td>
<td>0.011±0.037</td>
<td>0.082±0.049</td>
<td>0.10±0.02</td>
</tr>
<tr>
<td>110.471</td>
<td>0.004±0.029</td>
<td>0.172±0.039</td>
<td></td>
</tr>
<tr>
<td>120.434</td>
<td></td>
<td></td>
<td>0.00±0.04</td>
</tr>
<tr>
<td>130.384</td>
<td>0.012±0.026</td>
<td>0.158±0.034</td>
<td></td>
</tr>
<tr>
<td>150.250</td>
<td>0.114±0.054</td>
<td>0.043±0.073</td>
<td>-0.20±0.03</td>
</tr>
</tbody>
</table>
Fig. 4.4 Plot of the angular distribution of the analyzing powers $A_y(\theta)$ and $A_{yy}(\theta)$ for the $^3\text{He}(\vec{d},\gamma)^5\text{Li}$ reaction at $E_d = 0.8$ MeV where the deuteron beam was stopped in the target. The errors represent the statistical uncertainty of the measurement.
Fig. 4.5 Plot of the angular distribution of the analyzing power $T_{20}(\theta)$ for the $^3\text{He}(d,\gamma)^5\text{Li}$ reaction at $E_d = 0.8$ MeV where the deuteron beam was stopped in the target. The errors represent the statistical uncertainty of the measurement.
Table 4.3 The results of an angular distribution measurement of the analyzing powers for the $^3\text{He}(d,\gamma)^5\text{Li}$ reaction at $E_d = 0.6$ MeV where the deuteron beam lost 0.3 MeV in the target. The errors represent the statistical uncertainty of the measurement.

<table>
<thead>
<tr>
<th>$\theta_{\text{c.m.}}$</th>
<th>$A_y(\theta)$</th>
<th>$A_{yy}(\theta)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>35.289</td>
<td>-0.028±0.016</td>
<td>0.194±0.015</td>
</tr>
<tr>
<td>50.386</td>
<td>-0.017±0.022</td>
<td>0.218±0.020</td>
</tr>
<tr>
<td>70.473</td>
<td>-0.018±0.019</td>
<td>0.241±0.017</td>
</tr>
<tr>
<td>90.502</td>
<td>-0.047±0.023</td>
<td>0.223±0.021</td>
</tr>
<tr>
<td>110.471</td>
<td>0.000±0.019</td>
<td>0.222±0.017</td>
</tr>
<tr>
<td>130.384</td>
<td>0.020±0.019</td>
<td>0.169±0.018</td>
</tr>
<tr>
<td>145.287</td>
<td>0.033±0.020</td>
<td>0.212±0.019</td>
</tr>
</tbody>
</table>
Fig. 4.6 Plot of the angular distribution of the analyzing powers $A_y(\theta)$ and $A_{yy}(\theta)$ for the $^3\text{He}(d,\gamma)^5\text{Li}$ reaction at $E_d = 0.6$ MeV where the deuteron beam lost 0.3 MeV in the target. The errors represent the statistical uncertainty of the measurement.
Chapter 5: Legendre and Transition Matrix Element Analyses

5.1 Introduction

The analysis of the angular distributions of the cross section and analyzing powers proceeds in two steps. Initially, the observables are fitted with Legendre or associated Legendre polynomials. As well as yielding the Legendre coefficients, these fits provide two more very useful pieces of information. First, the sum of polynomials terminates at twice the highest multipole of radiation involved in the reaction. Second, interference between different multipoles of radiation creates asymmetries in the observables about 90° which are reflected in the odd coefficients of the fits.

The second method of analysis, which offers more insight than the Legendre fitting, is the transition matrix element (TME) analysis which uses the formalism of [Sey79]. In this method, the cross section and analyzing powers are expressed in terms of irreducible transition matrix elements which contain the dynamical physics involved in the reaction. The values of the TME's are extracted by fitting them directly to all of the data at one energy. The advantage of this method is that all observables can be fit simultaneously, thereby increasing the sensitivity of the analysis. However, since the number of possible TME's involved in a reaction generally exceeds the number of data points being fit, one must be careful to include in the fits all TME's which have a significant effect while keeping the total number of free parameters of the fit reasonably small.

5.2 Allowed Transitions

The decomposition of the incident $d+^3$He scattering wave function is expressed in the LS coupling scheme [Sey79]. For the reaction $a(x,\lambda)e$ (in this case $^3$He($d,\gamma)^5$Li), the angular momenta of the system are defined as follows:
a = the spin of the target,

x = the spin of the projectile,

L = the orbital angular momentum of the projectile,

b = the spin of the gamma emitting state,

\lambda = the multipolarity of the \( \gamma \)-ray,

c = the spin of the residual nucleus

and \( p = \) the mode of the \( \gamma \)-ray (1 = electric, 0 = magnetic).

Then in the LS coupling scheme:

\[ x + a = S, \quad [5.1a] \]

\[ L + s = b, \quad [5.1b] \]

\[ \lambda + c = b. \quad [5.1c] \]

For the \( ^3\text{He}(d,\gamma)^5\text{Li} \) reaction, \( a = 1/2 \) and \( x = 1 \).

Continuum states of \( J^\pi = 1/2^+, 3/2^+ \), and \( 5/2^+ \) can radiate to the \( J^\pi = 3/2^- \) ground state of \( ^5\text{Li} \) via E1 radiation which changes parity and transfers 1 unit of angular momentum.

In the continuum, the intrinsic spin of the deuteron (1\(^+\)) couples with the intrinsic spin of \( ^3\text{He} \) (1/2\(^+\)) to a total channel spin of \( S = 1/2^+ \) or 3/2\(^+\). Table 5.1 lists the channel spin \( S \), orbital angular momentum \( L \), and total angular momentum \( J = L + S \) for the continuum waves which can decay to the ground state of \( ^5\text{Li} \) via the E1, M1, and E2 operators.
Table 5.1 Some coupling schemes for L, S, and J quantum numbers which can couple to the $J^\pi = 3/2^-$ ground state of $^5\text{Li}$ via E1, M1, and E2 radiation.

<table>
<thead>
<tr>
<th>1</th>
<th>S</th>
<th>J</th>
<th>$2S+1L_J$</th>
<th>Eλ, Mλ</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1/2</td>
<td>1/2</td>
<td>$^2s_{1/2}$</td>
<td>E1</td>
</tr>
<tr>
<td>0</td>
<td>3/2</td>
<td>3/2</td>
<td>$^4s_{3/2}$</td>
<td>E1</td>
</tr>
<tr>
<td>1</td>
<td>1/2</td>
<td>1/2</td>
<td>$^2p_{1/2}$</td>
<td>M1, E2</td>
</tr>
<tr>
<td>1</td>
<td>1/2</td>
<td>3/2</td>
<td>$^2p_{3/2}$</td>
<td>M1, E2</td>
</tr>
<tr>
<td>1</td>
<td>3/2</td>
<td>1/2</td>
<td>$^4p_{1/2}$</td>
<td>M1, E2</td>
</tr>
<tr>
<td>1</td>
<td>3/2</td>
<td>3/2</td>
<td>$^4p_{3/2}$</td>
<td>M1, E2</td>
</tr>
<tr>
<td>1</td>
<td>3/2</td>
<td>5/2</td>
<td>$^4p_{5/2}$</td>
<td>M1, E2</td>
</tr>
<tr>
<td>2</td>
<td>1/2</td>
<td>3/2</td>
<td>$^2d_{3/2}$</td>
<td>E1</td>
</tr>
<tr>
<td>2</td>
<td>1/2</td>
<td>5/2</td>
<td>$^2d_{5/2}$</td>
<td>E1</td>
</tr>
<tr>
<td>2</td>
<td>3/2</td>
<td>1/2</td>
<td>$^4d_{1/2}$</td>
<td>E1</td>
</tr>
<tr>
<td>2</td>
<td>3/2</td>
<td>3/2</td>
<td>$^4d_{3/2}$</td>
<td>E1</td>
</tr>
<tr>
<td>2</td>
<td>3/2</td>
<td>5/2</td>
<td>$^4d_{5/2}$</td>
<td>E1</td>
</tr>
</tbody>
</table>
5.3 The Legendre Polynomial Expansions

5.3.1 Definitions of the Legendre Polynomial Expansions

According to [Sey79], for a tensor-polarized beam the associated Legendre polynomial coefficients $b_k$, $c_k$, $d_k$, and $e_k$ are defined by the equation,

$$\sigma(\theta, \phi) = \sum_k A_k P_k + B_k P_y^1 \text{Re}(t_{20}) + C_k P_k t_{20} + D_k P_k^1 \text{Re}(t_{21}) + E_k P_k^2 \text{Re}(t_{22}).$$  \[5.2\]

The unitless, lower-case coefficients are given by dividing the above coefficients by $A_0$, e.g. $\tilde{A}_k = A_k / A_0$. By comparison with

$$\sigma(\theta, \phi) = \sigma_u(\theta)(1 + \frac{3}{2}p_y A_y(\theta) + t_{20} T_{20}(\theta) + 2\text{Re}(t_{21}) T_{21}(\theta) + 2\text{Re}(t_{22}) T_{22}(\theta)),$$  \[5.3\]

it is found that:

$$\sigma_u(\theta) = \sum_k A_k Q_k P_k(\cos\theta),$$  \[5.4\]

$$\sigma_u(\theta) A_y(\theta) = \frac{2}{3} \sum_k B_k Q_k P_y^1(\cos\theta),$$  \[5.5\]

$$\sigma_u(\theta) T_{20}(\theta) = \sum_k C_k Q_k P_k(\cos\theta),$$  \[5.6\]

$$\sigma_u(\theta) T_{21}(\theta) = \frac{1}{2} \sum_k D_k Q_k P_k(\cos\theta),$$  \[5.7\]

and

$$\sigma_u(\theta) T_{22}(\theta) = \frac{1}{2} \sum_k E_k Q_k P_k^2(\cos\theta),$$  \[5.8\]

where the $Q_k$ terms account for the finite geometry of the detector. The tensor analyzing power $A_{yy}(\theta)$ can be expressed in terms of $T_{20}(\theta)$ and $T_{22}(\theta)$ as:

$$A_{yy}(\theta) = -\frac{1}{\sqrt{2}} T_{20}(\theta) + \sqrt{3} T_{22}(\theta).$$  \[5.9\]

In the experiments with $E_d=8.6$ MeV and $E_q=0.6-0.3$ MeV, $A_{yy}(\theta)$ has been measured but $T_{20}(\theta)$ and $T_{22}(\theta)$ have not. Unfortunately, the Legendre and second associated Legendre polynomials do not form a linearly-independent system, making it impossible to get a unique solution from a fit to Eqn. 5.8, with Eqns. 5.6 and 5.7 substituted for $T_{20}(\theta)$.
and $T_{22}(\theta)$ respectively. Therefore, $\sigma_u(\theta)A_{yy}(\theta)$ was simply fitted to an expansion in Legendre polynomials of the form:

$$\sigma_u(\theta)A_{yy}(\theta) = \sum_k C_k Q_k P_k(\cos\theta).$$ [5.10]

5.3.2 Legendre fits at $E_d = 0.8-0.0$ MeV

Fits to four observables ($\sigma_u(\theta)$, $\sigma_u(\theta)A_Y(\theta)$, $\sigma_u(\theta)T_{20}(\theta)$, and $\sigma_u(\theta)T_{22}(\theta)$) have been performed. The results are stated in Table 5.2. The finite odd-$k$ coefficients indicate E1-M1 and/or E1-E2 interference. The small $k=3$ and 4 coefficients suggest that the contributions from E2 radiation are small. Previous angular distributions of the cross section in the fusion resonance region (see Chapter 1) have indicated that the cross section is isotropic to within 10%. The present cross section has been determined to be consistent with isotropy to within 5% in a Legendre fit to $A_0$ only.

5.3.3 Legendre fits at $E_d = 0.6-0.3$ MeV

Fits to three observables ($\sigma_u(\theta)$, $\sigma_u(\theta)A_Y(\theta)$, and $\sigma_u(\theta)A_{yy}(\theta)$) have been performed. The results are stated in Table 5.2. Again, the finite odd-$k$ coefficients indicate E1-M1 and/or E1-E2 interference. As with the $E_d = 0.8$-MeV cross section, the present cross section has been determined to be consistent with isotropy to within 5% in a Legendre fit to $A_0$ only.

5.3.4 Legendre fits at $E_d = 8.6$ MeV

Fits to three observables ($\sigma_u(\theta)$, $\sigma_u(\theta)A_Y(\theta)$, and $\sigma_u(\theta)A_{yy}(\theta)$) have been performed. The results are stated in Table 5.2. The finite odd-$k$ coefficients indicate E1-M1 and/or E1-E2 interference with the non-negligible $k=3$ and 4 coefficients suggesting important contributions from E1-E2 interference.
Table 5.2 The results of the Legendre polynomial fits at \(E_d = 0.8-0.0\, \text{MeV},\)
\(E_d = 0.6-0.3\, \text{MeV},\) and \(E_d = 8.6\, \text{MeV}.

<table>
<thead>
<tr>
<th>(A_0 (\mu\text{b/sr}))</th>
<th>0.8-0.0, \text{MeV}</th>
<th>0.6-0.3, \text{MeV}</th>
<th>8.6, \text{MeV}</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a_1)</td>
<td>-0.033 ± 0.018</td>
<td>0.013 ± 0.027</td>
<td>-0.138 ± 0.013</td>
</tr>
<tr>
<td>(a_2)</td>
<td>-0.056 ± 0.024</td>
<td>-0.007 ± 0.048</td>
<td>-0.203 ± 0.017</td>
</tr>
<tr>
<td>(a_3)</td>
<td>0.004 ± 0.022</td>
<td>0.053 ± 0.030</td>
<td>-0.023 ± 0.016</td>
</tr>
<tr>
<td>(a_4)</td>
<td>0.010 ± 0.030</td>
<td>-0.029 ± 0.070</td>
<td>0.010 ± 0.034</td>
</tr>
<tr>
<td>(b_1)</td>
<td>0.029 ± 0.022</td>
<td>-0.022 ± 0.014</td>
<td>-0.015 ± 0.009</td>
</tr>
<tr>
<td>(b_2)</td>
<td>-0.015 ± 0.013</td>
<td>-0.022 ± 0.009</td>
<td>-0.020 ± 0.007</td>
</tr>
<tr>
<td>(b_3)</td>
<td>-0.002 ± 0.015</td>
<td>0.011 ± 0.008</td>
<td>0.007 ± 0.005</td>
</tr>
<tr>
<td>(b_4)</td>
<td>-0.025 ± 0.014</td>
<td>-0.007 ± 0.007</td>
<td>-0.006 ± 0.005</td>
</tr>
<tr>
<td>(c_0)</td>
<td>-0.029 ± 0.012</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(c_1)</td>
<td>0.008 ± 0.020</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(c_2)</td>
<td>-0.262 ± 0.026</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(c_3)</td>
<td>0.015 ± 0.041</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(c_4)</td>
<td>-0.008 ± 0.046</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(c_0')</td>
<td></td>
<td>0.216 ± 0.009</td>
<td>0.025 ± 0.009</td>
</tr>
<tr>
<td>(c_1')</td>
<td></td>
<td>0.006 ± 0.013</td>
<td>0.002 ± 0.012</td>
</tr>
<tr>
<td>(c_2')</td>
<td></td>
<td>-0.022 ± 0.025</td>
<td>-0.068 ± 0.023</td>
</tr>
<tr>
<td>(c_3')</td>
<td></td>
<td>-0.023 ± 0.026</td>
<td>-0.039 ± 0.032</td>
</tr>
<tr>
<td>(c_4')</td>
<td></td>
<td>0.033 ± 0.041</td>
<td>-0.034 ± 0.034</td>
</tr>
<tr>
<td>(c_2)</td>
<td>-0.045 ± 0.020</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(c_3)</td>
<td>-0.027 ± 0.011</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(c_4)</td>
<td>0.009 ± 0.005</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
5.4 The Transition Matrix Element Analysis

5.4.1 Introduction

The formalism of [Sey79] can be used to express the Legendre polynomial coefficients in terms of the reduced transition matrix elements (TME's), where the TME's are labelled by their continuum state quantum numbers \( ^{2S+1}L_J \) and the multipolarity \( (E \lambda \text{ or } M \lambda) \) of the outgoing \( \gamma \) ray. The relevant TME's can then be fit to all of the data simultaneously, giving a fairly model-independent interpretation of the reaction. In order to maintain a reasonable number of free parameters in the fits, the \( J \) dependence of the transition matrix elements has been suppressed in these relationships. Of course, for s-wave capture this assumption is always true because there is only one possible \( J \) value for a given \( S \). Appendix D contains the expansion of the Legendre polynomials in terms of the transition matrix elements under this no-\( J \)-dependence assumption.

5.4.2 Transition Matrix Element fits at \( E_d = 0.8-0 \) MeV

With the low-energy deuterons of this experiment, s-wave capture is expected to dominate the reaction. Considering s-wave capture only, the expressions for the observables in terms of the TME's are quite simple:

\[
\sigma(\theta) = 2 \left| s_{1/2}(E1) \right|^2 + 4 \left| s_{3/2}(E1) \right|^2,
\]

\[
A_y(\theta) = 0,
\]

and
\[
A_{yy}(\theta) = \frac{0.632 \left| s_{1/2}(E1) \right| \left| s_{3/2}(E1) \right| \cos(\delta) + 0.800 \left| s_{3/2}(E1) \right|^2}{2 \left| s_{1/2}(E1) \right|^2 + 4 \left| s_{3/2}(E1) \right|^2}.
\]

where \( \delta \) is the relative phase between the two transition matrix elements. In the \( 3/2^+ \) fusion resonance region, it is expected that the \( s_{3/2}(E1) \) transition matrix element will dominate the reaction. For pure \( s_{3/2}(E1) \) capture, \( A_{yy} = 0.2 \).
The results of a simultaneous fit to all the data are given in Table 5.3 and shown in Figs. 5.1 and 5.2. It was necessary to introduce a small amount of M1 radiation in order to fit the asymmetry of the data. Two statistically significant solutions with nearly-identical total \( \chi^2 = 25 \) have been found, one with dominant \( S=3/2 \) s-wave capture strength, the second with nearly equal strengths for the \( S=1/2 \) and \( S=3/2 \) s-wave transition matrix elements. (Note: upon re-analysis, a second solution has also been found for the \( ^3\text{H}(\vec{d},\gamma)^5\text{He} \) data of [Ril89] which also has nearly equal \( S=1/2 \) and \( S=3/2 \) s-wave capture strengths.) Unfortunately, from the analysis alone, it is impossible to choose one solution over the other.

Table 5.3 Transition matrix element relative contributions to the cross section from a TME fit to the cross section and analyzing powers of the \( ^3\text{He}(\vec{d},\gamma)^5\text{Li} \) reaction for an 0.8 MeV beam which was stopped in the target. Two solutions with nearly identical \( \chi^2 = 25 \) have been found. The phase of the \( ^4s_{3/2} \) (E1) matrix element has been arbitrarily set to 0.0.

<table>
<thead>
<tr>
<th>TME</th>
<th>Solution 1</th>
<th></th>
<th>Solution 2</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Intensity</td>
<td>Phase</td>
<td>Intensity</td>
<td>Phase</td>
</tr>
<tr>
<td>( ^2s_{1/2} ) (E1)</td>
<td>0.03±0.03</td>
<td>260°±3°</td>
<td>0.48±0.06</td>
<td>145°±3°</td>
</tr>
<tr>
<td>( ^4s_{3/2} ) (E1)</td>
<td>0.89±0.03</td>
<td>0°</td>
<td>0.44±0.05</td>
<td>0°</td>
</tr>
<tr>
<td>( ^2p ) (M1)</td>
<td>0.01±0.01</td>
<td>69°±2°</td>
<td>0.01±0.01</td>
<td>66°±4°</td>
</tr>
<tr>
<td>( ^4p ) (M1)</td>
<td>0.07±0.02</td>
<td>173°±2°</td>
<td>0.07±0.02</td>
<td>-10°±2°</td>
</tr>
</tbody>
</table>

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Fig. 5.1 $\sigma(\theta)$ and $A_y(\theta)$ from the $^3\text{He}(d,\gamma)^5\text{Li}$ reaction at $E_d=0.80$ MeV are shown along with the results of the two TME solutions. The solid line represents Solution 1 of Table 5.3. The dashed line (which is almost entirely underneath the solid line) represents Solution 2.
Fig. 5.2 $A_{yy}(\theta)$ and $T_{20}(\theta)$ from the $^3\text{He}(d,\gamma)^5\text{Li}$ reaction at $E_d = 0.8-0$ MeV are shown along with the results of the two TME solutions. The solid line represents Solutions 1 and 2 of Table 5.3.
5.4.3 Transition Matrix Element fits at $E_d = 0.6$-$0.3$ MeV

In order to understand more fully the physics of the $^3\text{He}(d,\gamma)^5\text{Li}$ reaction in the fusion resonance region, angular distributions of $\sigma(\theta)$, $A_y(\theta)$, and $A_{yy}(\theta)$ integrated over the FWHM of this resonance were measured. A transition matrix element analysis which uses the same choices of TME's as the $E_d=0.8$ MeV work finds, rather than the two previous solutions, one unique solution with a $\chi^2=19$ whose errors nearly span both previous solutions. If, instead of using M1 radiation to account for the asymmetry in the data, E2 radiation is introduced into the fits then the set of two solutions is recovered. These fits do not have quite as good a $\chi^2 (=25)$ although they are acceptable from a statistical point of view. The results of these fits are given in Table 5.4 and plotted in Fig. 5.3 and 5.4.
Table 5.4 Transition matrix element relative contributions to the cross section from a TME fit to the cross section and analyzing powers of the $^3$He($^3$He,$\gamma$)$^5$Li reaction for an 0.6 MeV deuteron beam which lost 0.3 MeV in the target. Three statistically acceptable solutions have been found. Note that for Solution 1, M1 radiation is included, whereas for Solutions 2 and 3, E2 radiation is included. In all cases, the phase of the $^4s_{3/2}$ (E1) matrix element has been arbitrarily set to 0.0.

<table>
<thead>
<tr>
<th>TME</th>
<th>Solution 1</th>
<th>Solution 2</th>
<th>Solution 3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\chi^2=19.5$</td>
<td>$\chi^2=25.5$</td>
<td>$\chi^2=25.2$</td>
</tr>
<tr>
<td></td>
<td>Intensity</td>
<td>Phase</td>
<td>Intensity</td>
</tr>
<tr>
<td>$^2s_{1/2}$ (E1)</td>
<td>0.19±0.17</td>
<td>175°±6°</td>
<td>0.01±0.07</td>
</tr>
<tr>
<td>$^4s_{3/2}$ (E1)</td>
<td>0.67±0.15</td>
<td>0°</td>
<td>0.99±0.08</td>
</tr>
<tr>
<td>$^2p$ (M1)</td>
<td>0.10±0.03</td>
<td>89°±5°</td>
<td>0.00±0.01</td>
</tr>
<tr>
<td>$^4p$ (M1)</td>
<td>0.04±0.02</td>
<td>-24°±9°</td>
<td>0.00±0.01</td>
</tr>
</tbody>
</table>
Fig. 5.3 $\sigma(\theta)$ from the $^3\text{He}(d,\gamma)^5\text{Li}$ reaction at $E_d = 0.6-0.3$ MeV is shown along with the results of the three TME solutions. The solid line represents Solution 1 of Table 5.4. The dashed-dotted line represents Solution 2, and the dashed line represents Solution 3.
\( ^3\text{He}(d,\gamma)^5\text{Li} \quad E_d = 0.60 \text{ MeV} \)

![Graph showing experimental points and curves for \( A_y(\theta) \) and \( A_{yy}(\theta) \).]

Fig. 5.4 \( A_y(\theta) \) and \( A_{yy}(\theta) \) from the \( ^3\text{He}(d,\gamma)^5\text{Li} \) reaction at \( E_d = 0.6-0.3 \text{ MeV} \) is shown along with the results of the three TME solutions. The solid line represents Solution 1 of Table 5.4. The dashed-dotted line represents Solution 2, and the dashed line represents Solution 3.
5.4.4 Transition Matrix Element fits at $E_d = 8.6$ MeV

At $E_d = 8.6$ MeV, only one solution (with $\chi^2 = 7$) was found from a fit to the cross section and analyzing powers $A_y(\theta)$ and $A_{yy}(\theta)$. The fit indicates that the capture cross-section strength is dominated by s-wave E1 capture with an admixture of $\sim 20\%$ d-wave E1 capture. The results of a simultaneous fit to all the data are given in Table 5.5 and shown in Figs. 5.5 and 5.6.

Table 5.5 Transition matrix element relative intensities from a TME fit to the cross section and analyzing powers of the $^3\text{He}(^3\text{He},\gamma)^5\text{Li}$ reaction at $E_d = 8.6$ MeV. Only one statistically significant solution (with $\chi^2 = 7$) has been found. The phase of the $^4s_{3/2}$ (E1) matrix element has been arbitrarily set to 0.0.

<table>
<thead>
<tr>
<th>TME</th>
<th>Intensity</th>
<th>Phase</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^2s_{1/2}$ (E1)</td>
<td>0.15±0.02</td>
<td>53°± 5°</td>
</tr>
<tr>
<td>$^4s_{3/2}$ (E1)</td>
<td>0.65±0.03</td>
<td>0°</td>
</tr>
<tr>
<td>$^2p$ (E2)</td>
<td>0.00±0.00</td>
<td>133°±6°</td>
</tr>
<tr>
<td>$^4p$ (E2)</td>
<td>0.00±0.00</td>
<td>179°±13°</td>
</tr>
<tr>
<td>$^2d$ (E1)</td>
<td>0.18±0.02</td>
<td>199°±3°</td>
</tr>
<tr>
<td>$^4d$ (E1)</td>
<td>0.02±0.01</td>
<td>72°±4°</td>
</tr>
</tbody>
</table>
Fig. 5.5 $\sigma(\theta)$ from the $^3\text{He}(d,\gamma)^5\text{Li}$ reaction at $E_d = 8.6 \text{ MeV}$ is shown along with the results of the TME solution of Table 5.5.
Fig. 5.6 $A_y(\theta)$ and $A_{yy}(\theta)$ from the $^3\text{He}(d,\gamma)^5\text{Li}$ reaction at $E_d = 8.6$ MeV are shown along with the results of the TME analysis. The solid lines represent the solution of Table 5.5.
5.4.5 Discussion

It should be noted that the results of all of the TME analyses, both in the fusion-resonance region and at \( E_d = 8.6 \) MeV, indicate that there are TME's with large intensities which arise from \( S=3/2 \) capture to the ground state via E1 radiation. As discussed previously, the dominant term of the E1 operator has a \( \Delta S=0 \) selection rule, meaning that it does not change the channel spin in the transition from the scattering state to the ground state. The most obvious mechanisms for an E1 \( \gamma \)-ray transition to the \( 3/2^- \) ground state of \( ^5\text{Li} \) is that the \( S=3/2 \) continuum state decays to the \( S=3/2 \) component of the ground state. However, in the single-particle model, the \( ^5\text{Li} \) ground state has \( S=1/2 \) since it consists predominantly of a proton in the \( p_{3/2} \) orbital above a closed \( s_{1/2} \) \( ^4\text{He} \) core (in LS coupling \( L=1, S=1/2 \)). This suggests that capture to the \( S=3/2 \) component of the ground state is a small effect unless the tensor force acts to produce a larger-than-expected \( S=3/2 \) component. Another mechanism which can cause large \( S=3/2 \) capture strengths is the mixing of the \( S=3/2 \) and \( S=1/2 \) scattering states by the tensor force, allowing the transition to the \( S=1/2 \) component of the ground state. In order to investigate this further, it is necessary to employ some nuclear model calculations. The theory and results from a Direct-Capture Model and a Multi-Channel Resonating Group Model will be discussed in the next two chapters.
Chapter 6: The Direct Capture Calculation at \( E_d = 0.45 \) MeV

Introduction

Our first effort to investigate the \(^3\text{He}(\vec{d},\gamma)^5\text{Li}\) reaction within a theoretical framework utilized the Direct Capture (DC) model. In the DC model, this reaction is a one-step process in which the scattering states and the ground state are described by a point-deuteron cluster coupled to a point-\(^3\text{He}\) cluster. Transition matrix elements (TME’s) for E1 transitions from the \(^2s_{1/2}\) and \(^4s_{3/2}\) continuum states to the P\(_{3/2}\) ground state of \(^5\text{Li}\) were calculated in the fusion resonance region. Since only the non-spin-flip E1 operator is considered, the S (=1/2 or 3/2) of the ground state must be the same as that of the scattering state in the E1-only calculation. In other words, the \(^2s_{1/2}\) scattering state captures into the \(^2\text{P}_{3/2}\) piece of the ground-state wave function. Likewise, the \(^4s_{3/2}\) scattering state captures into the \(^4\text{P}_{3/2}\) piece of the ground-state wave function. Therefore, in this model, the relative strengths of the experimentally determined transition matrix elements are a direct measure of the admixture of S=1/2 and S=3/2 in the ground state.

Wave Functions

The ground-state wave function is generated from an optical model potential of the form:

\[
V(r) = V_c(r) + V_n(r)
\]

where \(V_c(r)\) is the coulomb potential. \(V_n(r)\) is a real central potential with the Woods-Saxon form:

\[
V_n(r) = \frac{V_n}{1 + e^{(r-r_0)/a_0}}
\]

\(V_n\) is adjusted such that the calculated binding energy of \(^5\text{Li}\) agrees with experimental measurements. Although the \(^5\text{Li}\) nucleus is unbound in the p+\(\alpha\) channel, it is bound by
16.388 MeV in the d+³He channel which is the only channel included in this DC model. Since no optical model parameters at the fusion resonance energy were available, the optical model parameters $r_0$ and $a_0$ were taken to have average values as given in [Per76]. The values of the $V_n$, $r_0$, and $a_0$ which were used are:

$$V_n = -93.04 \text{ MeV}, \quad r_0 = 1.25 \text{ fm}, \quad a_0 = 0.65 \text{ fm}$$

The scattering-state wave function is described by a distorted wave which is derived from the same real optical model as the ground-state wave function.

As discussed in Chapter 1 and Section 5.4.5, the fact that the fusion resonance is seen in the γ-ray channel can be accounted for if a large admixture of the ground state of $^5\text{Li}$ is in an $S=3/2$ state. This is necessary since the $\Delta S=0$ selection rule for the dominant term of the E1 operator will only allow transitions from the $J^\pi=3/2^+$ fusion resonance (which has $L=0$, $S=3/2$) to the $S=3/2$ admixture of the ground state. The ground-state wave function can be written as:

$$\Psi_{\text{g.s.}} = \rho \mid S=1/2\rangle + \sqrt{1-\rho^2} \mid S=3/2\rangle,$$

and calculated as a function of $\rho$.

**Results**

Transition matrix elements were calculated for several values of $\rho$ using the above wave functions and the Siegert E1 operator. The Legendre coefficients $\tilde{a}_2$, $\tilde{b}_2$, $\tilde{c}_0$, $\tilde{c}_2$, and $\tilde{e}_2$ (see Section 5.3.1) were then computed as a function of $\rho$. It was found that only $\tilde{b}_2$, $\tilde{c}_2$, and $\tilde{e}_2$ are sensitive to the spin admixture in the ground state. Fig. 6.1 plots these coefficients as a function of $\rho$. Also plotted are the values of these coefficients which were extracted in a Legendre fit to the data taken at $E_d=0.8-0.0$ MeV. Together the DC model and the Legendre fits indicate that the fraction of $S=3/2$ in the ground state (denoted by $1-\rho^2$) of $^5\text{Li}$ is between 0.84 and 0.98.
Fig. 6.1 The Legendre coefficients $b_2$, $c_2$, and $e_2$. $1 - \rho^2$ represents the fraction of $S=3/2$ in the ground state of $^5\text{Li}$. The curves are from a Direct Capture calculation. The solid horizontal lines are the values of the coefficients obtained from the Legendre fits to the data taken at $E_0=0.8-0.0$ MeV. The dashed lines represent the errors on these fits.
Limitations of the model

The DC model prediction of > 80% $S=3/2$ in the ground-state wave function appears
to be unphysical. Certainly from a simple shell-model picture, the ground state of $^5\text{Li}$
should be predominantly $S=1/2$ since it consists of a proton in the $p_{3/2}$ shell above a
closed $\alpha$ core. So what has gone wrong in this model?

The first shortfall is in the application of the model. In a resonance region, direct
capture is not necessarily the dominant capture process. This poses a large problem if one
is trying to calculate the absolute cross section [Bro64], however the analyzing powers
and the dimensionless Legendre coefficients are not necessarily affected.

The second, more fundamental problem is that the model is confined to considering
only the $d+^3\text{He}$ cluster configuration. Certainly the ground state, which decays to a
proton and an $\alpha$ particle within $10^{-21}$ seconds, must include the $p+\alpha$ configuration in its
wave function. The continuum wave functions must also include $p+\alpha$ channels as
evidenced by the large cross section for the $^3\text{He}(d,p)^4\text{He}$ reaction [Ajz88]. In order to
include this channel in the theoretical framework, the more sophisticated Resonating
Group Model is needed. Chapter 7 presents the results of this calculation. It will be
shown that the admixture of $S=3/2$ in the ground state is small and that the large capture
cross section is due to the coupling of the $[d+^3\text{He}]_{S=3/2}^{J=3/2}$ scattering channel to the
$[p+^4\text{He}]_{S=1/2}^{J=3/2}$ scattering channel by the tensor force.
Chapter 7: The Multichannel Resonating Group Model

7.1 Introduction

One of the myriad of problems encountered when modeling nuclear reactions is that of effectively manipulating the large number of degrees of freedom. The Resonating Group Model (RGM), which was first proposed by Wheeler [Whe37] accomplishes this by clustering nucleons into bound substructures which have no internal degrees of freedom. For example, $^5$Li can be described as an $\alpha$-particle coupled with a proton, or as a deuteron coupled with a $^3$He nucleus. The total wave function of the $^5$Li nucleus is then described as a linear combination of cluster wave functions multiplied by their respective relative-motion wave functions [Hof86]. These wave functions are totally antisymmetrized in order to adhere to the Pauli Principle and optimized according to the prescription of the variational principle. Thus, the calculations can be improved by expanding the model-space to include more clusters or more sophisticated internal cluster wave functions.

7.2 Description of the cluster wave functions

In the current RGM calculation, $^5$Li is described using three cluster wave functions and the proton. The three clusters are the deuteron, the nucleus of $^3$He, and the $\alpha$-particle. The proton is described simply as a point particle. The three clusters have a more complicated radial expression of the form:

$$\Psi(r) = C_1e^{-b_1r^2} + C_2e^{-b_2r^2}.$$  \[7.1\]

The $C_1$, $C_2$, $b_1$, and $b_2$ coefficients for the present case were obtained by H.M. Hofmann using the Ritz variational principle [Mer86] and therefore yield upper bounds on the binding energy of each cluster. The coefficients for the three cluster wave functions as well as the calculated and experimental binding energies of each cluster are listed in Table 7.1.
Table 7.1 The parameters for the three clusters used to describe $^5\text{Li}$ are given along with the binding energy of each cluster obtained with these parameters. The experimentally determined binding energy of each cluster is also given for comparison.

<table>
<thead>
<tr>
<th>cluster</th>
<th>$C_1$</th>
<th>$C_2$</th>
<th>$b_1$ (fm$^{-2}$)</th>
<th>$b_2$ (fm$^{-2}$)</th>
<th>B.E. (RGM) (MeV)</th>
<th>B.E. (Exp.) (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>deuteron</td>
<td>0.5320</td>
<td>0.1492</td>
<td>0.3204</td>
<td>0.0403</td>
<td>-1.35</td>
<td>-2.23</td>
</tr>
<tr>
<td>$^3\text{He}$</td>
<td>0.1428</td>
<td>0.0184</td>
<td>0.293</td>
<td>0.084</td>
<td>-3.98</td>
<td>-7.72</td>
</tr>
<tr>
<td>$\alpha$-particle</td>
<td>5.2018</td>
<td>0.0658</td>
<td>0.324</td>
<td>0.084</td>
<td>-21.43</td>
<td>-28.30</td>
</tr>
</tbody>
</table>

7.3 The ground-state wave function of $^5\text{Li}$

Now that the cluster wave functions have been described, they can be combined along with relative-motion wave functions to generate the total ground-state wave function of $^5\text{Li}$. In general, this wave function, as described by [Mer86], is:

$$\phi(\psi_J^i) = \phi \sum_{a\alpha \sigma} c_{a}^{L,S,J} (LmS\sigma|Jj) |\alpha Lm> |\alpha S\sigma>$$  \[7.2\]

where $L$ is the relative orbital angular momentum of the cluster pairs, $S$ is their total spin, and $J$ is their total angular momentum $L+S$. Their projections are denoted by $m$, $\sigma$, and $j$ respectively. All other quantum numbers and all of the different cluster decompositions are denoted by the symbol $\alpha$. The $c_{a}^{L,S,J}$ coefficients are the free parameters that are minimized in the variational calculation.

For the description of the $^5\text{Li}$ ground state, three cluster configurations have been used: $[p+^4\text{He}]^{S=1/2}$, $[d+^3\text{He}]^{S=1/2}$, and $[d+^3\text{He}]^{S=3/2}$, where the cluster pairs are in a relative P-state and their total angular momentum is $J=3/2$. The radial wave function for each cluster pair is then described by

$$|\alpha Lm> = (\Psi_1 \Psi_2) Y_{Lm} e^{-\gamma r^2},$$  \[7.3\]
where $\alpha$ denotes the cluster configuration, $L=1$, and the $\Psi_i$ represent the internal cluster wave functions from Eqn. 7.1. The coefficient $\gamma$ in the exponential determines the distance between the two clusters (for example, between the $\alpha$-particle and the proton). For a bound nucleus, the $\gamma$'s can also be used as free parameters in the variational calculation. However, the $^5$Li ground state is unbound in the $\alpha+p$ channel. If the distance between $\alpha$ and proton clusters is used as a free parameter in the variational calculation, the result will be an infinite separation between them. It was therefore necessary to fix the distance between the clusters to values which reproduced the binding energy of $^5$Li reasonably well. The calculated binding energy is -20.7476, which underestimates the experimental value of -26.331 MeV. This variational calculation was carried out using the phenomenological soft-core nucleon-nucleon potential of [Mer86].

The overlap integral of each cluster configuration with the total ground-state wave function is defined as:

$$\text{Overlap Integral} = \frac{\langle \alpha | ^5\text{Li} \rangle}{\sqrt{\langle \alpha | \alpha \rangle} \sqrt{\langle ^5\text{Li} | ^5\text{Li} \rangle}}$$

[7.4]

where $\langle \alpha |$ represents one of the three cluster-configuration wave functions and $\langle ^5\text{Li} |$ represents the total $^5\text{Li}$ ground-state wave function. Table 7.2 gives the values for $\gamma$, the coefficients $c^L_{\alpha}^S$ and the overlap integral for each cluster configuration. According to this calculation, the $^5\text{Li}$ nucleus is almost entirely in the $[^\alpha+p]^S=1/2$ configuration, which is consistent with the shell model description. The non-zero value for the $[^d+^3\text{He}]^S=1/2$ overlap is due to the non-orthogonality of the two $S=1/2$ configurations.
Table 7.2 For each of the three cluster configurations, the coefficients \( c^{L,S,J}_{\alpha} \) and the parameters \( \gamma \) of Eqns. 7.2 and 7.3 are listed. The values of the \( c^{L,S,J}_{\alpha} \) coefficients were obtained from a variational calculation which used the effective soft-core nucleon-nucleon potential of [Mer86]. Also given are the overlap integrals of each cluster-configuration wave function with the total ground-state wave function of \(^5\text{Li}\) as defined in Eqn. 7.4.

<table>
<thead>
<tr>
<th>Configuration</th>
<th>( c^{L,S,J}_{\alpha} )</th>
<th>( \gamma ) (fm(^{-2}))</th>
<th>overlap</th>
</tr>
</thead>
<tbody>
<tr>
<td>([p+^4\text{He}]_{S=1/2}^8)</td>
<td>(1.89 \times 10^{-5})</td>
<td>0.005</td>
<td>0.9996</td>
</tr>
<tr>
<td>([d+^3\text{He}]_{S=1/2}^8)</td>
<td>(-23.70 \times 10^{-5})</td>
<td>0.07</td>
<td>0.1424</td>
</tr>
<tr>
<td>([d+^3\text{He}]_{S=3/2}^8)</td>
<td>(1.98 \times 10^{-5})</td>
<td>0.07</td>
<td>0.0010</td>
</tr>
</tbody>
</table>

7.4 The scattering wave function

The scattering wave function is described by the same cluster configurations which were used to describe the ground state of \(^5\text{Li}\) and is constructed according to Eqn. 7.2, however now all values of \( J^\pi \) which are allowed by the angular-momentum coupling algebra are used. The scattering wave function includes an interaction region where all channels are open and a tail which reproduces the long-range Coulomb piece of the wave function. The interaction region is constructed as a sum of regular and irregular Coulomb waves and square-integrable functions of the form \( R^{L+1} e^{-\alpha R^2} \), where \( R \) is the relative-motion coordinate between the two clusters.

In order to simplify the calculation, the regular and irregular Coulomb waves are approximated by 20 square-integrable functions which are chosen in such a way that different partial waves (predominantly s, p, and d waves) can be reproduced by the 20 Gaussians over a wide range of energies. If the Gaussians are not carefully chosen, some terms may become linearly dependent within the numerical accuracy of the computer. Therefore, a trial-and-error procedure which was previously performed by H.M.
Hofmann [HofPri] started with a relatively small number of Gaussians and added more as needed. The 20 values of $\bar{\delta}$ (fm$^{-2}$) used are:

$$5.0,\ 1.7,\ 0.7,\ 0.35,\ 0.15,\ 0.07,\ 0.04,\ 0.025,\ 0.015,\ 0.01,\ 0.007,\ 0.005,\ 0.0035,\ 0.0025,\ 0.0015,\ 0.001,\ 0.0007,\ 0.0004,\ 0.0002,\ \text{and}\ 0.0001.$$ 

Accurately describing the long-range behavior of the scattering wave function is important in low-energy reactions which are less sensitive to the interior of the wave function and in radiative-capture reactions where the electric-dipole operator weights the scattering wave function by $r$ in the transition matrix element, thereby increasing the sensitivity to its long-range behavior.

7.5 The nuclear potential and phase shifts of the d+$^3$He and p+$\alpha$ reactions

The Resonating Group Model calculations performed for this experiment use the phenomenological local soft-core nucleon-nucleon potential of [Mer86] which is described by the same formalism as discussed in [Eik71]. This potential is derived from a realistic potential and transformed such that no D-state is necessary to bind the deuteron. The four states of the two-nucleon system have $S = 0$ or 1, and $T = 0$ or 1. Therefore, the two-nucleon potential is decomposed into four different potentials, with each operating on one of these four two-nucleon states. More specifically, the potential is written as:

$$V = V^{1+}p^{1+} + V^{1-}p^{1-} + V^{3+}p^{3+} + V^{3-}p^{3-}$$  \hspace{1cm} [7.5]

where the $P$'s are projection operators for the four states. In the potential of [Eik71] no singlet-odd term is used. The three potentials are given by:

$$V^{1+} = V^{1+}_C(r),$$  \hspace{1cm} [7.6a]

$$V^{3+} = V^{3+}_C(r) + V^{3+}_{LS}(r)(L\cdot S) + r^2V^{3+}_{T}(r)S_{12},$$  \hspace{1cm} [7.6b]

$$V^{3-} = V^{3-}_C(r) + V^{3-}_{LS}(r)(L\cdot S) + r^2V^{3-}_{T}(r)S_{12},$$  \hspace{1cm} [7.6c]
where $V_C(r)$, $V_{LS}(r)$, and $V_T(r)$ give the radial dependence of the central, spin-orbit, and tensor pieces of the potential, respectively. The terms in the potentials are defined by:

$$V_{C}^{1+}(r) = \sum_{i=1}^{N} A_{C}^{1+} \exp(-B_{C}^{1+} r^2),$$ \[7.7\]

$$\mathbf{L} \cdot \mathbf{S} = \frac{1}{2\hbar} \mathbf{r} \times \mathbf{p} \cdot (\mathbf{\sigma}_1 + \mathbf{\sigma}_2),$$ \[7.8\]

and $$S_{12} = \frac{3(\mathbf{\sigma}_1 \cdot \mathbf{r})(\mathbf{\sigma}_2 \cdot \mathbf{r})}{r^2} - \mathbf{\sigma}_1 \cdot \mathbf{\sigma}_2,$$ \[7.9\]

where the other radial functions $V(r)$ are expressed analogously to $V_{C}^{1+}(r)$. The parameters $A$ and $B$ of the potential are given in Table 7.3.

In order to determine how well this potential describes the $d+^3\text{He}$ reactions, phase shifts were calculated for the $d+^3\text{He}$ and $p+\alpha$ elastic scattering reactions. Only one phase shift analysis [Jen80] of the $d+^3\text{He}$ reaction exists. Unfortunately, multiple solutions were found. Therefore, it was necessary to compare the calculated $p+\alpha$ phase shifts to the experimentally determined phase shifts of [Hou78], [Pla72], and [Sch71]. Two phase shifts were found to be particularly sensitive to variations of the strengths of the spin-orbit and tensor forces. The $^2d_{3/2}$ phase shift clearly exhibits the $3/2^+$ fusion resonance. Although the energy of the fusion resonance is calculated to be too low (a general failing of most RGM models), the strength of the resonance was found to depend almost exclusively on the tensor force. By increasing the tensor force to 1.75 times its two-body value, agreement between the theory and experimental values was obtained. The spin-orbit force was also adjusted to 0.75 times its two-body value in order to reproduce the $^2d_{5/2}$ phase shift. Figs 7.1 through 7.3 plot the experimental $^4\text{He}(p,p)^4\text{He}$ phase shifts of [Hou78], [Pla72], and [Sch71] against three sets of RGM-calculated phase shifts. These three RGM calculations used the potential of [Mer86] with different
strengths of the spin-orbit and tensor forces, i.e. the two-body potential, the phase-shift adjusted strengths described above, and the two-body potential with the tensor-force strength set to 0. Figs. 7.4 through 7.6 plot the experimental $^3\text{He}(d,d)^3\text{He}$ phase shifts of [Jen80] against the three sets of RGM-calculated phase shifts.

Table 7.3 The coefficients of the phenomenological local soft-core nucleon-nucleon potential of [Mer86] are listed. The width parameters $B$ are given in units of fm$^{-2}$, and the strength parameters $A$ in units of MeV.

<table>
<thead>
<tr>
<th>$A_C^{1+,1}$</th>
<th>$A_C^{3+,1}$</th>
<th>$A_C^{1+,2}$</th>
<th>$A_C^{3+,2}$</th>
<th>$A_C^{1+,3}$</th>
<th>$A_C^{3+,3}$</th>
<th>$A_C^{1+,4}$</th>
<th>$A_C^{3+,4}$</th>
<th>$B_C^{1+,1}$</th>
<th>$B_C^{3+,1}$</th>
<th>$B_C^{1+,2}$</th>
<th>$B_C^{3+,2}$</th>
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<th>$B_C^{3+,3}$</th>
<th>$B_C^{1+,4}$</th>
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<tr>
<td>-692.60</td>
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<td>904.72</td>
<td>681.06</td>
<td>-77.031</td>
<td>-8.004</td>
<td>-692.60</td>
<td>-510.32</td>
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<td>4.8092</td>
<td>0.6409</td>
<td>7.1199</td>
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<td></td>
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<td></td>
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<tr>
<td>$A_{LS}^{3+,1}$</td>
<td>$A_{LS}^{3+,2}$</td>
<td>$B_{LS}^{3+,1}$</td>
<td>$B_{LS}^{3+,2}$</td>
<td>$B_{LS}^{3+,3}$</td>
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<tr>
<td>$A_T^{3+,1}$</td>
<td>$A_T^{3+,2}$</td>
<td>$A_T^{3+,3}$</td>
<td>$B_T^{3+,1}$</td>
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</tbody>
</table>

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Fig. 7.1 The experimental $^4\text{He}(p,p)^4\text{He}$ phase shifts of [Hou78], [Pla72], and [Sch71] are plotted against the three sets of RGM-calculated phase shifts as described in the text.
Fig. 7.2 More $^4\text{He}(p,p)^4\text{He}$ phase shifts from [Hou78], [Pla72], and [Sch71] and the three sets of RGM-calculated phase shifts as described in the text.
Fig 7.3 Still more $^4\text{He(p,p)}^4\text{He}$ phase shifts from [Hou78], [Pla72], and [Sch71] and the three sets of RGM-calculated phase shifts as described in the text.
Fig. 7.4 The $^3\text{He}(d,d)^3\text{He}$ phase shifts of [Jen80] are plotted against the three sets of RGM-calculated phase shifts as described in the text.
Fig. 7.5 More $^3$He(d, d)$^3$He phase shifts from [Ien80] are plotted against the three sets of RGM-calculated phase shifts as described in the text.
Fig. 7.6 Still more $^{3}\text{He}(d,d)^{3}\text{He}$ phase shifts from [Jen80] are plotted against the three sets of RGM-calculated phase shifts as described in the text.
7.6 The electromagnetic transition operators

A multipole expansion of the electromagnetic field in the long-wavelength limit yields the electromagnetic transition operators:

\[ O(E,L\mu) = \sum_{j=1}^{A} e g_{lj} \frac{e}{2mc} \frac{k}{L+1} L_{L}(r_{ij}^{L} Y_{L\mu}(r_{ij})) \cdot S_j \]  \[ \text{where } L = -i r \times \vec{V} \]  \[ O'(E,L\mu) = -\sum_{j=1}^{A} g_{lj} \frac{e}{2mc} \frac{1}{L+1} \nabla (r_{ij}^{L} Y_{L\mu}(r_{ij})) \cdot l_j. \]  \[ O(M,L\mu) = \sum_{j=1}^{A} 2 g_{lj} \frac{e}{2mc} \frac{1}{L+1} \nabla (r_{ij}^{L} Y_{L\mu}(r_{ij})) \cdot l_j. \]  \[ O'(M,L\mu) = \sum_{j=1}^{A} g_{lj} \frac{e}{2mc} \nabla (r_{ij}^{L} Y_{L\mu}(r_{ij})) \cdot S_j. \]

where \( L \) and \( \mu \) give the multipolarity of the transition and the electric and magnetic operators are labeled \( E \) and \( M \), respectively. The \( g \)-factors for free nucleons are \( g_l(\text{proton}) = 1, g_l(\text{neutron}) = 0 \), \( g_s(\text{proton}) = 5.586 \), and \( g_s(\text{neutron}) = -3.826 \). Eqn. 7.10 is derived using Siegert's theorem and contains most meson-exchange current effects. The other operators do not include these effects.

7.7 Transition Matrix Element calculation at \( E_d = 0.45 \text{ MeV} \)

Transition matrix elements were calculated at the \( 3/2^+ \) fusion resonance (see Tables 7.4 and 7.5) for E1, M1, and E2 transitions using both the two-body potential (Calculation A) and the phase-shift adjusted potential (Calculation B). As shown in Figs. 7.7 through 7.10, both calculations reproduce the measured analyzing powers of the \( E_d = 0.8-0.0 \text{ MeV} \) and \( E_d = 0.6-0.3 \text{ MeV} \) experiments. Although the calculations were performed at the peak of the fusion resonance, energy averaging over the resonance (in
accordance with the data) does not change the results substantially since almost identical results were obtained at other energies in the averaging region. Of course, the calculated on-resonance absolute cross section is larger than the energy-averaged experimental values. However, even compared to the measured on-resonance cross sections of [Bla54] and [Bus68], Calculation A still overestimates the cross section by a factor of 2 and 5, respectively. It is interesting to note that a factor of 2 reduction in the calculated cross section can be obtained by reducing the tensor force strength to 0.75 times its two-body value (Calculation C). This reduction does not lead to any appreciable discrepancies with the measured analyzing powers.

In calculations A and B, the $^4S_{3/2}$ (E1) transition matrix element dominates the capture cross section, contributing 98.5% and 99.8% of the cross section strength, respectively. The tensor force couples the $[d+^3He]^{S=3/2}_{J=3/2}$ scattering channel to the $[p+^4He]^{S=1/2}_{J=3/2}$ scattering channel, allowing the transition to the $S=1/2$ ground state to proceed via the dominant $\Delta S=0$ term of the E1 operator. To further examine the sensitivity of the calculation to the tensor force, a third calculation (Calculation D) was performed with the tensor force turned off. In this case (see Figs. 7.7 to 7.10), the tensor analyzing powers nearly vanish, the vector analyzing power becomes quite large, and the calculated cross section underestimates both measured cross sections by a factor of 16. These results differ from those of Calculations A and B due to the dramatic reduction in the strength of the $^4S_{3/2}$ (E1) transition matrix element which now contributes less than 1% of the calculated cross section strength. This clearly demonstrates the importance of the tensor force in driving the $d+^3He$ fusion reaction in the region of the $3/2^+$ resonance.
Table 7.4 Transition matrix element relative intensities from a TME fit to the \( E_d = 0.8 \) MeV data (two statistically significant solutions with \( \chi^2 = 25 \)) and the three RGM calculations. Calculation A uses the two-body potential. Calculation B uses the modified potential which has been adjusted to reproduce the \(^4\text{He}(p,p)^4\text{He} \) phase shifts. Calculation C uses the two-body potential with the tensor force strength set to 0.75 times its two-body value. Calculation D uses the two-body potential with no tensor force.

<table>
<thead>
<tr>
<th></th>
<th>TME fits</th>
<th>RGM calculations</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Solution 1</td>
<td>Solution 2</td>
</tr>
<tr>
<td>(^2s_{1/2}) (E1)</td>
<td>0.03±0.03</td>
<td>0.48±0.06</td>
</tr>
<tr>
<td>(^4s_{3/2}) (E1)</td>
<td>0.89±0.03</td>
<td>0.44±0.05</td>
</tr>
<tr>
<td>(^2p)</td>
<td>0.01±0.01</td>
<td>0.01±0.01</td>
</tr>
<tr>
<td>(^4p)</td>
<td>0.07±0.02</td>
<td>0.07±0.02</td>
</tr>
</tbody>
</table>
Table 7.5 Legendre polynomial coefficients from a direct fit to the $E_d=0.8$ MeV data, a TME fit to this data, and the RGM calculations. Calculation A uses the two-body potential. Calculation B uses the modified potential which has been adjusted to reproduce the $^4\text{He}(p,p)^4\text{He}$ phase shifts. Calculation C uses the two-body potential with the tensor force strength set to 0.75 times its two-body value. Calculation D uses the two-body potential with no tensor force. $A_0$ is given in units of $\mu$b/sr.

<table>
<thead>
<tr>
<th></th>
<th>Legendre fit</th>
<th>TME fit</th>
<th>RGM calculation</th>
<th></th>
</tr>
</thead>
<tbody>
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<td></td>
<td>$A_0$</td>
<td>1.855 ± 0.012</td>
<td>1.85 ± 0.13</td>
<td>A</td>
</tr>
<tr>
<td></td>
<td>$a_1$</td>
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<td>-0.035 ± 0.011</td>
<td>B</td>
</tr>
<tr>
<td></td>
<td>$a_2$</td>
<td>-0.061 ± 0.016</td>
<td>-0.065 ± 0.017</td>
<td>C</td>
</tr>
<tr>
<td></td>
<td>$b_1$</td>
<td>0.032 ± 0.022</td>
<td>0.031 ± 0.020</td>
<td>D</td>
</tr>
<tr>
<td></td>
<td>$b_2$</td>
<td>-0.009 ± 0.013</td>
<td>-0.010 ± 0.003</td>
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</tr>
<tr>
<td></td>
<td>$c_0$</td>
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<td>-0.001 ± 0.001</td>
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</tr>
<tr>
<td></td>
<td>$c_1$</td>
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<td>-0.020 ± 0.006</td>
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<td>$c_2$</td>
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<td>-0.242 ± 0.025</td>
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<td></td>
<td>$c_3$</td>
<td>-0.046 ± 0.006</td>
<td>-0.099 ± 0.010</td>
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</tbody>
</table>
Fig. 7.7 $\sigma(\theta)$ and $A_y(\theta)$ from the $^3\text{He}(d,\gamma)^5\text{Li}$ reaction at $E_d=0.8\text{-}0$ MeV are shown along with the results of the three RGM calculations. The solid curve is Calculation A (two-body forces), the dashed curve is Calculation B ($^4\text{He}(p,p)^4\text{He}$ phase shift adjusted forces), and the dashed-dotted curve is Calculation D which uses no tensor force.
Fig. 7.8 $A_{yy}(\theta)$ and $T_{20}(\theta)$ from the $^3\text{He}(\vec{d},\gamma)^5\text{Li}$ reaction at $E_r=0.8-0$ MeV are shown along with the results of the three RGM calculations. The solid curve is Calculation A (two-body forces), the dashed curve is Calculation B ($^4\text{He}(p,p)^4\text{He}$ phase shift adjusted forces), and the dashed-dotted curve is Calculation D which uses no tensor force.
Fig. 7.9 $\sigma(\theta)$ from the $^3\text{He}(\bar{d},\gamma)^5\text{Li}$ reaction at $E_d=0.6-0.3$ MeV is shown along with the results of the three RGM calculations. The solid curve is Calculation A (two-body forces), the dashed curve is Calculation B ($^4\text{He}(p,p)^4\text{He}$ phase shift adjusted forces), and the dashed-dotted curve is Calculation D which uses no tensor force.
Fig. 7.10 $A_y(\theta)$ and $A_{yy}(\theta)$ from the $^3\text{He}(\vec{d},\gamma)^5\text{Li}$ reaction at $E_d = 0.6-0.3$ MeV are shown along with the results of the three RGM calculations. The solid curve is Calculation A (two-body forces), the dashed curve is Calculation B ($^4\text{He}(p,p)^4\text{He}$ phase shift adjusted forces), and the dashed-dotted curve is Calculation D which uses no tensor force.
7.8 Transition Matrix Element calculation at $E_d = 8.6$ MeV

Transition matrix elements have also been calculated for $E_d(\text{lab}) = 8.6$ MeV using the two-body potential (Calculation E), the phase-shift adjusted potential (Calculation F), and the two-body potential with the tensor-force strength set to 0 (Calculation G). All of the parameters of the calculation are exactly the same as were used for the calculation at $E_d(\text{lab}) = 0.45$ MeV. The results of these calculations are listed in Tables 7.6 and 7.7, and are plotted in Figs 7.11 and 7.12. At this energy, qualitative agreement with the measured cross section and analyzing powers was improved by turning off the tensor force (Calculation G). However, a comparison of transition matrix elements reveals that although these calculated observables reproduce the data better, the relative intensities of the transition matrix elements do not agree with those extracted from a TME fit to the data. In fact, a non-zero tensor force is necessary to get agreement between data and theory for the relative strengths of the TME's. As with the $E_d(\text{lab}) = 0.45$ MeV calculation, the $^4s_{3/2}(E1)$ transition matrix element dominates the capture cross section due to the coupling of the $[d + ^3\text{He}]^{S=3/2}_{J=3/2}$ scattering channel to the $[p + ^4\text{He}]^{S=1/2}_{J=3/2}$ scattering channel by the tensor force. In Calculation E, the $^4d_{3/2}(E1)$ TME contributes 14% to the cross section strength via this same tensor-force coupling mechanism and the $^2d_{5/2}(E1)$ TME contributes 22% of the cross section strength by coupling directly to the $\alpha + p$ channel. Only about 6% E2 radiation is present in the calculation and virtually no M1 radiation.
Table 7.6  Transition matrix element relative intensities from a TME fit to the $E_d=8.6$ MeV data (two statistically significant solutions) and the three RGM calculations.

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<td></td>
<td></td>
<td>E</td>
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<tr>
<td>$^2s_{1/2}$ (E1)</td>
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<td>$^2d_{3/2}$ (E1)</td>
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<td>$^2d_{5/2}$ (E1)</td>
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<td>$^4d_{1/2}$ (E1)</td>
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<td>$^2p_{1/2}$ (M1)</td>
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<td>0.000</td>
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<td>$^2p_{3/2}$ (M1)</td>
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<td>$^4p_{1/2}$ (M1)</td>
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<td></td>
<td>0.000</td>
</tr>
<tr>
<td>$^2p_{1/2}$ (E2)</td>
<td></td>
<td>0.003</td>
</tr>
<tr>
<td>$^2p_{3/2}$ (E2)</td>
<td></td>
<td>0.127</td>
</tr>
<tr>
<td>Total: $^2p$(E2)</td>
<td>$0.004 \pm 0.004$</td>
<td>0.131</td>
</tr>
<tr>
<td>$^4p_{1/2}$ (E2)</td>
<td></td>
<td>0.004</td>
</tr>
<tr>
<td>$^4p_{3/2}$ (E2)</td>
<td></td>
<td>0.000</td>
</tr>
<tr>
<td>$^4p_{5/2}$ (E2)</td>
<td></td>
<td>0.000</td>
</tr>
<tr>
<td>Total: $^4p$(E2)</td>
<td>$0.005 \pm 0.001$</td>
<td>0.004</td>
</tr>
</tbody>
</table>

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Table 7.7 Legendre polynomial coefficients from a direct fit to the $E_d=8.6$ MeV data, a TME fit to this data, and the RGM calculations. Calculation E uses the two-body potential. Calculation F uses the modified potential which has been adjusted to reproduce the $^4\text{He}(p,p)^4\text{He}$ phase shifts. Calculation G uses the two-body potential with no tensor force.

<table>
<thead>
<tr>
<th></th>
<th>Legendre fit</th>
<th>TME fit</th>
<th>RGM calculation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_0$ (µb/sr)</td>
<td>$2.147 \pm 0.013$</td>
<td>$2.15 \pm 0.082$</td>
<td>E: 4.96, F: 7.82, G: 3.07</td>
</tr>
<tr>
<td>$a_1$</td>
<td>$-0.138 \pm 0.013$</td>
<td>$-0.140 \pm 0.014$</td>
<td>E: 0.018, F: 0.025, G: 0.002</td>
</tr>
<tr>
<td>$a_2$</td>
<td>$-0.203 \pm 0.017$</td>
<td>$-0.205 \pm 0.024$</td>
<td>E: -0.086, F: 0.070, G: -0.189</td>
</tr>
<tr>
<td>$b_1$</td>
<td>$-0.015 \pm 0.009$</td>
<td>$-0.015 \pm 0.018$</td>
<td>E: -0.107, F: -0.053, G: 0.021</td>
</tr>
<tr>
<td>$b_2$</td>
<td>$-0.020 \pm 0.007$</td>
<td>$-0.023 \pm 0.009$</td>
<td>E: 0.122, F: 0.127, G: -0.034</td>
</tr>
<tr>
<td>$c_0$</td>
<td>$0.229 \pm 0.019$</td>
<td>$0.004$</td>
<td>E: -0.004, F: -0.143, G: 0.018</td>
</tr>
<tr>
<td>$c_1$</td>
<td>$-0.036 \pm 0.005$</td>
<td>$-0.036$</td>
<td>E: -0.036, F: -0.059, G: -0.004</td>
</tr>
<tr>
<td>$c_2$</td>
<td>$-0.198 \pm 0.015$</td>
<td>$-0.273$</td>
<td>E: -0.273, F: -0.344, G: -0.006</td>
</tr>
<tr>
<td>$d_2$</td>
<td>$-0.111 \pm 0.006$</td>
<td>$-0.004$</td>
<td>E: -0.004, F: -0.024, G: -0.005</td>
</tr>
</tbody>
</table>
Fig. 7.11 $\sigma(\theta)$ from the $^3\text{He}(d,\gamma)^5\text{Li}$ reaction at $E_d = 8.6\text{ MeV}$ is shown along with the results of the three RGM calculations. The solid curve is Calculation E (two-body forces), the dashed curve is Calculation F ($^4\text{He}(p,p)^4\text{He}$ phase shift adjusted forces), and the dashed-dotted curve is Calculation G which uses no tensor force.
Fig. 7.12 $A_y(\theta)$ and $A_{yy}(\theta)$ from the $^3\text{He}(d,\gamma)^5\text{Li}$ reaction at $E_d = 8.6$ MeV are shown along with the results of the three RGM calculations. The solid curve is Calculation E (two-body forces), the dashed curve is Calculation F ($^4\text{He}(p,p)^4\text{He}$ phase-shift adjusted forces), and the dashed-dotted curve is Calculation G which uses no tensor force.
Chapter 8: Conclusions

In conclusion, the absolute differential cross section and vector and tensor analyzing powers of the $^3\text{He}(\vec{d},\gamma)^5\text{Li}$ reaction have been measured in two energy regions. The first energy region spans the fusion resonance which lies at $E_d(\text{lab}) = 0.45$ MeV. The second energy region is at $E_d(\text{lab}) = 8.6$ MeV.

Transition matrix element analyses of the fusion resonance region observables indicated that the reaction proceeds predominantly through s-wave (E1) capture, with small admixtures of M1 or E2 radiation. While two solutions exist for the TME fits of the $E_d(\text{lab}) = 0.8-0.0$ MeV observables, and three solutions exist for the TME fits of the $E_d(\text{lab}) = 0.6-0.3$ MeV observables, the multichannel Resonating Group Model calculations unambiguously select the dominant $^4s_{3/2}(\text{E1})$ solution (where the fitted $^4s_{3/2}(\text{E1})$ TME contributes >89% of the cross section strength) as the physically preferred one. In the RGM picture, the strong capture cross section in the fusion resonance region is a consequence of the tensor force, which couples the $[^d+^3\text{He}]^{S=3/2}_{J=3/2}$ channel to the $[^p+^4\text{He}]^{S=1/2}_{J=3/2}$ channel, enabling the capture reaction to proceed via the $\Delta S=0$ (E1) transition to the ground state.

At $E_d = 8.6$ MeV, only one TME solution was found from a fit to the cross section and analyzing powers $A_y(\theta)$ and $A_{yy}(\theta)$. The fit indicates that the capture cross-section strength is dominated by s-wave E1 capture with an admixture of \~20% d-wave E1 capture. In an RGM calculation at this energy, qualitative agreement with the measured cross section and analyzing powers was improved by turning off the tensor force. However, a comparison of transition matrix elements revealed that although the calculated observables reproduce the data better, the relative intensities of the calculated transition matrix elements do not agree with those extracted from a TME fit to the data. In fact, a non-zero tensor force is necessary to get agreement between data and theory for the relative strengths of the TME's. As with the $E_d(\text{lab}) = 0.45$ MeV calculation, the strong
$^4s_{3/2}$ (E1) capture strength is due to the coupling of the $[d + ^3\text{He}]_{J=3/2}^{S=3/2}$ scattering channel to the $[p + ^4\text{He}]_{J=3/2}^{S=1/2}$ scattering channel by the tensor force.

As shown, the tensor force has a dominating effect in this reaction. In the fusion resonance region it is especially important since it enhances the cross section by a factor of 16 (in a comparison of the measured cross section to the RGM calculation with no tensor force). Since the branching ratio for the $^3\text{He}(d,\gamma)^5\text{Li}$ and $^3\text{He}(d,p)^4\text{He}$ reactions is flat about the fusion resonance [Cec85b], the present study shows that the tensor force actually drives the energy producing $3\text{He}(d,p)^4\text{He}$ reaction. Furthermore, these results indicate that it is the tensor force which dramatically increases the feasibility of using the $^3\text{He}(d,\gamma)^5\text{Li}$ reaction as a diagnostic tool for deuterium-tritium reactors.
Appendix A

A.1 Ground-State Widths of $^5\text{Li}$ and $^5\text{He}$

In the analysis of the data taken for the $^3\text{He}(d,\gamma)^5\text{Li}$ reaction, it was necessary to fit the peak in the $\gamma$-ray spectra in order to extract consistent sums with which to calculate the angular distributions. In order to perform this fitting procedure, the line-shape response function of the NaI spectrometers were convoluted with the energy distribution of the $\gamma$ rays which come from this capture reaction to an unbound ground state. An important biproduct of this fitting procedure is the width of the ground state of $^5\text{Li}$. This fitting procedure was also performed on $\gamma$-ray spectra obtained for the $^3\text{H}(d,\gamma)^5\text{He}$ reaction in order to compare the widths of the two nuclei which are related through the principle of charge symmetry. The following is a discussion of the results of these analyses which resolves some disagreements in the literature. The FORTRAN subroutine which performed the fitting procedure is also given.

The ground states of $^5\text{He}$ and $^5\text{Li}$ have $J^\pi = 3/2^-$ and are most easily pictured as a single nucleon in the p$_{3/2}$ shell-model orbital above a closed s$_{1/2}$ core. They are unbound by 0.89 and 1.97 MeV, respectively, in the n-α and p-α channels [Ajz88]. Previous charged-particle reactions established the widths of the ground states to be $\sim 1$ MeV, corresponding to lifetimes of $\sim 10^{-22}$ seconds. These experimental widths varied widely depending on the experimental set-up and reactions used. For a review of experiments see [Ohl64] and [Ajz88]. The simplest measurement, nucleon scattering from $^4\text{He}$ at energies just above threshold, is complicated by potential scattering interference in the energy-dependent cross section. Warburton and McGruer [War57] used the $^4\text{He}(d,p)^5\text{He}$ reaction and fit a Gaussian to the high-energy side of the peak in their proton spectrum obtaining $\Gamma_{5\text{He}} = 0.55 \pm 0.03$ MeV. This fit the high-energy side well but failed to reproduce the long low-energy tail and therefore underestimated the width. Ohlsen and Young [Ohl64] used the same reaction and measured both the Full Width at Half
Maximum (FWHM) of a Gaussian fit and the FWHM of the observed spectrum. The Gaussian fit gave a value of $\Gamma_{5\text{He}} = 0.57 \pm 0.02$ MeV in agreement with Warburton and McGruer. The observed spectrum had a FWHM of $\Gamma_{5\text{He}} = 0.85 \pm 0.05$ MeV. Cerny et al. [Cer66] have studied both $^5\text{He}$ and $^5\text{Li}$ via the $^7\text{Li}(p,^3\text{He})^5\text{He}$ and $^7\text{Li}(p,t)^5\text{Li}$ reactions respectively. Widths of $\Gamma_{5\text{He}} = 0.80 \pm 0.04$ MeV and $\Gamma_{5\text{Li}} = 1.55 \pm 0.15$ MeV are quoted although it is not stated how these widths were obtained. [Ajz88] quotes a best value for the width of $^5\text{He}$ from the work above and others to be $\Gamma_{5\text{He}} = 0.6 \pm 0.02$ MeV. For $^5\text{Li}$ only a value of $\Gamma_{5\text{Li}} \sim 1.5$ MeV is quoted. Table 1 summarizes these and other [Smi63 and Bus68] various experimental widths and the methods of measurement used.

According to the theory of resonance reactions, the energy dependent shape of an isolated resonance for an electric dipole transition and an $N$-$\alpha$ breakup channel (where $N=p$ or $n$) can be described in terms of a Breit-Wigner form:

$$BW(E_\gamma) \propto \frac{1}{k^2} \frac{E_\gamma^3 \Gamma_N(E_N)}{\left[ \left( E_\gamma - E_{\text{res}} + \Delta(E_N) \right)^2 + \left( \frac{\Gamma_N}{2} \right)^2 \right]}$$  \hspace{1cm} [A.1]

where $\Gamma_N$ is the width of the $N$-$\alpha$ channel, $E_N$ is the nucleon emission energy, and $E_\gamma^3$ is the energy-dependent piece of the $\gamma$-ray width $\Gamma_\gamma$. We have assumed that $\Gamma_\gamma$ is much less than $\Gamma_N$ so that the total width $\Gamma = \Gamma_N + \Gamma_\gamma = \Gamma_N$. The energy dependence of the nucleon widths $\Gamma_N$ is determined by the relation $\Gamma_N = 2P_L(E_N)\gamma^2$, where $\gamma^2$ is the reduced width and $P_L(E_N)$ is the penetrability of the nucleon, which for this case has an orbital angular momentum $L=1$. $P_1(E_N)$ was calculated directly from the confluent hypergeometric functions, $F_1$ and $G_1$, for several choices of channel radius. Similarly, the energy dependence of the level shift, $\Delta(E_N)$, is determined from the shift function $S_1(E_N)$ in the relation $\Delta(E_N) = -(S_1(E_N) - B)\gamma^2$ where $B$, the boundary condition, was chosen such that $\Delta(E_{\text{res}}) = 0.0$. Both the penetrability and the shift function are defined according to the Lane and Thomas definitions [Lan58]. The $\gamma$-ray energy is simply related to the nucleon

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emission energy by kinematic considerations for both the $^3\text{H}(d,\gamma)^5\text{He} \rightarrow ^4\text{He} + n$ and the $^3\text{He}(d,\gamma)^5\text{Li} \rightarrow ^4\text{He} + p$ reactions. For the $^5\text{He}$ reaction, $E_{\text{res}}$ corresponds to a neutron energy of $E_n = 0.89$ MeV and a $\gamma$-ray energy of $E_\gamma = 21.8$ MeV. For the $^5\text{Li}$ reaction, $E_{\text{res}}$ corresponds to a proton energy of $E_p = 1.97$ MeV and a $\gamma$-ray energy of $E_\gamma = 21.5$ MeV. These values were taken from [Ajz88] and were not measured in this experiment.

The $\gamma$-ray spectra from our NaI spectrometer were fit using the minimization code MINUIT with a convolution of the measured energy-dependent response function of the NaI spectrometer (See Section 3.5) and the Breit-Wigner resonance shape described above. The functional form used is:

$$F(E) \propto \int \text{NaI}(E, E_\gamma) \ B-W(E_\gamma) \ dE_\gamma$$  \hspace{1cm} [A.2]

where $\Gamma_N(E_{\text{res}})$ and the overall height of $F(E)$ were varied, and the entire convoluted form was shifted up or down in energy to allow for spectrum calibration errors. The lack of a precise spectrum calibration prevented us from extracting values for $E_{\text{res}}$. From the extracted widths, we reconstructed the resonance shape using Eqn. A.1. Figure A1 shows the best fits to the $^5\text{He}$ and $^5\text{Li}$ spectra (solid curve), as well as the NaI response function and the B-W shape used in the convolution.

Table A2 summarizes the results of the best fits to the data. Using a channel radius of $r = 1.4 \ (4^{1/3} + 1^{1/3}) = 3.6$ fm, the FWHM of the resonance curves that we have extracted are comparable to the values of previous measurements as quoted in Table A1, but they differ from the average values quoted by [Ajz88]. The dependence of the widths, $\Gamma_p$ and $\Gamma_n$, on the channel radius is plotted in Fig. A2. It was found that while the individual widths vary with radius, the ratio of the widths $\Gamma_p/\Gamma_n$ is insensitive to the radius. We can make a simple prediction of this ratio by setting $\gamma_p^2 = \gamma_n^2$, as expected from the charge symmetry of the nuclear force. With this assumption, the ratio of widths is given by:

$$\Gamma_p(E_{\text{res}})/\Gamma_n(E_{\text{res}}) = 2P_1(E_{\text{res}})\gamma_p^2/2P_1(E_{\text{res}})\gamma_n^2 = P_1(E_{\text{res}})/P_1(E_{\text{res}}) = 1.79.$$
This prediction is in agreement with the measured ratio $\Gamma_p/\Gamma_n = 1.79 \pm 0.29$.

Within the framework of resonance reaction theory, we have measured the ground-state widths of $^5$He and $^5$Li to be $\Gamma_n = 1.36 \pm 0.19$ MeV and $\Gamma_p = 2.44 \pm 0.21$ MeV, assuming pure E1 radiation. These values vary significantly from previously published widths due to the energy dependence of the penetrability and shift functions. The FWHM of the Breit-Wigner functions corresponding to these widths, however, do fall within the range of previous measurements. From our results, the relative magnitudes of $\Gamma_n(E_{\text{res}})$ and $\Gamma_p(E_{\text{res}})$ can be explained simply in terms of the relative penetrability of the neutron and proton through their respective Coulomb and angular momentum barriers. Furthermore, the ratio of these widths is consistent with the property of charge symmetry in the nuclear force.
Fig. A1 Typical $\gamma$-ray spectra for the $^3$H(d,$\gamma$)$^5$He and $^3$He(d,$\gamma$)$^5$Li reactions at $E_d=8.6$ MeV and $\theta_{lab}=90^\circ$. The top spectrum was obtained by subtracting a pure titanium foil spectrum from a tritiated titanium foil spectrum. No background was subtracted in the bottom spectrum. The convolutions were fitted in the range $20.0 \text{ MeV} \leq E_\gamma \leq 24.0 \text{ MeV}$ and $19.0 \text{ MeV} \leq E_\gamma \leq 24.0 \text{ MeV}$ respectively.
Fig. A2  Upper panel: The neutron resonance width $\Gamma_n$ at the resonance energy $E_n = 0.89$ MeV and the proton resonance width $\Gamma_p$ at the resonance energy $E_p = 1.97$ MeV are plotted as a function of the channel radius. Lower panel: The ratio of the widths demonstrates the insensitivity of this ratio to the channel radius. The solid curve is the ratio of penetrabilities $P^p(E_{res})/P^p(E_{res})$ as discussed in the text.
as the reactions and methods of measurement used.

Table A1: A summary of various measurements of the $^{3}$He and $^{4}$Li ground-state widths, as well

<table>
<thead>
<tr>
<th>Bus68</th>
<th>FWHM of Resonance Curve</th>
<th>$^{3}$He($d,p$)$^{4}$Li</th>
<th>2.6±0.4</th>
<th>0.2±0.030</th>
</tr>
</thead>
<tbody>
<tr>
<td>Smit63</td>
<td>FWHM of Proton Spectrum</td>
<td>$^{4}$He</td>
<td></td>
<td>0.80±0.4</td>
</tr>
<tr>
<td>Cer66</td>
<td>no scaled</td>
<td>$^{3}$He($d,p$)$^{4}$Li</td>
<td>1.55±0.15</td>
<td>0.3±0.03</td>
</tr>
<tr>
<td>Wae77</td>
<td>Gaussian fit to Proton Spectrum</td>
<td>$^{4}$He</td>
<td></td>
<td>0.8±0.05</td>
</tr>
<tr>
<td>Oth64</td>
<td>FWHM of Proton Spectrum</td>
<td>$^{4}$He</td>
<td></td>
<td>0.7±0.02</td>
</tr>
<tr>
<td>Oth64</td>
<td>Gaussian fit to Proton Spectrum</td>
<td>$^{4}$He</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Atr88</td>
<td>Average value of several studies</td>
<td>$^{4}$He</td>
<td></td>
<td>1.2±0.03</td>
</tr>
<tr>
<td></td>
<td>FWHM of Resonance Curve</td>
<td>$^{3}$He($d,p$)$^{4}$Li</td>
<td>1.3±0.19</td>
<td>2.4±0.2</td>
</tr>
<tr>
<td></td>
<td>Single-particle width in B-W shape</td>
<td>$^{3}$He($d,p$)$^{4}$Li</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
FWHM measurements were obtained by constructing a Breit-Wigner resonance for $\frac{N}{A}$. The errors for our $\chi^2$, and FWHM were measured in this experiment as described in the text. The errors for our energy at the resonance $E^\gamma$ and the nucleon energy at the resonance $E^N$ are from [A978].

The $\gamma$-ray dimensions and were calculated using the continuous hypergeometric functions. The $\gamma$-ray energies and shift functions are quoted in units of $M_{\gamma\gamma}$ in the center of mass system. The penetrability and shift functions are

<table>
<thead>
<tr>
<th>$\chi$</th>
<th>PFWHM</th>
<th>$\gamma$</th>
<th>$E^N(E^\gamma)$</th>
<th>$S(E^\gamma)$</th>
<th>$E^\gamma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.24 ± 0.03</td>
<td>5.06 + 0.29</td>
<td>3.33 ± 0.21</td>
<td>2.44 + 0.24</td>
<td>9.656</td>
<td>0.97</td>
</tr>
<tr>
<td>1.18 ± 0.03</td>
<td>5.96 ± 0.46</td>
<td>3.32 ± 0.19</td>
<td>9.647</td>
<td>0.99</td>
<td>2.8</td>
</tr>
</tbody>
</table>

Table A2. The results of the present experiment. All energies, widths, and reduced widths are
A.2 The Convolution Code

In order to fit the spectra from the $^3\text{He}(d,\gamma)^5\text{Li}$ reaction, it was necessary to write a code that could convolute the NaI line-shape response function as measured with the $^3\text{H}(p,\gamma)^4\text{He}$ reaction (see Section 3.5) with a one-level Breit-Wigner function corresponding to the ground-state resonance. The MINUIT subroutine which accomplishes this is given below. The convolution is performed on a mesh corresponding to the channels of the spectrum which is being fit. It is accomplished simply by constructing response functions for every $\gamma$-ray energy on the mesh, weighting each by the value of the Breit-Wigner function at that channel energy, and summing all the weighted response functions. The fitted parameters are WIDTH, HEIGHT, ISHIFT, and GWIDTH. WIDTH is defined in equation A1 as $\Gamma_N$, Height is the overall height of the convoluted function, ISHIFT is the number of channels the convoluted function is shifted in order to line up with the spectrum, and GWIDTH is the Gaussian width as in the form $e^{-(x/GWIDTH)^2}$. This version of the subroutine was used to fit the fusion resonance spectra. An alternate version which does not perform the additional Gaussian convolution was used to fit the spectra at $E_d=8.6$ MeV and to extract the widths quoted in Appendix A.1.

In both versions, the user supplies information about the projectile and target charges and masses, the projectile energy, and the angle of the detector so that the code can perform a kinematics calculation to determine the energy of the proton which is emitted from the residual nucleus. In order to calculate the penetrability and shift functions, the user also supplies L, R0, and RMAX, which are the relative angular momentum of the decaying system ($L=1$ for $^5\text{Li}$ breaking up into a proton and $\alpha$ particle) and the channel radius (where $RMAX=R0*A^{1/3}$, and R0 is not used if RMAX is supplied) which was taken to be $RMAX=3.6$ in this case. The parameters for the line-shape response function of the NaI spectrometer is also supplied by the user.
SUBROUTINE FCN(NPAR, GG, FF, X, IFLAG)
C
C
C THIS PROGRAM IS DESIGNED TO BE A GENERAL FITTING PROGRAM
C THAT FITS THE LINESHAPE OF A NaI DETECTOR
C
Mark Balbes 11/28/89

C
C Minuit subroutine
C Inputs: npar= no. of ADJUSTABLE parameters
C
GG = derivative vector (not used here)
C
FF = value of function to be minimized
C
X = parameter values
C
IFLAG = control value 1===> set up constants
C
2===> not used here
C
3===> write out information
C
4===> calculate F (CHI-SQR)
C
>4===> not used here
C
C
C----------------------------------------

C234567

DOUBLE PRECISION GG(50),X(50),ALIM(50),BLIM(50)
REAL FOREBCI BACKBCI,FOREDTC,FOREACC,BACKDTC,BACKACC
REAL LOWB, UPB
REAL ERES,WIDTH,WIDTHOLD,HEIGHT,CHISQR,CHIPER
INTEGER IPENFLAG,ISHIFTFLAG,ISHIFT,IRESFLAG
CHARACTER*50 FILENAME,BACKNAME
REAL*8 T1, TM1, TM2, TM3, TM4, EX, ANG
REAL*8 ANG3P, T3P, ANG4P, T4P, RATIOP, SHIFTP, QP
REAL*8 ANG3M, T3M, ANG4M, T4M, RATIOM, SHIFTM, QM
REAL*8 ECM, ANGL4P
REAL APROJ1, ATARG1, ZPROJ1, ZTARG1
REAL APROJ2, ATARG2, ZPROJ2, ZTARG2
INTEGER IAProj1, IATARG1, IZPROJ1, IZTARG1
INTEGER IAProj2, IATARG2, IZPROJ2, IZTARG2
INTEGER IANUC1, IZNUC1, IANUC2, IZNUC2
CHARACTER TITLE*60, ATIME*10, ADATE*10, AATIME*20
DOUBLE PRECISION C1,C2,C3,C4,C5,C6,C7,C8,C9,C10,RJOIN
COMMON/RESPONSE/C1,C2,C3,C4,C5,C6,C7,C8,C9,C10,RJOIN
&
COMMON/REACTION/APROJ1,ZPROJ1,ATARG1,ZTARG1,EBEAM,ANGL
&
AB,QGAM

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COMMON/RESONANCE/APROJ2,ZPROJ2,ATARG2,ZTARG2,L,R0,RMAX,
QRES,
&       ISHIFTFLAG  
DOUBLE PRECISION U(50),WERR(50),XS(50),XTS1(50),WTS(50)  
COMMON/PAREXN/U,WERR  
COMMON/FIX/IPFIX(50),XS,XTS1,WTS,NPFIX  
COMMON/PARM/P,SH  
COMMON/ITITLE/TITLE,FDATE  
REAL GWIDTH,GAUSS(512,512),SHAPE2(2000)  
GO TO (10,20,30,40,20),IFLAG

C
C Initialize
C
10 CONTINUE
C
READ IN SPECTRUM DATA AT THIS POINT
C
READ(5,555)IFILE,Filename
555 FORMAT(I1,A50)
READ(5,*),FOREBCI,FOREDTC,FOREACC
OPEN(UNIT=20,FILE=FILENAME,STATUS='OLD')

IF(IFILE.EQ.2) THEN
   I = 1
   WRITE(6,22)FILENAME
   22 FORMAT(’READING 2 COLUMNS OF DATA FROM FILE’,A50)
   READ(20,*,END=101)XFORE(I),YFORE(I)
   YFORE(I)=YFORE(I)*FOREDTC*FOREACC
   ERRFORE(I)=SQRT(YFORE(I))
   IF(ERRFORE(I),LE.0.) ERRFORE(I)=1.
   I=I+1
   GOTO 100
   101 NDATA=I-1   !NDATA IS NOW #OF DATA POINTS
ELSE IF(IFILE.EQ.3) THEN
   I = 1
   WRITE(6,23)FILENAME
   23 FORMAT(’READING 3 COLUMNS OF DATA FROM FILE’,A50)
   READ(20,*,END=103)XFORE(I),YFORE(I),ERRFORE(I)
   YFORE(I)=YFORE(I)*FOREDTC*FOREACC
   IF(ERRFORE(I),LE.0.) ERRFORE(I)=1.
   I=I+1
   GOTO 102
   103 NDATA=I-1   !NDATA IS NOW #OF DATA POINTS
ELSE
   WRITE(6,*),’DATA FILE FORMAT UNKNOWN. CHECK UNIT 5’
   WRITE(6,*),’EXECUTION TERMINATED’
   STOP
ENDIF
CLOSE(20)
WRITE(6,*)'NDATA=',NDATA
READ(5,555)IFILEB,BACKNAME
IF(IFILEB.EQ.2.OR.,IFILEB.EQ.3) THEN
   READ(5,*)BACKBCI,BACKDTC,BACKACC
   OPEN(UNIT=20,FILE=BACKE NAME,STATUS='OLD')
   IF(IFILEB.EQ.2) THEN
      I = 1
      WRITE(6,24)BACKNAME
      FORMAT(' READING 2 COLUMNS OF BKGRND FROM FILE',A50)
   ENDIF
   READ(20,*,END=201)XBACK(I),YBACK(I)
   IF(XBACK(I).NE.XFORE(I)) THEN
      WRITE(6,*)'BKGRND AND FOREGRND ENERGIES DIFFERENT'
      STOP
   ENDIF
   YBACK(I)=YBACK(I)*BACKDTC*BACKACC
   ERRBACK(I)=SQRT(YBACK(I))
   IF(ERRBACK(I).LE.0.) ERRBACK(I)=1.
   I=I+1
   GOTO 200
   201 NBACK=I-1 !NBACK IS NOW #OF BKGRND POINTS
 ELSE IF(IFILEB.EQ.3) THEN
      I = 1
      WRITE(6,25)BACKNAME
      FORMAT(' READING 3 COLUMNS OF BKGRND FROM FILE',A50)
      READ(20,*,END=203)XBACK(I),YBACK(I),ERRBACK(I)
      IF(XBACK(I).NE.XFORE(I)) THEN
         WRITE(6,*)'BKGRND AND FOREGRND ENERGIES DIFFERENT'
         STOP
      ENDIF
      YBACK(I)=YBACK(I)*BACKDTC*BACKACC
      IF(ERRBACK(I).LE.0.) ERRBACK(I)=1.
      I=I+1
      GOTO 202
   203 NBACK=I-1 !DATA IS NOW #OF DATA POINTS
 ELSE
      WRITE(6,*)'BKGRND FILE FORMAT UNKNOWN. CHECK UNIT'
 ENDIF
 WRITE(6,*)'EXECUTION TERMINATED'
 STOP
ENDIF
CLOSE(20)
WRITE(6,*)'NBACK=',NBACK
ELSE
   BACKBCI=1.
ENDIF
C Subtract the background and calculate the new errors.
DO I=1,NDATA
   XSPEC(I)=XFORE(I)
   YSPEC(I)=YFORE(I)-(FOREBCI/BACKBCI)*YBACK(I)
   126
ERR(I)=SQRT(ERRFORE(I)**2+((FOREBCI/BACKBCI)*ERRBACK(I))**2)
ENDDO

C READ CARDS.INFO (UNIT 5)
READ(5,*)
READ(5,*)APROJ1,ZPROJ1,ATARG1,ZTARG1,EBEAM,ANGLAB
IAPROJ1=APROJ1
IZPROJ1=ZPROJ1
IATARG1=ATARG1
IZTARG1=ZTARG1
READ(5,*)
READ(5,*)C1,C2,C3
READ(5,*)C4,C5
READ(5,*)C6,C7,C8
READ(5,*)C9,C10
READ(5,*)RJOINT,CUTLO,CUTHI,STNCENT
READ(5,*)
READ(5,*)
READ(5,*)IRESFLAG
READ(5,*)APROJ2,ZPROJ2,ATARG2,ZTARG2
IAPROJ2=APROJ2
IZPROJ2=ZPROJ2
IATARG2=ATARG2
IZTARG2=ZTARG2
READ(5,*)LR,RO,RMAX
READ(5,*)PENFLAG
READ(5,*)ISIFITFLAG
READ(5,*)
READ(5,*)LOWB,UPB
READ(5,*)

C Calculate the gamma ray resonance energy.
TM1=IAPROJ1 + EMASS(IAPROJ1,IZPROJ1)/931.5016
TM2= IATARG1 + EMASS(IATARG1,IZTARG1)/931.5016
TM3=0.
TM4=IATARG1+IAPROJ1+
& EMASS(IATARG1+IAPROJ1,IZTARG1+IZPROJ1)/931.5016
EX=0.
ANG=ANGLAB
WRITE(6,*)IAPROJ1,IZPROJ1=’, APROJ1,ZPROJ1
WRITE(6,*)IATARG1,IZTARG1=’,ATARG1,ZTARG1
WRITE(6,*)EBEAM=’,EBEAM
WRITE(6,*)’ANGLAB=’,ANGLAB
CALL SRKIN (EBEAM, TM1, TM2, TM3, TM4, EX, ANG, &
ANG3P, T3P, ANG4P, T4P, RATIOP, SHIFTP, QP, &
ANG3M, T3M, ANG4M, T4M, RATIOM, SHIFTM, QM, &
ECM,ANGL4P)
ERES=T3P
WRITE(6,*)'T3P=',T3P
WRITE(6,*)'ERES=',ERES

C Calculate Q-Values of gamma ray reaction and exit channel reaction
IANUC1=IATARG1+IAPROJ1
IZNUC1=IZTARG1+IZPROJ1
IANUC2=IATARG2+IAPROJ2
IZNUC2=IZTARG2+IZPROJ2

WRITE(6,*)'IATARG1,ZTARG1,IATARG1,IZTARG1'
WRITE(6,*)'IATARG2,ZTARG2,IATARG2,IZTARG2'
WRITE(6,*)'IANUC1,ZNUC1,IANUC1,ZNUC1'
WRITE(6,*)'IANUC2,ZNUC2,IANUC2,ZNUC2'
QGAM=EMASS(IATARG1,IZTARG1)+EMASS(IAPROJ1,IZPROJ1)
& -EMASS(IANUC1,ZNUC1)
QRES=EMASS(IANUC2,ZNUC2)
& -EMASS(IATARG2,IZTARG2)-EMASS(IAPROJ2,IZPROJ2)
WRITE(6,*)'QGAM=',QGAM
WRITE(6,*)'QRES=',QRES

C Calculate the needed penetrabilities.
IF(PENFLAG.NE.0) THEN
  WRITE(6,*)'CALCULATING PENETRABILITIES'
  CALL GETPEN(PEN,SHIFT,XSPEC,NDATA)
  WRITE(6,*)'PENETRABILITIES HAVE BEEN CALCULATED'
ELSE
  OPEN(2,FILE='PEN.DAT',STATUS='OLD')
  OPEN(3,FILE='SHIFT.DAT',STATUS='OLD')
  DO 464 I=1,NDATA
       READ(2,*)PENX(I),PEN(I)
       READ(3,*)SHIFTX(I),SHIFT(I)
       IF(PENX(I).NE.XSPEC(I)) THEN
         WRITE(6,*)'WARNING: PENETRABILITIES & ARE FOR THE WRONG ENERGIES!!!'
         ENDIF
         IF(SHIFTX(I).NE.XSPEC(I)) THEN
           WRITE(6,*)'WARNING: SHIFT FUNCTIONS & ARE FOR THE WRONG ENERGIES!!!'
           ENDIF
       CONTINUE
   CLOSE(2)
   ENDIF

IF(PENFLAG.NE.0) THEN
  OPEN(19,FILE='PEN.DAT',STATUS='NEW')
  OPEN(18,FILE='SHIFT.DAT',STATUS='NEW')
  DO 465 I=1,NDATA
       WRITE(19,*)XSPEC(I),PEN(I)
       WRITE(18,*)XSPEC(I),SHIFT(I)
  CONTINUE
  CLOSE(19)
  ENDIF

128
C Calculate the NaI response function for all the gamma ray energies going into the detector.
WRITE(6,*)'ENTERING DETR CALCULATION'
DO 13 I=1,NDATA
   CALL MKRESPSUB(NAIR,XSPEC,NDATA,XSPEC(I))
DO 14 J=1,NDATA
   DETR(I,J)=NAIR(J)
13 CONTINUE
WRITE(6,*)'EXITING DETR CALCULATION'
C OPEN(2,FILE='DETR.OUT',STATUS='NEW')
C DO 141 I=1,NDATA
   WRITE(2,*)XSPEC(I),DETR(NDATA-100,I)
C141 CONTINUE
C DO 142 I=1,NDATA
   WRITE(2,*)XSPEC(I),DETR(NDATA-50,I)
C142 CONTINUE
C DO 143 I=1,NDATA
   WRITE(2,*)XSPEC(I),DETR(NDATA-10,I)
C143 CONTINUE
C DO 144 I=1,NDATA
   WRITE(2,*)XSPEC(I),DETR(NDATA,I)
C144 CONTINUE
RETURN
C

C---------------------------------------------------------------

20 CONTINUE
RETURN
C

C---------------------------------------------------------------

30 CONTINUE
C THIS IS THE LAST CALL TO FCN -- THE FITTING IS FINISHED BY NOW
   WIDTH = X(1)
   HEIGHT = X(2)
   ISHIFT = X(3)
   GWIDTH=X(4)
C FIRST CALCULATE THE BREIT-WIGNER SHAPE
   CALL SIMPRESSUB(RES,W,PEN,SHIFT,ERES,WIDTH,XSPEC,NDATA)
C NOW DO THE CONVOLUTION
   CALL SPREADSUB(SHAPE,XSPEC,DETR,RES,NDATA)
C Calculate a gaussian to allow for beam energy straggling.
   DO I=1,NDATA
      DO J=1,NDATA
         GAUSS(I,J)=EXP(-1.*((XSPEC(J)-XSPEC(I))/GWIDTH)**2)
      ENDF
   ENDF
   RETURN
   C
ENDDO
ENDDO

C Now convolute in the gaussian
CALL SPREADSUB(SHAPE2,XSPEC,GAUSS,SHAPE,NDATA)

DO 12 I=1,NDATA
    SHAPE2(I)=HEIGHT*SHAPE2(I)
12 CONTINUE

OPEN(1,FILE="FITSPEC.FIT",STATUS="NEW")
DO 35 I=1,NDATA
    WRITE(1,*)XSPEC(I),SHAPE2(I+ISHIFT)
35 CONTINUE
CLOSE(1)

C FIND PEAK OF SHAPE2(I) FOR B-W NORMALIZATION PURPOSES.
CALL FINDMAX(XSPEC,SHAPE2,NDATA,IMAXS)

C CALCULATE THE BREIT-WIGNER RESONANCE
CALL SIMPRESSUB(RES,W,PEN,SHIFT,ERES,WIDTH,XSPEC,NDATA)

C FIND ITS MAX VALUE
CALL FINDMAX(XSPEC,RES,NDATA,IMAXR)

OPEN(20,FILE="RESPONSE.OUT",STATUS="NEW")
DO 41 I=1,NDATA
    WRITE(20,*)XSPEC(I),RES(I+IMAXR-IMAXS)*
               (SHAPE2(IMAXS)/RES(IMAXR))
41 CONTINUE
CLOSE(20)

C CALCULATE THE NaI RESPONSE AT THE RESONANCE
CALL MKRESPSUB(NAIR,XSPEC,NDATA,XSPEC(IMAXR))
OPEN(21,FILE="RESPONSE.OUT",STATUS="NEW")
DO 43 I=1,NDATA
    WRITE(21,*)XSPEC(I),NAIR(I+IMAXR-IMAXS)*
               (SHAPE2(IMAXR)/NAIR(IMAXR))
43 CONTINUE

C calculate the reduced width
AION=APROJ2
ZION=ZPROJ2
ATARG=ATARG2
ZTARG=ZTARG2
L=L
LMAX=L+3
R0=1.4
R=RMAX
NEN=1
E=(AION+ATARG)/ATARG * QRES
CALL PENETSUB(E)
PENRES = P(L+1)
RW = WIDTH/(2.*PENRES)

C Spit out the data being fit.
OPEN(15,FILE='DATA.OUT',STATUS='NEW')
DO 11 I=1,NDATA
   IF((I,GE.0.AND.(I,LE.NDATA) THEN
      WRITE(15,*)XSPEC(I),YSPEC(I),ERR(I)
   ENDF
11 CONTINUE

C Sum the calculated fit and data to get the yields for the spectrum.
   NDATA2=0
   YFITALL = 0.
   YFITRANGE = 0.
   YIELDFORE = 0.
   YIELDBACK = 0.
   DO I = 1,NDATA
      YFITALL = YFITALL + SHAPE2(I)
      IF((XSPEC(I),GE.LOWB).AND.(XSPEC(I),LE.UPB)) THEN
         YFITRANGE = YFITRANGE+SHAPE2(I+ISHELF)
         YIELDFORE = YIELDFORE+YFORE(I)
         YIELDBACK = YIELDBACK+YBACK(I)
      NDATA2 = NDATA2 + 1
   ENDF
ENDO
   YIELD = YIELDFORE-(FOREBCI/BACKBCI)*YIELDBACK
   YIELDERR = SQRT(YIELDFORE+(FOREBCI/BACKBCI)**2*YIELDBACK)

C Write the FITSPEC.OUT file with the final results of the fit.
OPEN(99,FILE='FITSPEC.OUT',STATUS='NEW')
CALL TIME(ATIME)
CALL DATE(ADATE)
ENCODE (20,'(10A10A)',AATIME)ADATE,ATIME
WRITE(99,8000)TITLE,AATIME
8000 FORMAT(' ',4X,A50,5X,A20)
   NDGFREE = NDATA2-(4-NPFIX)
   WRITE(99,*)CHISQR=.,CHISQR
   IF (NDGFREE,NE.0.) THEN
      CHIPER=CHISQR/FLOAT(NDGFREE)
      WRITE(99,*)'CHISQR/DGFREE=',CHIPER
   ENDF
WRITE(99,293)LOWB,UPB
293 FORMAT(' FITTING REGION: ',2F8.3)
WRITE(99,'(99,*)# OF DATA POINTS=',NDATA2
WRITE(99,'(99,*)# OF VARIED PARAMETERS=',NPFIX
WRITE(99,'(99,*)# OF DEGREES OF FREEDOM=',NDGFREE
IF(IPFIX(1).EQ.1.OR.IPFIX(2).EQ.1.OR.IPFIX(3).EQ.1)THEN
WRITE(99,*)'WIDTH =',X(1),'FIXED'
ELSE
WRITE(99,*)'WIDTH =',X(1),'+-',WERR(1)
ENDIF
IF(IPFIX(1).EQ.2.OR.IPFIX(2).EQ.2.OR.IPFIX(3).EQ.2)THEN
WRITE(99,*)'HEIGHT =',X(2),'FIXED'
ELSE
WRITE(99,*)'HEIGHT =',X(2),'+-',WERR(2)
ENDIF
IF(IPFIX(1).EQ.3.OR.IPFIX(2).EQ.3.OR.IPFIX(3).EQ.3)THEN
WRITE(99,*)'ISHIFT =',X(3),'FIXED'
ELSE
WRITE(99,*)'ISHIFT =',X(3),'+-',WERR(3)
ENDIF
IF(IPFIX(1).EQ.4.OR.IPFIX(2).EQ.4.OR.IPFIX(3).EQ.4.OR.
& IPFIX(4).EQ.4)THEN
WRITE(99,*)'GWIDTH =',X(4),'FIXED'
ELSE
WRITE(99,*)'GWIDTH =',X(4),'+-',WERR(3)
ENDIF
WRITE(99,*)'PENETRABILITY AT ERES=',PENRES
WRITE(99,*)'REduced WIDTH AT ERES=',RW
WRITE(99,*)'YIELD OF DATA =',YIELD,'+-',YIELDERR
WRITE(99,199)YFITRANGE/YFITALL
FORMAT(' RATIO OF FIT YIELD IN FITTING RANGE TO TOTAL FIT
YIELD ='
& ,F9.4)
CLOSE(99)

WRITE (6,*)'FINAL CALL'
RETURN
C -----------------------------------------------------------------------------

40 CONTINUE

C--------------------------------------------------------------------------------
C
FF = 0.
C
C--------------------------------------------------------------------------------
C FITTING THE SPECTRUM WITH A CONVOLUTED LINESHAPE AND
C BREIT-WIGNER
C--------------------------------------------------------------------------------
WIDTH = X(1)
HEIGHT = X(2)
ISHIFT=X(3)
GWIDTH=X(4)
C FIRST CALCULATE THE BREIT-WIGNER SHAPE

CALL SIMPRESSUB(RES,W,PEN,SHIFT,ERES,WIDTH,XSPEC,NDATA)

C NOW DO THE CONVOLUTION

CALL SPREADSUB(SHAPE,XSPEC,DETR,RES,NDATA)

C Calculate a gaussian to allow for beam energy straggling.
DO I=1,NDATA
  DO J=1,NDATA
    GAUSS(I,J)=EXP(-1.*((XSPEC(J)-XSPEC(I))/GWIDTH)**2)
  ENDDO
ENDDO

C Now convolute in the gaussian
CALL SPREADSUB(SHAPE2,XSPEC,GAUSS,SHAPE,NDATA)

C Vary the height of the convoluted shape.
DO 98 I=1,NDATA
  SHAPEOLD(I)=SHAPE2(I)
  SHAPE2(I)=HEIGHT*SHAPE2(I)
98 CONTINUE

DO 332 I=1,NDATA
  SHAPE2(I)=HEIGHT*SHAPEOLD(I)
332 CONTINUE

DO 99 I =1,NDATA
  IF(XSPEC(I),GE.,LOWB.AND.XSPEC(I),LE.,UPB) THEN
    FF = FF + ((YSPEC(I)-SHAPE2(I+ISHIFT))/ERR(I))**2
  ENDFI
99 CONTINUE

CHISQR = FF

C
C*******************************************************************************
RETURN
END

C*******************************************************************************

C This is a subroutine to find the maximum value of a function given C to it in x,y coordinate pairs.
C Written by Mark Balbes
C 3/14/90

SUBROUTINE FINDMAX(X,Y,NDATA,IMAX)
REAL YMAX
INTEGER IMAX

IMAX = 0
YMAX = 0

DO 10 I = 1, NDATA
  IF(Y(I),GT,YMAX) THEN
    YMAX=Y(I)
    IMAX=I
  ENDIF
10 CONTINUE
END

SUBROUTINE GETPEN(PEN,SHIPT,XSPEC,NDATA)

REAL E,P(51),SH(51),T(100,10),RHO(100),ETA(100),FF(100,10)
1,F(150),G(147),GG(100,10),MU,EPRIME(100)
  REAL LIEX,QHEP
  REAL APROJ1,ATARG1,ZPROJ1,ZTARG1
  REAL APROJ2,ATARG2,ZPROJ2,ZTARG2
  INTEGER I, NUMBER, L

  REAL XSPEC(2000)
  REAL WIDTH,ERES,REDW,EI,EF,INC
  COMMON/REACTION/APROJ1,ZPROJ1,ATARG1,ZTARG1,EBEAM,ANGL
  AB, GQGM
  COMMON/RESONANCE/APROJ2,ZPROJ2,ATARG2,ZTARG2,L,R0,RMAX,
  QRES
  COMMON / PARM / P,SH
  COMMON F,G,FP,GP,SIGO
  COMMON /COM/AION,ZION,ATARG,ZTARG,LMAX,RO,R,E

C SET UP THE INITIAL VALUES
  I=1
C-----------------------------------------------
C Calculate the energy dependent penetrability and width.

  AION=APROJ2
  ZION=ZPROJ2
  ATARG=ATARG2
  ZTARG=ZTARG2
  LMAX=L+3
  R=RMAX
  NEN=1
  QHEP=QRES ! Q VALUE FOR P+ALPHA=5Li REACTION IN C.O.M.

134
100   EGAM(I)=XSPEC(I)
     CALL CALCLIEXSUB(EGAM(I),LIEX) !LIEX=EX. ENERGY OF LI ABOVE
     G.S.
     E= QHEP + LIEX      !C.O.M. ENERGY OF ALPHA+PROTON SYSTEM
     E=((ATARG+AI0N)/ATARG)*E      !E IN LAB FRAME
     IF(E.GT.0.)THEN
       CALL PENETSUB(E)
       PEN(I)=P(L+1)
       SHIFT(I)=SH(L+1)
     ELSE
       PEN(I)=0.
       SHIFT(I)=0.
     ENDF

C SET UP I AND EGAM(I) FOR NEXT PASS
     I =I+1
     IF(I.LE.NDATA) GOTO 100
     RETURN
END

C----------------------------------------------------------------------

SUBROUTINE CALCLIEXSUB(EGAMLAB,EXLI)

REAL*8 T1, TM1, TM2, TM3, TM4, EX, ANGLAB
REAL*8 ANG3P, T3P, ANG4P, T4P, RATIOP, SHIFTP, QP
REAL*8 ANG3M, T3M, ANG4M, T4M, RATIOM, SHIFTM, QM
REAL*8 ECM, ANGL4P
REAL*8 Q, EGAMLAB, EEX, BETA, GAMMA, EGAMCM, RM4
REAL*8 E1CM, EK1CM, E2CM, EK2CM
REAL ANUC, ZNUC
REAL APROJ1, ATARG1, ZPROJ1, ZTARG1
REAL APROJ2, ATARG2, ZPROJ2, ZTARG2

INTEGER IAPROJ1, IATARG1, IZPROJ1, IZTARG1
INTEGER IAPROJ2, IATARG2, IZPROJ2, IZTARG2

COMMON/REACTION/APROJ1, ZPROJ1, ATARG1, ZTARG1, EBEAM, ANGLAB, QGAM
COMMON/RESONANCE/APROJ2, ZPROJ2, ATARG2, ZTARG2, L, R0, RMAX, QRES

C WRITE(6,*)'IN CALCLIEX SUBROUTINE'
C WRITE(6,*)'EGAMLAB='.,EGAMLAB
C WRITE(6,*)'APROJ1,ZPROJ1,ATARG1,ZTARG1,EBEAM,ANGLAB,QGAM
C WRITE(6,*)'APROJ2,ZPROJ2,ATARG2,ZTARG2,L,R0,RMAX,QRES
IAPROJ1=APROJ1
IATARG1=ATARG1

135
IZPROJ1=ZPROJ1
IZTARG1=ZTARG1
IAPROJ2=APROJ2
IATARG2=ATARG2
IZPROJ2=ZPROJ2
IZTARG2=ZTARG2

T1=EBEAM
TM1= IAPROJ1 + EMASS(IAPROJ1,IZPROJ1)/931.5016
TM2= IATARG1 + EMASS(IATARG1,IZTARG1)/931.5016
TM3=0.
IANUC=IATARG1+IAPROJ1
IZNUC=IZTARG1+IZPROJ1
TM4=IANUC + EMASS(IANUC,IZNUC)/931.5016
Q = (TM1+TM2-TM3-TM4)*931.5016

C Set up to do relativistic transformations.
E1LAB = T1+TM1*931.5016
PC1LAB = SQRT(E1LAB**2 - (TM1*931.5016)**2)
BETA = PC1LAB/(E1LAB+TM2*931.5016)
GAMMA = 1/SQRT(1-BETA**2)

C Calculate the kinetic energy available in the center of mass frame (ECM)
E1CM = GAMMA*(E1LAB-BETA*PC1LAB)
EK1CM = E1CM - TM1*931.5016

E2CM = GAMMA*TM2*931.5016
EK2CM = E2CM - TM2*931.5016
ECM = EK1CM+EK2CM
EEX = ECM +Q
EGAMCM= GAMMA*(1.-BETA*COSD(ANGLAB))*EGAMLAB

RM4=TM4*931.5016
EKLIICM=(EEX+RM4-EGAMCM)-SQRT((EEX+RM4-EGAMCM)**2-
EGAMCM**2)

C Calculate the mass of the excited Li nucleus
RMLIEX=(EGAMCM**2-EKLIICM**2)/(2.*EKLIICM)
EKLIILAB =((EKLIICM+RMLIEX)+BETA*SQRT((EKLIICM+RMLIEX)**2-
RMLIEX**2))
& *GAMMA-RMLIEX

EXLI=RMLIEX-RM4
RETURN
END

C-----------------------------------------------

SUBROUTINE MKRESPSUB(NAIR,XSPEC,N,CENT)

136
DOUBLE PRECISION C1,C2,C3,C4,C5,C6,C7,C8,C9,C10,RJOINT
DOUBLE PRECISION XCH, FUNCTION, SLOPE
COMMON/RESPONSE/C1,C2,C3,C4,C5,C6,C7,C8,C9,C10,
 & RJOINT, CUTLO, CUTHI, STNCENT

DO 10 I=1,N
XCH = XSPEC(I)*STNCENT/CENT
IF (XCH .GE. RJOINT .AND. XCH .LE. CUTHI) THEN

FUNCTION = DEXP(C6+C7*XCH+C8*XCH**2+C9*XCH**3+C10*XCH**4)

NAIR(I) = FUNCTION
ELSE
NAIR(I) = 0.
ENDIF
IF (XCH .LT. RJOINT .AND. XCH .GE. CUTLO) THEN

FUNCTION = DEXP(C1+C2*XCH+C3*XCH**2+C4*XCH**3+C5*XCH**4)

NAIR(I) = FUNCTION
ENDIF
IF (XCH .LT. CUTLO) THEN
SLOPE = DEXP(C1+C2*CUTLO+C3*CUTLO**2+
 & C4*CUTLO**3+C5*CUTLO**4)/CUTLO

NAIR(I) = SLOPE*XCH
ENDIF
10 CONTINUE

C Normalize the response function to have a total area of one.

SUM = 0.
DO 20 I=1,N
SUM = SUM + NAIR(I)
20 CONTINUE
DO 30 I=1,N
NAIR(I) = NAIR(I)/SUM
30 CONTINUE
RETURN
END

C THIS PROGRAM WILL SIMPLY CALCULATE A SIMPLE NATURAL
LINESHAPE
C USING THE ONE LEVEL SMALL WIDTH APPROXIMATION OF THE BREIT-
WIGNER
C FORMULA.

SUBROUTINE SIMPRESSUB(RES,W,PEN,SHIFT,ERES,WIDTH,XSPEC,NDATA)
REAL E,P(51),SH(51),T(100,10),RHO(100),ETA(100),FF(100,10)
1),F(150),G(147),GG(100,10),MU,EPRIME(100)
COMMON F,G,FP,GP,SIGO
COMMON /COM/AION,ZION,ATARG,ZTARG,LMAX,RO,R,E
COMMON/RESONANCE/APROJ2,ZPROJ2,ATARG2,ZTARG2,L,RO,RMAX,
QRES,
& ISHIFTFLAG
COMMON/PARM/P,SH
INTEGER I, NUMBER, L

REAL WIDTH,ERES,REDW,EL,EF,INC,NORM

C---------------------------------------------------------------
C calculate the reduced width and normalization to make value
C of resonance 1 at the peak.
C
AION=APROJ2
ZION=ZPROJ2
ATARG=ATARG2
ZTARG=ZTARG2
L=L+3
R=RMAX
NEN=1
E=((ATARG+AION)/ATARG)*QRES !E IN LAB FRAME

CALL PENETSUB(E)
RW = WIDTH/(2.*P(L+1))
SHIFTRES=SH(L+1)
C WRITE(6,*)'RW IN SIMPRES=',RW
NORM= (ERES**3*WIDTH)/(WIDTH/2)**2/ERES
C WRITE(6,*)'CALCULATED NORM'

C SET UP THE INITIAL VALUES
I=1
C Calculate the energy dependent penetrability and width.

100 EGAM(I)=XSPEC(I)
W(I) =2.*PEN(I)*RW
IF(ISHIFTFLAG,NE.0) THEN
  DEL(I)=1.*(SHIFT(I)-SHIFTRES)*RW
ELSE
  DEL(I)=0.
ENDIF
RES(I)=(EGAM(I)**3*W(I))/
& ((EGAM(I)-ERES-DEL(I))**2+(W(I)/2)**2)
& /EGAM(I)
RES(I) = RES(I)/NORM

C SET UP I AND EGAM(I) FOR NEXT PASS
   I = I+1
   IF(I.LE.NDATA) GOTO 100
   RETURN
END

C This is a modified convolution to take into
C account the change of the NaI response function with energy
C The each point of the function RES is
C smeared out according to the NaI response
C function for that energy.

SUBROUTINE SPREADSUB(SPREAD,XSPEC,DETR,RES,NDATA)
INTEGER I,PENFLAG
COMMON/RESPONSE/C1,C2,C3,C4,C5,C6,C7,C8,C9,C10,
& RJOIN,CUTLO,CUTHI,STNCENT

DO 5 I=1,NDATA
   SPREAD(I)=0.
5 CONTINUE

DO 10 I=1,NDATA
   DO 15 J=1,NDATA
      SPREAD(J)=SPREAD(J)+DETR(I,J)*RES(I)
15 CONTINUE
10 CONTINUE

C Normalize curve to have a max height of 1.
CALL FINDMAX(XSPEC,SPREAD,NDATA,IMAX)
YMAX=SPREAD(IMAX)
DO 30 I=1,NDATA
      SPREAD(I)=SPREAD(I)/YMAX
30 CONTINUE

C WRITE(6,*)'spread(imax)='&,SPREAD(IMAX)
RETURN
END
Appendix B: Polarimeter Code

As discussed in Section 2.7, the polarization of the beam from the ABPIS was measured with a polarimeter which employed the $^3\text{He}(\alpha,p)^4\text{He}$ reaction. The code which follows reads in yields from user-defined gates in a spectrum which contains peaks from each of the five detectors. It then carries out a search of $\chi^2$ space using the MINUIT code to determine the beam polarization.

C This program uses MINUIT to calculate the beam moments
C of the TUNL Atomic Beam Source using the Stephenson polarimeter.
C WRITTEN BY MARK BALBES 2/26/90
C
C This program was modified to read previously set data gates directly from
C XSYS data areas.
C Modified by Zandy Williams 5/90

SUBROUTINE FCN(NPAR, GG, FF, X, IFLAG)

C
C
C
C Minuit subroutine
C Inputs: npar=number of ADJUSTABLE parameters
C GG = derivative vector (not used here)
C FF = value of function to be minimized
C X = parameter values
C IFLAG = control value 1 ==> set up constants
C 2 ==> not used here
C 3 ==> write out information
C 4 ==> calculate F (CHI-SQR)
C >4 ==> not used here
C
C ---------------------------------------------------------------
C234567

DOUBLE PRECISION EBEAM
DOUBLE PRECISION AP(9) !REACTION MOMENTS FROM APCALC
DOUBLE PRECISION DAP(9) !ERROR IN MEASURED MOMENTS
DOUBLE PRECISION TR(5,4) !MOMENTS OF REACTION Ti
DOUBLE PRECISION BETA(5) !YIELD RATIO (SEE STEPHENSON REF.)
DOUBLE PRECISION DBETA(5) !ERROR IN BETA
DOUBLE PRECISION CBETA(5) !MINUIT'S GUESS FOR THE BETA'S

140
REAL PAREA(36),PNETAREA(36),PERROR(36),PCHAN(2,36)
REAL UAREA(36),UNETAREA(36),UERROR(36),UCHAN(2,36)

DOUBLE PRECISION GG(50),X(50),ALIM(50),BLIM(50)
CHARACTER*50 FILENAME,BACKNAME
CHARACTER TITLE*60,AFILEM*10,ADATE*10,AATIME*20
DOUBLE PRECISION U(50),WERR(50),XS(50),XTS1(50),WTS(50)
COMMON/PAREXN/U,WERR
COMMON/FIX/IPFIX(50),XS,XTS1,WTS,NPFIX
COMMON/PARM/P,SH
COMMON/TITLEC/TITLE,FDATE
GO TO (10,20,30,40,20),IFLAG

C
C Initialize
C
10 CONTINUE

WRITE(6,*)'WHAT IS THE DEUTERON BEAM ENERGY?'
READ(5,*)EBEAM
C Get the reaction moments from the literature.

CALL APCALC(AP,DAP,EBEAM)

T11A=AP(8) !T11 for alpha particles
T20A=AP(2) !T20 for alpha particles
T21A=AP(4) !T21 for alpha particles
T22A=AP(6) !T22 for alpha particles

T11P=AP(9) !T11 for protons
T20P=AP(3) !T20 for protons
T21P=AP(5) !T21 for protons
T22P=AP(7) !T22 for protons

T200=AP(1) !T20 for 0 degree protons

C Get yields from the detectors and calculate
C beta= yield(pol)/yield(unpol) - 1.

C WRITE(6,*)'POL BCI'
C READ(7,*)POLBCI
C WRITE(6,*)'LEFT SSD POL YIELD'
C READ(7,*)SSDLPOL
C WRITE(6,*)'RIGHT SSD POL YIELD'
C READ(7,*)SSDRPOL
C WRITE(6,*)'LEFT CsI POL YIELD'
C READ(7,*)CSILPOL
C WRITE(6,*)'RIGHT CsI POL YIELD'
C READ(7,*)CSIRPOL
C WRITE(6,*)'0 DEGREE CsI POL YIELD'
C READ(7,*)CSI0POL
CALL PARSE
CALL SUM(100,0,PAREA,PNETAREA,PERROR,PICCHAN)
CALL SUM(101,0,UAREA,UNETAREA,UERROR,UICHAN)

IFLAG = 1
DO 1000,IJ=1,6
  IF(PAREA(IJ) .EQ. -100) THEN
    WRITE(6,1010)IJ
  1010 FORMAT(' GATE ',I1,' NOT SET IN DATA AREA 8')
  IFLAG = 0
END IF
  IF(UAREA(IJ) .EQ. -100) THEN
    WRITE(6,1020)IJ
  1020 FORMAT(' GATE ',I1,' NOT SET IN DATA AREA 9')
  IFLAG = 0
END IF
1000 CONTINUE

IF (IFLAG .EQ. 0) STOP

POLBCI = PAREA(1)
SSDLPOL = PAREA(2)
SSDRPOL = PAREA(3)
CSILPOL = PAREA(4)
CSIRPOL = PAREA(5)
CSI0POL = PAREA(6)

UNPOLBCI = UAREA(1)
SSDLUNPOL = UAREA(2)
SSDRUNPOL = UAREA(3)
CSILUNPOL = UAREA(4)
CSIRUNPOL = UAREA(5)
CSI0UNPOL = UAREA(6)

SSDLPOLN = SSDLPOL/POLBCI
SSDRPOLN = SSRDPOL/POLBCI
CSILPOLN = CSILPOL/POLBCI
CSIRPOLN = CSIRPOL/POLBCI
CSI0POLN = CSI0POL/POLBCI

C WRITE(6,*),'UNPOLBCI'
C READ(7,*)UNPOLBCI
C WRITE(6,*),'LEFT SSD UNPOL YIELD'
C READ(7,*)SSDLUNPOL
C WRITE(6,*),'RIGHT SSD UNPOL YIELD'
C READ(7,*)SSDRUNPOL
C WRITE(6,*),'LEFT CsI UNPOL YIELD'
C READ(7,*)CSILUNPOL
C WRITE(6,*),'RIGHT CsI UNPOL YIELD'
C READ(7,*)CSIRUNPOL
C WRITE(6,*),'0 DEGREE CsI UNPOL YIELD'

142
C READ(7,*)CSI0UNPOL

SSDLUNPOLN = SSDLUNPOL/UNPOLBCI
SSDRUNPOLN = SSDRUNPOL/UNPOLBCI
CSILUNPOLN = CSILUNPOL/UNPOLBCI
CSIRUNPOLN = CSIRUNPOL/UNPOLBCI
CSI0UNPOLN = CSI0UNPOL/UNPOLBCI

BETA(1)=SSDLPOLN/SSDLUNPOLN -1.
BETA(2)=SSDRPOLN/SSDRUNPOLN -1.
BETA(3)=CSILPOLN/CSILUNPOLN -1.
BETA(4)=CSIRPOLN/CSIRUNPOLN -1.
BETA(5)=CSI0POLN/CSI0UNPOLN -1.

RA = UNPOLBCI/POLBCI
DBETA(1) = RA*SQR((SSDLPOLN*(SSDLPOLN+SSDLUNPOLN))/SSDLUNPOLN**1.5
DBETA(2) = RA*SQR((SSDRPOLN*(SSDRPOLN+SSDRUNPOLN))/SSDRUNPOLN**1.5
DBETA(3) = RA*SQR((CSILPOLN*(CSILPOLN+CSILUNPOLN))/CSILUNPOLN**1.5
DBETA(4) = RA*SQR((CSIROPOLN*(CSIROPOLN+CSIRUNPOLN))/CSIRUNPOLN**1.5
DBETA(5) = RA*SQR((CSI0POLN*(CSI0POLN+CSI0UNPOLN))/CSI0UNPOLN**1.5

WRITE(6,*)'BETA1=',BETA(1),'+-',DBETA(1)
WRITE(6,*)'BETA2=',BETA(2),'+-',DBETA(2)
WRITE(6,*)'BETA3=',BETA(3),'+-',DBETA(3)
WRITE(6,*)'BETA4=',BETA(4),'+-',DBETA(4)
WRITE(6,*)'BETA5=',BETA(5),'+-',DBETA(5)
RETURN

20 CONTINUE
RETURN

30 CONTINUE

CBETA(1) = 2.*T11A*X(1)+T20A*X(2)+2.*T21A*X(3)+2.*T22A*X(4)
CBETA(2) = -2.*T11A*X(1)+T20A*X(2)-2.*T21A*X(3)+2.*T22A*X(4)
CBETA(3) = 2.*T11P*X(1)+T20P*X(2)+2.*T21P*X(3)+2.*T22P*X(4)
CBETA(4) = -2.*T11P*X(1)+T20P*X(2)-2.*T21P*X(3)+2.*T22P*X(4)
CBETA(5) = T200*X(2)

OPEN(1,FILE='POL.OUT',STATUS='NEW')
FF= 0.
DO I = 1,5
    FF = FF + ((BETA(I)-CBETA(I))/DBETA(I))**2
C WRITE(1,*)'BETA, CBETA, DBETA, CHI'
C WRITE(1,*)BETA(I),CBETA(I),DBETA(I),
C & ((BETA(I)-CBETA(I))/DBETA(I))**2

143
ENDDO

WRITE(1,*)'CHISQR=','FF
WRITE(1,*)'CHISQR/DF=','FF/4.
WRITE(1,21)X(1),WERR(1)
WRITE(1,22)X(2),WERR(2)
WRITE(1,23)X(3),WERR(3)
WRITE(1,24)X(4),WERR(4)
WRITE(1,*)
21 FORMAT(1t11 = 'F7.4,' +/- 'F7.4)
22 FORMAT(1t20 = 'F7.4,' +/- 'F7.4)
23 FORMAT(1t21 = 'F7.4,' +/- 'F7.4)
24 FORMAT(1t22 = 'F7.4,' +/- 'F7.4)
T20IND = (CSIOPOLN/CSIOUNPOLN -1.)/T200
WRITE(1,*)'t20ind =',T20IND
RETURN

40 CONTINUE

c Calculate the betas (CALLED CBETA) from the reaction analyzing powers and
C Minuit's guess for the beam moments
C X(1)= it11
C X(2)= t20
C X(3)= t21
C X(4)= t22

CBETA(1) = 2.*T11A*X(1)+T20A*X(2)+2.*T21A*X(3)+2.*T22A*X(4)
CBETA(2) = -2.*T11A*X(1)+T20A*X(2)-2.*T21A*X(3)+2.*T22A*X(4)
CBETA(3) = 2.*T11P*X(1)+T20P*X(2)+2.*T21P*X(3)+2.*T22P*X(4)
CBETA(4) = -2.*T11P*X(1)+T20P*X(2)-2.*T21P*X(3)+2.*T22P*X(4)
CBETA(5) = T200*X(2)

FF = 0.
DO I = 1,5
   FF = FF + ((BETA(I)-CBETA(I))/DBETA(I))**2
ENDDO
END

C******************************************************************************
SUBROUTINE APCALC (AP,DAP,ECELL)
IMPLICIT REAL*8 (A-H,O-Z)
C SEPTEMBER 11, 1979  APC CHANGED BY HRB
C UPDATED BY JK ON NOV 12, 1980
C CALCULATES ANALYZING POWERS FOR STEPHENSON POLARIMETER
C GIVEN ECELL, THE ENERGY OF THE BEAM AT THE CENTER OF THE 3HE
C GAS CELL. PARAMETERIZATION DESCRIBED IN INTERNAL REPORT ON
C POLARIMETER
C ANALYZING POWER ERRORS (DAP) NOT IMPLEMENTED IN THIS
C DIMENSION AP(9),DAP(9),ESEG(2,9),TH(5,9)

144
DATA ESEG(1,1),ESEG(2,1)/5.74320,9.73664/
DATA ESEG(1,2),ESEG(2,2)/5.66486,8.46368/
DATA ESEG(1,3),ESEG(2,3)/7.16195,9.71793/
DATA ESEG(1,4),ESEG(2,4)/3.57236,7.38535/
DATA ESEG(1,5),ESEG(2,5)/5.00022,8.87300/
DATA ESEG(1,6),ESEG(2,6)/5.25266,8.48731/
DATA ESEG(1,7),ESEG(2,7)/3.94564,8.62899/
DATA ESEG(1,8),ESEG(2,8)/4.78185,7.18520/
DATA ESEG(1,9),ESEG(2,9)/5.31735,9.04752/
DATA TH(1,1),TH(2,1),TH(3,1),TH(4,1),TH(5,1)
   */.5678260E-1,.6339732E-2,.3626511E+1,-.1288532E+1,-.1357468E-2/
DATA TH(1,2),TH(2,2),TH(3,2),TH(4,2),TH(5,2)
   */-.1386417E-1,-.2157343E-1,.6658048E+1,.3954708E+0,.5704074E-2/
DATA TH(1,3),TH(2,3),TH(3,3),TH(4,3),TH(5,3)
   */.1768071E-2,.101657E-1,.8875718E+1,-.5852313E+0,.2791001E-2/
DATA TH(1,4),TH(2,4),TH(3,4),TH(4,4),TH(5,4)
   */.9926582E-1,.9827271E-3,.43.18327,-1.269458,.1376273E-1/
DATA TH(1,5),TH(2,5),TH(3,5),TH(4,5),TH(5,5)
   */-.3334001E-1,-.1531519E-1,.7052077E+1,.6524656,-.1869513E-2/
DATA TH(1,6),TH(2,6),TH(3,6),TH(4,6),TH(5,6)
   */-.3873436E-1,.2417044E-1,.7998671E+1,-.5987587E+0,-.3338909E-3/
DATA TH(1,7),TH(2,7),TH(3,7),TH(4,7),TH(5,7)
   */.1956599E-1,.4379229E-2,.9937902E+1,-.3045045E+0,.4836683E-2/
DATA TH(1,8),TH(2,8),TH(3,8),TH(4,8),TH(5,8)
   */-.2020095E-1,.3360658E-1,.7457660E+1,-.5700812E+0,.3487879E-2/
DATA TH(1,9),TH(2,9),TH(3,9),TH(4,9),TH(5,9)
   */-.1833195E-1,.1267094E-1,.1108175E+2,-.5992863E+0,.5430263E-2/
DO 10 N=1,9
A1=TH(1,N)
A2=TH(2,N)
B2=TH(3,N)
C2=TH(4,N)
A3=TH(5,N)
IF ECELL-ESEG(1,N)) 20,30,30
30 IF ECELL-ESEG(2,N)) 40,40,50
40 AP(N)=A2*(ECELL-B2)**2+C2
C TYPE *, 'IN 40'
GO TO 10
50 EM=ECELL-ESEG(2,N)
C TYPE *, 'IN 50'
X=A2/A3*(ESEG(2,N)-B2)
AP(N)=A3*(2.*X*EM+EM**2)+(X*A3)**2/A2+C2
GO TO 10
20 EM=ECELL-ESEG(1,N)
C TYPE *, 'IN 20'
X=A2/A1*(ESEG(1,N)-B2)
AP(N)=A1*(2.*X*EM+EM**2)+(X*A1)**2/A2+C2
10 CONTINUE
DO 60 N=1,9
60 DAP(N)=0.
RETURN
END
Appendix C: $^3$H(p,$\gamma$)$^4$He Line-shape Fitting Code

This appendix contains the FORTRAN code for a MINUIT subroutine which fits a $^3$He(p,$\gamma$)$^4$He $\gamma$-ray spectrum with the composite function described in Section 3.5. Also included is a code which takes the parameters of the composite function and reconstructs the line-shape.

```
SUBROUTINE FCN(NPAR, GG, FF, X, IFLAG)

C
C THIS PROGRAM IS DESIGNED TO BE A GENERAL FITTING PROGRAM
C THAT FITS THE LINESHAPE OF A NaI DETECTOR
C
Mark Balbes 11/28/89

C Minuit subroutine
C Inputs: npar=no. of ADJUSTABLE parameters
C GG = derivative vector (not used here)
C FF = value of function to be minimized
C X = parameter values
C IFLAG = control value 1==> set up constants
C 2==> not used here
C 3==> write out information
C 4==> calculate F (CHI-SQR)
C >4==> not used here
C
C -----------------------------------------------------------------------------
C234567

DOUBLE PRECISION GG(50),X(50),XDATA(2000),YDATA(2000),W(50),
* FNCT,ERR(2000),A0,XP(50),AP,BP,EN
DOUBLE PRECISION UPB,LOWB,INTLO,INTHI,CENT,FUNC
CHARACTER*80 CHAR(30)
COMMON /KEEP/XDATA,YDATA,NDATA,ERR
COMMON /ZERO/INTHI,A,B,C,D
GO TO (10,20,30,40,20),IFLAG

C
C Initialize
C
10 CONTINUE
C
146
C READS IN DATA AT THIS POINT
C
C Read in lower and upper bound of data to be fit.
    READ(5,*,END=101)LOWB,UPB  !Read from Cards.info file

C Read in lineshape spectrum to be fit.
    OPEN(UNIT=20,STATUS='OLD')
    I = 1
100 READ(20,*,END=101)TEMPX,TEMPY,TEMPE  !Read one line of data.
    IF(TEMPX.GE.LOWB .AND. TEMPX.LE.UPB) THEN  !Take log of the data
        XDATA(I)=TEMPX
        IF(TEMPY.NE.0.) THEN
            ERR(I)=TEMPE/TEMPY  !Avoid divide by zero error
            YDATA(I)=LOG(TEMPY)  !Avoid log(0) error
        ELSE
            ERR(I)=0.
            YDATA(I)=0.
        ENDIF
        IF(ERR(I).EQ.0.) ERR(I)=I.  !Avoid divide by zero error
            IF(ydata not zero but err is
                !(this is almost redundant)
        I=I+1
    ENDIF
GOTO 100
!101 NDATA=I-1  !NDATA IS NOW #OF DATA POINTS
C WRITE(6,*)'EXITED LOOP'
    CLOSE(20)
C WRITE(6,*)'CLOSED 20'

RETURN

C -----------------------------------------------

20 CONTINUE
    RETURN
C -----------------------------------------------

30 CONTINUE
C THIS IS THE LAST CALL TO FCN -- THE FITTING IS FINISHED BY NOW
C
C Define the constants in the Exponential functions

    C=X(1)
    D=X(2)
    F=X(3)
    G=X(4)
    H=X(5)
    J=X(6)
    CENT=X(7)
    JNTHI=X(8)
S=X(9)
T=X(10)
C To get matching function and first derivative at the
C jnthi two parameters become dependent. I've chosen A and B.

B=G+2*H*(JNTHI-CENT)+3*I*(JNTHI-CENT)**2
&  -2*C*(JNTHI-CENT)-3*D*(JNTHI-CENT)**2
&  +4.*T*(JNTHI-CENT)**3-4.*S*(JNTHI-CENT)**3

A=F+G*(JNTHI-CENT)+H*(JNTHI-CENT)**2+J*(JNTHI-CENT)**3
&  -B*(JNTHI-CENT)-C*(JNTHI-CENT)**2-D*(JNTHI-CENT)**3
&  +T*(JNTHI-CENT)**4-S*(JNTHI-CENT)**4

C The data has been fit to F(x-cent) because it is
C easier for Minuit. It would
C be nice to have the parameters that give F(x) so
C that centroid doesn't need to be known to reconstruct
C the curve. Also Mulfit needs this form of the parameters.

AP=A-B*CENT+C*CENT**2-D*CENT**3+S*CENT**4
BP=B-2*C*CENT+3*D*CENT**2-4.*S*CENT**3
XP(1)=C-3*D*CENT+6*S*CENT**2
XP(2)=D-4.*S*CENT

XP(3)=F-G*CENT+H*CENT**2-J*CENT**3+T*CENT**4
XP(4)=G-2*H*CENT+3*J*CENT**2-4.*T*CENT**3
XP(5)=H-3*J*CENT+6.*T*CENT**2
XP(6)=J-4.*T*CENT
XP(7)=S
XP(8)=T

open (unit=44,name='FitLINE4.out',type='new',disp='save')
WRITE(44,*)AP
WRITE(44,*)BP
WRITE(44,*)XP(1)
WRITE(44,*)XP(2)
WRITE(44,*)XP(3)
WRITE(44,*)XP(4)
WRITE(44,*)XP(5)
WRITE(44,*)XP(6)
WRITE(44,*)XP(7)
WRITE(44,*)JNTHI
WRITE(44,*)
WRITE(44,*)'LOWB=',LOWB
WRITE(44,*)'UPB=',UPB
write(44,169)CHISQR
169 format (/,'Total chi-squared = ',F9.3)
CHIMER=CHISQR/(NDATA-7.)
write(44,168)CHIMER
168 format (/,'Chi-squared/degree of freedom = ',F9.3)
C Calculate the function and its slope at the joint to
C see that they match.
C EN=JNTHI
C FNCT1=A+B*(EN-CENT)+C*(EN-CENT)**2+D*(EN-CENT)**3+S*(EN-
C CENT)**4
C FNCT2=F+G*(EN-CENT)+H*(EN-CENT)**2+J*(EN-CENT)**3+T*(EN-
C CENT)**4
C WRITE(44,*)'F(JOINT LOW)=','FNCT1
C WRITE(44,*)'F(JOINT HIGH)=','FNCT2
C WRITE(44,*)'DIF=','FNCT1-FNCT2
C
C FP1=B+2.*C*(EN-CENT)+3.4*D*(EN-CENT)**2+4.5*S*(EN-CENT)**3
C FP2=G+2.*H*(EN-CENT)+3.4*J*(EN-CENT)**2+4.5*T*(EN-CENT)**3
C WRITE(44,*)'SLOPE(JOINT LOW)=','FP1
C WRITE(44,*)'SLOPE(JOINT HIGH)=','FP2
C WRITE(44,*)'DIF=','FP1-FP2
C Calculate the function and its slope at the joint to
C see that they match.
C EN=JNTHI
FNCT1=AP+BP*EN+XP(1)*EN**(2)+XP(2)*EN**(2)+XP(7)*EN**(2)
FNCT2=XP(3)+XP(4)*EN+XP(5)*EN**(2)+XP(6)*EN**(2)+XP(8)*EN**(2)
WRITE(44,*)'F(EN,JOINT LOW)=','FNCT1
WRITE(44,*)'F(EN,JOINT HIGH)=','FNCT2
WRITE(44,*)'DIF=','FNCT1-FNCT2
C Spit out the data being fit
OPEN(2,FILE='DATA.OUT',STATUS='NEW')
DO 23 I=1,NDATA
WRITE(2,*)XDATA(I),EXP(YDATA(I)),ERR(I)*EXP(YDATA(I))
23 CONTINUE
C Make new cards.out file to be able to continue minimizing with
OPEN(i,FILE='CARDS4.OUT',STATUS='OLD')
I=1
24 READ(1,163-END=25)CHAR(I)
163 FORMAT(A80)
I=I+1
GOTO 24
25 CLOSE(I)
INUM = I-1
INUM=# OF LINES IN CARDS.OUT
OPEN(1,FILE='CARDS4.OUT',STATUS='NEW')
WRITE(1,163)CHAR(I)
WRITE(1,167)'C',X(I),'.0001'

149
WRITE(1,167)'D',X(2),'.0001'
WRITE(1,167)'F',X(3),'.0001'
WRITE(1,167)'G',X(4),'.0001'
WRITE(1,167)'H',X(5),'.0001'
WRITE(1,167)'J',X(6),'.0001'
WRITE(1,171)'CENT',X(7),'.0001'
WRITE(1,172)'JNTHI',X(8),'.0001'
WRITE(1,167)'S',X(9),'.0001'
WRITE(1,167)'T',X(10),'.0001'
DO 26 I=12,INUM
   WRITE(1,163)CHAR(I)
26 CONTINUE
167   FORMAT(A1,3X,F15.6,3X,A5)
171   FORMAT(A4,3X,F15.6,3X,A5)
172   FORMAT(A5,3X,F15.6,3X,A5)
CLOSE(1)
WRITE(6,*)'FINAL CALL'
RETURN

C ---------------------------------------------------------------

40 CONTINUE

C*************************************************************************
C FF = 0.
C*************************************************************************
C LINESHAPE EQUATION
C*************************************************************************
C WRITE(6,*)'ENTERED LINESHAPE EQN'
C
C C Define the constants in the Exponential functions

C=X(1)
D=X(2)
F=X(3)
G=X(4)
H=X(5)
J=X(6)
CENT=X(7) !Not true centroid, but is used to help Minuit fit
JNTHI=X(8) !Matching energy located between the two functions.
 !Value should be just less than the true centroid.
S=X(9)
T=X(10)
C To get matching function and first derivative at the
C jnthi two parameters become dependent. I've chosen A and B.

B=G+2*H*(JNTHI-CENT)+3*J*(JNTHI-CENT)**2+4*T*(JNTHI-CENT)**3
&  -2*C*(JNTHI-CENT)-3*D*(JNTHI-CENT)**2-4*S*(JNTHI-CENT)**3

150

DO 211 I=1, NDATA     !Loop calculates lineshape.
EN=XdData(I)
IF(EN.LT.JNTHI) THEN
  FNCT=A+B*(EN-CENT)+C*(EN-CENT)**2+D*(EN-CENT)**3 & +S*(EN-CENT)**4
ELSE
  FNCT=F+G*(EN-CENT)+H*(EN-CENT)**2+J*(EN-CENT)**3 & +T*(EN-CENT)**4
ENDIF

211   FF = FF + ((YDATA(I)-FNCT)/ERR(I))**2     !Calculate chi squared

CHISQR = FF

C WRITE(6,*)'ABOUT TO EXIT LINESHAPE EQN'
C
C*******************************************************************************
  RETURN
  END

DOUBLE PRECISION ILO,IHI,INC
DOUBLE PRECISION C1,C2,C3,C4
DOUBLE PRECISION C5,C6,C7,C8
DOUBLE PRECISION C9,JNTHI,S,T
DOUBLE PRECISION X,Y
C TYPE *, 'INPUT LINESHAPE PARAMETERS (TAIL AND LOW ENERGY) C1-C4'
  READ (11,*)C1,C2,C3,C4,S
C TYPE *, 'INPUT LINESHAPE PARAMETERS (PEAK AND HIGH ENERGY) C5-C8'
  READ (11,*)C5,C6,C7,C8,T
C TYPE *, 'INPUT MATCHING POINT C9 (JOINT)'
  READ (11,*)JNTHI
  TYPE *, JNTHI = ',JNTHI
  TYPE *, 'INPUT PLOTTING RANGE FOR LINESHAPE (LOW,HIGH,INC)'
  READ (5,*)ILO,IHI,INC
  OPEN (UNIT=9,NAME='LINESHAPE4.OUT',TYPE='NEW',DISP='SAVE')
  WRITE(6,*)C1,C2,C3,C4,C5,C6,C7,C8,S,T,JNTHI
  X=ILO
10   IF (X.LT. JNTHI) THEN
      Y=EXP(C1+C2*X+C3*X*X+C4*X*X*X+S*X*X*X*X)
    ELSE
      Y=EXP(C5+C6*X+C7*X*X+C8*X*X*X+T*X*X*X*X)
    ENDIF
  C type *,x,y

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WRITE (9,*) X, Y
X = X + INC
IF (X.LE.IHJ) GOTO 10
C WRITE (9,*) C1, C2, C3, C4
C WRITE (9,*) C5, C6, C7, C8
C WRITE (9,*) IC9
99 STOP
END
Appendix D: The J-Independent TME Coefficients

Listed below are the J-independent TME coefficients that have been used in the analysis of the current experiment. The coefficients are identified by the spin and orbital angular momentum quantum numbers of the two transition matrix elements which it multiplies and the mode and multipolarity of the radiation. The Legendre coefficients $A_i, C_i,$ and $E_i$ are given as the real part of the sum of complex transition matrix element products. The $B_i$ coefficients are given as the imaginary part of the sum. For example:

\[
A_0 = 2 (^2s(E1))^2 + 12 (^4d(E1))^2 + 10 (^2d(E1))^2 + 4 (^4s(E1))^2 \\
+ 6 (^2p(M1))^2 + 12 (^4p(M1))^2 + 6 (^2p(E2))^2 + 12 (^4p(E2))^2
\]

\[
A_1 = 8.325 ^2s(E1) ^2p(M1) \cos(\theta_{2p} - \theta_{2s}) + 8.363 ^2s(E1) ^2p(E2) \cos(\theta_{2p} - \theta_{2s}) \\
+ 4.731 ^4d(E1) ^4p(E2) \cos(\theta_{4p} - \theta_{4d}) + 13.543 ^2d(E1) ^2p(M1) \cos(\theta_{2p} - \theta_{2d}) \\
- 1.989 ^2d(E1) ^2p(E2) \cos(\theta_{2p} - \theta_{2d}) + 16.617 ^4s(E1) ^4p(E2) \cos(\theta_{4p} - \theta_{4s})
\]

\[
B_3 = 3.957 ^4d(E1) ^4p(E2) \sin(\theta_{4p} - \theta_{4d}) + 1.056 ^2d(E1) ^2p(E2) \sin(\theta_{2p} - \theta_{2d}) \\
- 4.383 ^2d(E1) ^4p(E2) \sin(\theta_{4p} - \theta_{2d}) + 3.063 ^4d(E1) ^2p(E2) \sin(\theta_{2p} - \theta_{4d})
\]

Table D.1 A lists of all of the coefficients for the TME’s which were used in the analysis of the data. The code which calculates these coefficients from the J-dependent coefficients follows the table.

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PROGRAM MODLS

C THIS PROGRAM IS DESIGNED TO READ THE FOR022 OUTPUT FILE FROM
C LS (OR TEST2 OR COEF) AND MODIFY THE FILE TO A NO J DEPENDENCE
C FORMAT. I.E. ALL TERMS WITH LIKE S,S',L,L' QUANTUM #'S BUT
DIFFERENT
C J AND J' VALUES WILL HAVE THEIR COEFFICIENTS ADDED.
C
C EXAMPLE:
C A0 = (3.5)(2S2)(2P2) + (1.5)(2S2)(2P4) = (5.0)(2S)(2P)
C
C WHERE NOTATION = 2S+1 L 2J+1
C
C INTEGER COUNT(160,100)
DIMENSION A(0:4),B(0:4),C(0:4),D(0:4),E(0:4),FRAC(20)
DIMENSION TME(10,10,10,2,4),PHASE(10,10,10,2,4)
DIMENSION ISS(20),ILL(20),IJJ(20),IEEMM(20),IMM(20)
CHARACTER*80 TITLE
CHARACTER*1 NAME,EM1,EM2

DOUBLE PRECISION G(50),X(50),XDATA(50),YDATA(50),W(50),
* FNCT,ERR(50),A0
COMMON /KEEP/XDATA,YDATA,NDATA1,NDATA2
* ERR, tme, phase, radtodeg, NDATA3, NDATA4,ndata5
COMMON /LS/COEF,I,MAX(5,0:4),COEFF(5,0:4,500),IEM1(5,0:4,500),
& IMULT1(5,0:4,500),IS1(5,0:4,500),IL1(5,0:4,500),IJ1(5,0:4,500),
& IEM2(5,0:4,500),IMULT2(5,0:4,500),IS2(5,0:4,500),IL2(5,0:4,500),
& IJ2(5,0:4,500)

C
C---------------------------------------------------------------------
C READ IN THE TME QUANTUM #'S
OPEN(UNIT=22,STATUS='OLD')
READ(22,200,END=99) BLANK
READ(22,200,END=99) TITLE
READ(22,200,END=99) BLANK
200 FORMAT(A80)
C
C *** First time through sets values of ICF and IK and IMAX(ICF,IK)
C *** where ICF and IK give the ang. dist. coeff. (A1 or B3 or C2 etc.)
C *** and IMAX tells the number of terms in the LS eqn. for that coeff.
C
199 READ (22,201,END=99)NAME,IK,IM,M,XA,D,DCOEF,EM1,IMUL1,IDS1,
     &IDJ1,EM2,IMUL2,IDS2,IF,IDS2,IDS2,IF,IDS2,IF
201 FORMAT (2X,A1,I1,3X,I3,2X,I3,3X,F7.3,5X,A1,I1,4X,I2,4X,I2,4X,I2,
     &6X,A1,I1,4X,I2,4X,I2,4X,I2)
IF (NAME .EQ. 'A')ICF=1
IF (NAME .EQ. 'B')ICF=2
IF (NAME .EQ. 'C')ICF=3
IF (NAME .EQ. 'D')ICF=4
IF (NAME .EQ. 'E')ICF=5
IMAX(ICF,IK)=MAX
COEFF(ICF,IK,1)=DCOEF
IMULT1(ICF,IK,1)=IMUL1
IEM1(ICF,IK,1)=1
IF (EM1 .EQ. 'M')IEM1(ICF,IK,1)=2
IS1(ICF,IK,1)=IDS1
IL1(ICF,IK,1)=IDJ1
IJ1(ICF,IK,1)=IDJ1
IMULT2(ICF,IK,1)=IMUL2
IEM2(ICF,IK,1)=1
IF (EM2 .EQ. 'M')IEM2(ICF,IK,1)=2
IS2(ICF,IK,1)=IDS2
IL2(ICF,IK,1)=IDJ2
IJ2(ICF,IK,1)=IDJ2
C
C *** Succeeding reads, values of ICF, IK, IMAX already set from above
C
51 CONTINUE
GO TO 199
99 CLOSE(22)
C
C---------------------------------------------------------------------
C
C ADD COEFFICIENTS WITH THE SAME S AND L QUANTUM #S

DO 100 ICF=1,5
  DO 250 IK=0,4
    DO 300 II=1,IMAX(ICF,IK)-1
      S1 = IS1(ICF,IK,II)
      L1 = IL1(ICF,IK,II)
      REM1 = IEM1(ICF,IK,II)
100 CONTINUE
250 CONTINUE
300 CONTINUE
MULT1 = IMULT1(ICF, IK, I1)
S2 = IS2(ICF, IK, I1)
L2 = IL2(ICF, IK, I1)
REM2 = IEM2(ICF, IK, I1)
MULT2 = IMULT2(ICF, IK, I1)

IF (COEFF(ICF, IK, I1), EQ, 0.) GOTO 300

DO 400 I2 = I1 + 1, IMAX(ICF, IK)
S1P = IS1(ICF, IK, I2)
L1P = IL1(ICF, IK, I2)
REM1P = IEM1(ICF, IK, I2)
MULT1P = IMULT1(ICF, IK, I2)
S2P = IS2(ICF, IK, I2)
L2P = IL2(ICF, IK, I2)
REM2P = IEM2(ICF, IK, I2)
MULT2P = IMULT2(ICF, IK, I2)
COEP = COEFF(ICF, IK, I2)

IF (S1.EQ.S1P .AND. L1.EQ.L1P .AND. REM1.EQ.REM1P)
& .AND. MULT1.EQ.MULT1P .and.,
& S2.EQ.S2P .AND. L2.EQ.L2P .AND. REM2.EQ.REM2P
& .AND. MULT2.EQ.MULT2P) then
  COEFF(ICF, IK, I1) = COEFF(ICF, IK, I1) + COEFF(ICF, IK, I2)
  COEFF(ICF, IK, I2) = 0.
ELSEIF (S1.EQ.S2P .AND. L1.EQ.L2P .AND. REM1.EQ.REM2P)
& .AND. MULT1.EQ.MULT2P .and.,
& S2.EQ.S1P .AND. L2.EQ.L1P .AND. REM2.EQ.REM1P
& .AND. MULT2.EQ.MULT1P) THEN
  IF (ICF.EQ.2) COEFF(ICF, IK, I2) = -1. * COEFF(ICF, IK, I2)
  COEFF(ICF, IK, I1) = COEFF(ICF, IK, I1) + COEFF(ICF, IK, I2)
  COEFF(ICF, IK, I2) = 0.
ENDIF

400  CONTINUE
300  CONTINUE
250  CONTINUE
100  CONTINUE

C---------------------------------------------------------------
C COUNT HOW MANY NONZERO TERMS THERE ARE PER COEFFICIENT

DO 500 ICF = 1, 5
DO 600 IKP = 1, 5
IK = IKP - 1
COUNT(ICF, IK) = 0
DO 700 I = 1, IMAX(ICF, IK)
  IF (COEFF(ICF, IK, I) .NE. 0.) COUNT(ICF, IK) = COUNT(ICF, IK) + 1

700  CONTINUE
600  CONTINUE
500  CONTINUE
C
C WRITE A NEW LS FILE IN UNIT 23
  open(unit=23,status='NEW')
  WRITE (23,200) BLANK
  WRITE (23,200) TITLE
  WRITE (23,200) BLANK
  DO 10 ICF = 1,5
      DO 20 IK=0,4
          J=0
          DO 30 I = 1,IMAX(ICF,IK)
C IF (ABS(COEFF(ICF,IK,I)).LE..02) COEFF(ICF,IK,I)=0.
    IF (ICF,EQ.1) NAME='A'
    IF (ICF,EQ.2) NAME='B'
    IF (ICF,EQ.3) NAME='C'
    IF (ICF,EQ.4) NAME='D'
    IF (ICF,EQ.5) NAME='E'
    EM1 = 'E'
    IF (IEM1(ICF,IK,I),EQ.2) EM1 = 'M'
    EM2 = 'E'
    IF (IEM2(ICF,IK,I),EQ.2) EM2 = 'M'
    IF (COEFF(ICF,IK,I),NE.0.) THEN
        J=J+1
        WRITE(23,201) NAME,IK,J,COUNT(ICF,IK),COEFF(ICF,IK,I),EM1,
            &IMULT1(ICF,IK,I),IS1(ICF,IK,I),IL1(ICF,IK,I),
            &-1,EM2,IMULT2(ICF,IK,I),
            &IS2(ICF,IK,I),IL2(ICF,IK,I),-1
      ENDIF
  30 CONTINUE
  20 CONTINUE
  10 CONTINUE
  CLOSE(23)
C
END
References


[CleTBP] T.B. Clegg et al., to be published.


[HayPri] E. Hayward (private communication).

[HofPri] H.M. Hofmann (private communication).


Biography

Mark Joseph Balbes


Positions: Research Assistant
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Selected Publications:


Ground-State Widths of \(^5\)He and \(^5\)Li Determined in the \(^3\)H(d,\(\gamma\))\(^5\)He and the \(^3\)He(d,\(\gamma\))\(^5\)Li Reactions, M.J. Balbes, G. Feldman, L.H. Kramer, D.R. Tilley, H.R. Weller, Physical Review C 43 343 (1991).


