LOW ENERGY PROTON CAPTURE STUDY OF THE $^{14}\text{N}(p,\gamma)^{15}\text{O}$ REACTION

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ABSTRACT

Stephen Michael Daigle: Low Energy Proton Capture Study of the $^{14}\text{N}(p, \gamma)^{15}\text{O}$ Reaction.
(Under the direction of Arthur E. Champagne.)

The $^{14}\text{N}(p, \gamma)^{15}\text{O}$ reaction regulates the rate of energy production for stars slightly more massive than the sun throughout stable hydrogen burning on the main sequence. The $^{14}\text{N}(p, \gamma)^{15}\text{O}$ reaction rate also determines the luminosity for all stars after leaving the main sequence when their cores have exhausted hydrogen fuel, and later when they become red giant stars. The significant role that this reaction plays in stellar evolution has far-reaching consequences, from neutrino production in our Sun, to age estimates of globular clusters in our Galaxy. The weak cross section and inherent coincidence summing in the $^{15}\text{O}$ $\gamma$-ray decay scheme make a precision measurement of the astrophysical S-factor especially challenging, particularly for the ground-state transition.

The present study, performed in the Laboratory for Experimental Nuclear Astrophysics (LENA), was aimed at measuring the ground-state transition at low energy by utilizing a new 24-element, position-sensitive, NaI(Tl) detector array. Because the array is highly segmented, the $^{14}\text{N}(p, \gamma)^{15}\text{O}$ S-factor was evaluated for transitions to the ground, 5.18, 6.18, and 6.79 MeV states without the need for coincidence summing corrections. Additionally, the position-sensitivity of the detector was exploited to measure the angular correlation of the two-photon cascades. Software cuts were made to the data in order to identify single and coincident $\gamma$-ray events and a fraction fit analysis technique was used to extract the characteristic $^{15}\text{O}$ peaks from the composite $\gamma$-ray spectrum. The results from the current work demonstrated a new approach to measuring weak nuclear cross sections near astrophysically relevant energies that, with refinements, has broader applications in $\gamma$-ray spectroscopy.
Dedicated to the most influential teachers in my life – my mom and dad.
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# TABLE OF CONTENTS

LIST OF TABLES ......................................................................................... xi
LIST OF FIGURES ...................................................................................... xii
1 INTRODUCTION ...................................................................................... 1
   1.1 Astrophysical Motivation ................................................................. 1
      1.1.1 Proton-proton Chain ............................................................... 1
      1.1.2 CN Cycle .............................................................................. 3
      1.1.3 Solar Metallicity ............................................................... 5
      1.1.4 Globular Cluster Ages ....................................................... 9
   1.2 Nuclear Reactions .......................................................................... 14
      1.2.1 Nonresonant Cross Sections ............................................... 16
      1.2.2 Resonant Cross Sections ................................................... 19
      1.2.3 Astrophysical S-factor and Reaction Rates ....................... 21
   1.3 Previous Measurements .................................................................. 26
      1.3.1 Coincidence Summing Considerations ................................. 29
   1.4 The Project Goals ........................................................................... 33
2 ACCELERATOR AND DETECTOR ............................................................ 34
   2.1 The Laboratory for Experimental Nuclear Astrophysics ............... 34
   2.2 JN Van de Graaff Accelerator .................................................... 34
   2.3 Target Chamber .......................................................................... 37
   2.4 APEX Trigger Detector ............................................................. 38
      2.4.1 Assembly .......................................................................... 41
      2.4.2 Characterization ............................................................... 47


## LIST OF TABLES

1.1 Branching ratios for the decay of the $E_{cm}^{\gamma} = 259$ keV resonance  

2.1 Tuning parameter settings for the JN Van de Graaff accelerator  

2.2 Resonances used for calibrating the LENA analyzing magnet  

2.3 Photomultiplier tube comparison  

2.4 Assembled APEX configuration  

2.5 APEX NaI(Tl) attenuation coefficients  

3.1 Contaminant proton-induced capture reactions  

3.2 Tuning parameter settings for implanting $N^+_2$ ions into tantalum  

3.3 Summary of nitrogen targets analyzed with RBS  

4.1 Energies of $\gamma$ rays used for calibrating the APEX detector array  

4.2 Run time summary of the $S$-factor measurement  

4.3 $S$-factor statistical uncertainty  

4.4 $S$-factor systematic uncertainty  

4.5 Summary of $S$-factor results  

5.1 Angular correlation functions measured at the $E_{cm}^{\gamma} = 259$ keV resonance  

A.1 Photonis XP2012 PMT specifications  

A.2 Hamamatsu R580 PMT specifications  

B.1 Bias voltage, amplifier, and CFD settings
# List of Figures

1.1 Diagram of the \textit{pp} chain reactions in the sun .............................................. 3
1.2 Stellar energy production for the \textit{pp} chain and CN cycle ................................. 4
1.3 Sequence of reactions in the CNO bi-cycle .............................................................. 5
1.4 Reaction times in the CN cycle .................................................................................. 6
1.5 Solar neutrino energy spectrum .................................................................................. 8
1.6 Hubble Space Telescope image of NGC 6397 ............................................................. 10
1.7 Simulated time sequence of a globular cluster ............................................................ 12
1.8 H-R diagram of NGC 6397 ......................................................................................... 13
1.9 Direct capture level scheme of the \(^{14}\text{N}(p, \gamma)^{15}\text{O}\) reaction .......................... 17
1.10 Resonant capture level scheme of the \(^{14}\text{N}(p, \gamma)^{15}\text{O}\) reaction ........................ 20
1.11 Energy dependence of the cross section and \(S\)-factor ............................................. 22
1.12 Dominant energy-dependent functions for charged-particle reactions ...................... 25
1.13 Previous ground-state \(S\)-factor measurements ......................................................... 28
1.14 Coincidence summing of \(\gamma\) rays in the \(^{14}\text{N}(p, \gamma)^{15}\text{O}\) reaction .................. 31

2.1 Laboratory for Experimental Nuclear Astrophysics ..................................................... 35
2.2 Photograph of the JN Van de Graaff ion source ......................................................... 36
2.3 Schematic drawing of the LENA target chamber ....................................................... 38
2.4 Photograph of the assembled APEX detector ............................................................. 39
2.5 Exploded view drawing of a NaI(Tl) segment ............................................................ 40
2.6 Photograph of the APEX detector photomultiplier tubes .......................................... 42
2.7 Photograph of a NaI(Tl) scintillation crystal .............................................................. 43
2.8 Photograph of the APEX detector assembly ............................................................... 44
2.9 Drawing of the NaI(Tl) segment configuration in APEX ........................................... 46
2.10 Schematic drawing of a NaI(Tl) segment ................................................................. 47
2.11 Drawing of the APEX radial collimator .................................................................... 49
<table>
<thead>
<tr>
<th>Section</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.12</td>
<td>Photograph of APEX and the radial collimator on the beamline</td>
</tr>
<tr>
<td>2.13</td>
<td>Drawing of the relative position of APEX and radial collimator</td>
</tr>
<tr>
<td>2.14</td>
<td>Reconstructed positions of $\gamma$ rays from a $^{60}$Co source</td>
</tr>
<tr>
<td>2.15</td>
<td>Reconstructed position versus source position</td>
</tr>
<tr>
<td>2.16</td>
<td>Difference between the reconstructed and actual position</td>
</tr>
<tr>
<td>2.17</td>
<td>Schematic diagram of the APEX electronics setup</td>
</tr>
<tr>
<td>2.18</td>
<td>Reconstructed energy versus position in a 2-d histogram</td>
</tr>
<tr>
<td>2.19</td>
<td>G4ANT4 geometry used in the APEX Monte Carlo simulation</td>
</tr>
<tr>
<td>2.20</td>
<td>Full-energy peak efficiencies of the APEX and HPGe detectors</td>
</tr>
<tr>
<td>3.1</td>
<td>Polycarbonate target box</td>
</tr>
<tr>
<td>3.2</td>
<td>The UNC ion implanter</td>
</tr>
<tr>
<td>3.3</td>
<td>Schematic of the ion implantation system</td>
</tr>
<tr>
<td>3.4</td>
<td>Photograph of Ta$_2$N$_3$ and TiN targets</td>
</tr>
<tr>
<td>3.5</td>
<td>Schematic of the RBS experimental setup</td>
</tr>
<tr>
<td>3.6</td>
<td>Photograph of the RBS target wheel</td>
</tr>
<tr>
<td>3.7</td>
<td>Silicon detector RBS energy spectra</td>
</tr>
<tr>
<td>3.8</td>
<td>Yield curves of a nitrogen implanted target</td>
</tr>
<tr>
<td>3.9</td>
<td>Yield curve comparison between Ta$_2$N$_3$ and TiN targets</td>
</tr>
<tr>
<td>4.1</td>
<td>Photograph of the APEX and HPGe detector configuration</td>
</tr>
<tr>
<td>4.2</td>
<td>Drawing of the APEX array divided into pixels</td>
</tr>
<tr>
<td>4.3</td>
<td>APEX total efficiency</td>
</tr>
<tr>
<td>4.4</td>
<td>Drawing of multiplicity events detected by the APEX detector</td>
</tr>
<tr>
<td>4.5</td>
<td>Resonance data with imposed multiplicity 1 and 2 cuts</td>
</tr>
<tr>
<td>4.6</td>
<td>Fit of multiplicity 2 resonance data</td>
</tr>
<tr>
<td>4.7</td>
<td>Fit of multiplicity 1 resonance data</td>
</tr>
<tr>
<td>4.8</td>
<td>Overlay of 180 keV direct capture data with Monte Carlo simulation</td>
</tr>
<tr>
<td>4.9</td>
<td>Fit of multiplicity 2 $E_{\text{eff}} = 235$ keV data</td>
</tr>
<tr>
<td>4.10</td>
<td>Fit of multiplicity 2 $E_{\text{eff}} = 216$ keV data</td>
</tr>
</tbody>
</table>
4.11 Fit of multiplicity 2 $E_{\text{eff}} = 195$ keV data .................................................. 100
4.12 Measured $S$-factor for the 5.18, 6.18, and 6.79 MeV transitions .............................. 102
4.13 Fit of multiplicity 1 $E_{\text{eff}} = 235$ keV data ....................................................... 103
4.14 Fit of multiplicity 1 $E_{\text{eff}} = 216$ keV data ....................................................... 104
4.15 Fit of multiplicity 1 $E_{\text{eff}} = 195$ keV data ....................................................... 105
4.16 Residuals for fraction fits to multiplicity 1 data ....................................................... 107
4.17 Measured $S$-factor for the ground-state transition .................................................... 108
4.18 Measured total $S$-factor ......................................................................................... 112

5.1 Drawing of correlated $\gamma$ rays detected by APEX ..................................................... 114
5.2 Two-dimensional energy cuts used in the angular correlation measurement .......... 117
5.3 Schematic level diagram of the resonant capture angular correlation ...................... 118
5.4 Angular correlation of $\gamma$ rays in the resonant capture 5.18 MeV transition .......... 124
5.5 Angular correlation of $\gamma$ rays in the resonant capture 6.18 MeV transition .......... 125
5.6 Angular correlation of $\gamma$ rays in the resonant capture 6.79 MeV transition .......... 126
5.7 Angular correlation of $\gamma$ rays in the direct capture 6.18 MeV transition .......... 127
CHAPTER 1: INTRODUCTION

1.1 Astrophysical Motivation

The mechanism by which stars generate energy is dependent upon their evolutionary stage and composition. The oldest stars observed, referred to as Population II stars, consist mostly of hydrogen, and thus their energy is produced primarily by the fusion of protons into helium. This process is known as the proton-proton chain [Bethe and Critchfield, 1938]. The stars observed today are at least second-generation Population I stars, formed from hydrogen and material ejected during the explosive deaths of Population II stars. In stars slightly more massive than the sun, higher density and temperature inside the core will favor a chain of nuclear reactions, known as the carbon nitrogen (CN) cycle [von Weizsäcker, 1938, Bethe, 1939], that still converts four protons into helium but involves heavier elements that act as catalysts. The rate of energy production in the CN cycle is governed by the slowest step, the $^{14}\text{N}(p, \gamma)^{15}\text{O}$ reaction, which acts as a “bottleneck” in the cycle. This work will examine the $^{14}\text{N}(p, \gamma)^{15}\text{O}$ reaction and its impact on the energy generation in our Sun and age estimates of some of the oldest clusters of stars in our Galaxy.

1.1.1 Proton-proton Chain

The process during which four hydrogen nuclei fuse together to form helium is called “hydrogen burning” and is responsible for the observed luminosity of stars for the greater part of their lives. Hydrogen burning takes place in the cores of stars, and in later stages of stellar evolution, in spherical shells surrounding the helium core or helium-burning shells.
This fusion of hydrogen into helium is characterized by the nuclear reaction

$$4p \rightarrow ^4\text{He} + 2e^+ + 2\nu + 26.73\text{ MeV}$$

(1.1)

which produces 2 positrons $e^+$, 2 neutrinos $\nu$, and 26.73 MeV of energy in addition to $^4\text{He}$. The probability of four protons fusing simultaneously into helium is essentially zero. Instead, a series of two-body interactions take place as part of the proton-proton ($pp$) chain, and produce the same end result in Equation 1.1. A schematic diagram of the nuclear reactions involved in the $pp$ chain is shown in Figure 1.1. The first two steps of the main $pp$ chain, $ppI$, must occur twice before the last step can proceed and involves a total of six protons, two of which are released in the final step. The competing reaction

$$^3\text{He} + ^4\text{He} \rightarrow ^7\text{Be}$$

(1.2)

may occur instead of the last step of the $ppI$ chain resulting in two possible branches from $^7\text{Be}$, labeled as the $ppII$ and $ppIII$ chains. All three $pp$ chains operate simultaneously in a star but the $ppI$, $ppII$, and $ppIII$ chains are the main producers of $^4\text{He}$ at temperatures of $T < 18\text{ MK}$, $T = 18 - 25\text{ MK}$, and $T > 25\text{ MK}$ respectively [Iliadis, 2007]. The branching percentages shown in Figure 1.1 correspond to those in our sun, and thus the $ppII$ and $ppIII$ chains contribute a minor amount to the sun’s luminosity. The least probable branch

$$^3\text{He} + p \rightarrow ^4\text{He} + e^+ + \nu_e$$

(1.3)

burns only about $10^{-7}$ of $^3\text{He}$, but produces the most energetic neutrinos [Adelberger et al., 2011]. Nearly all of the neutrinos produced by the $pp$ chain escape the sun without interacting with solar material and their flux has been detected on earth. The solar neutrino energy spectrum serves as a probe of the interior of the sun, and can verify solar model parameters such as the temperature of the core, and the abundance of elements other than hydrogen and helium or “metallicity”. The significant role that the $^{14}\text{N}(p, \gamma)^{15}\text{O}$ reaction plays in the solar neutrino spectrum is discussed later in this chapter.
The $pp$ chain dominates energy generation at temperatures lower than 20 MK ($T_6 = 20$). Stars with masses $M > 1.5M_\odot$, that contain carbon and nitrogen, predominantly burn hydrogen via the CN cycle. The higher coulomb barriers of the carbon and nitrogen nuclei in the CN cycle compared to hydrogen and helium require higher temperatures to proceed. For comparison, the temperature dependence goes as approximately $T^4$ for the $pp$ chain and as $T^{18}$ for the CN cycle [Rolfs and Rodney, 1988]. The energy generation as a function of central temperature for the $pp$ chain and CN cycle is shown in Figure 1.2. For the core temperature of our sun, $T_\odot = 15.7$ MK, it is estimated that the $pp$ chain is responsible for nearly 99% of the luminosity while the CN cycle makes up the remainder. The details of the nuclear reactions involved in the CN cycle are outlined in the following section.

### 1.1.2 CN Cycle

The carbon nitrogen (CN) cycle consists of the following sequence of proton captures and $\beta^+$-decays

$$^{12}\text{C}(p, \gamma)^{13}\text{N}(e^+ \nu)^{13}\text{C}(p, \gamma)^{14}\text{N}(p, \gamma)^{15}\text{O}(e^+ \nu)^{15}\text{N}(p, \alpha)^{12}\text{C} \quad (1.4)$$
where the formation of $^4$He, is the same net result as for the $pp$ chain. In the CN cycle however, only hydrogen nuclei are consumed while the total abundance of carbon, nitrogen and oxygen remain unchanged. For example, if the cycle begins with $^{12}$C as in Equation 1.4, the last step ends with $^{12}$C, and thus the same nuclei can be used over and over again as catalysts.

The competing reaction $^{15}$N$(p, \gamma)^{16}$O may occur instead of the $^{15}$N$(p, \alpha)^{12}$C reaction and results in a branch from $^{15}$N into a secondary, oxygen nitrogen (ON) cycle. The hydrogen burning process in this cycle is similar to the CN cycle but involves additional isotopes of oxygen as catalyst nuclei. The ON cycle is characterized by the following sequence of reactions

\[
^{16}$O$(p, \gamma)^{17}$F$(e^+\nu)^{17}$O$(p, \alpha)^{14}$N \tag{1.5}
\]

which restores catalytic material back to the CN cycle. The CN and ON cycles together are commonly referred to as the CNO bi-cycle and are shown in Figure 1.3. The branching ratio between the $^{15}$N$(p, \gamma)^{16}$O : $^{15}$N$(p, \alpha)^{12}$C reactions is about 1 : 1000 for temperatures between 20 – 80 MK [Iliadis, 2007]. The $\beta^+$-decay half-lives of $^{13}$N, $^{15}$O, and $^{17}$F are 9.965(4) minutes,
122.24(16) seconds, and 64.49(16) seconds respectively [Ajzenberg-Selove, 1991, Tilley et al., 1993].

The rate of energy production in the CN cycle or any sequence of nuclear reactions, is governed by the slowest reaction. As an example, the CN cycle reaction times for a $2M_\odot$ star with a temperature $T_6 = 25$ and density $\rho = 100 \text{ g/cm}^3$ are shown in Figure 1.4. The $^{14}\text{N}(p, \gamma)^{15}\text{O}$ reaction proceeds at a rate nearly 1000 times slower than the next slowest reaction, $^{12}\text{C}(p, \gamma)^{13}\text{N}$. For the temperatures present in hydrostatic hydrogen burning, the $\beta^+$-decays of unstable nuclei proceed at a faster rate than competing proton capture reactions on the unstable nuclei and thus do not play a role in energy generation. The slow rate of the $^{14}\text{N}(p, \gamma)^{15}\text{O}$ reaction forms a bottleneck for the flow of material through the CN cycle and effectively limits the speed at which the cycle can operate. The significance of the $^{14}\text{N}(p, \gamma)^{15}\text{O}$ reaction and its impact on neutrino production in our Sun and age estimates of globular clusters in our Galaxy is detailed in the following two sections.

1.1.3 Solar Metallicity

The current model of the Sun, referred to as the standard solar model (SSM), includes all the physics principles that reproduce the observed macroscopic features for the Sun’s present
Figure 1.4: Reaction times in the CN cycle for a 2M⊙ star with a temperature and density of $T_6 = 25$ and $\rho = 100$ g/cm$^3$ respectively [Champagne, 2013]. The $^{14}$N($p, \gamma$)$^{15}$O reaction is by far the slowest reaction and in turn, regulates the rate of energy production of the CN cycle.

age of approximately 4.57 G.y. The SSM assumes local hydrostatic equilibrium, energy generation by hydrogen burning, a homogeneous zero-age sun, and boundary conditions imposed by the known mass, radius, and luminosity of the present sun [Haxton and Serenelli, 2008]. The assumption of a homogeneous zero-age sun constrains the primordial core metallicity to the surface abundances observed today. The chemical composition is characterized by the mass fractions of hydrogen ($X$), helium ($Y$), and all heavier elements or metals ($Z$). The composition of a star is expressed as

$$\mu(r) = [2X + (3/4)Y + (1/2)Z]^{-1}$$

(1.6)

where $\mu(r)$ is the mean molecular weight for a given stellar model. Assuming the sun was chemically homogeneous when it formed, the composition of the present solar surface would be identical to the abundances of elements present in the pre-stellar cloud, $X = 0.73$, $Y = 0.25$, and $Z = 0.02$. The composition of the core on the other hand, is believed to have evolved to $X = 0.42$, $Y = 0.56$, and $Z = 0.02$ as a result of the $pp$ chain and CNO cycles converting hydrogen into helium [Rolfs and Rodney, 1988].

One way to accurately determine the chemical composition of the solar interior is through
helioseismology, or the study of the propagation of wave oscillations in the sun, similar to the way terrestrial seismic waves are used to probe the interior of Earth. The oscillations are periodic fluctuations in the Doppler shifts of spectral lines in the solar atmosphere, and have periods ranging from less than 5 min to 2 h 40 min [Zeilik and Gregory, 1998]. The 5 min oscillations are observed as vertical motions of areas of the sun that result from sound waves traversing the solar interior. These radial motions of gases at the solar surface agree with a metallicity content of approximately $Z = 0.02$ as predicted in 1-d solar models. In contrast, the most recent 3-d solar models that include an improved modeling of the sun’s atmosphere, indicate a lower metallicity content of $Z=0.0134$ [Asplund et al., 2009]. This disagreement between low-Z solar model predictions and helioseismic constraints is known as the “solar abundance problem”.

As an independent test of the SSM, the solar neutrino spectrum can be measured to probe the interior of the sun. The 26.73 MeV energy released in hydrogen burning in Equation 1.1 is shared among the resulting nuclei, $\gamma$ rays, and neutrinos, and thus the energy of the neutrino is characteristic to the process in the $pp$ chain or CNO cycles that produces it. The CNO neutrinos have a different endpoint energy than $pp$ neutrinos and can be distinguished on earth using large neutrino detectors filled with liquid scintillator. The predicted solar neutrino energy spectrum for the SSM is shown in Figure 1.5 and comparing this spectrum to the measured solar neutrino flux would provide an independent test of whether the zero-age sun was in fact homogeneous.

Neutrino detectors such as Borexino [Alimonti et al., 2009] and the future Sudbury Neutrino Observatory (SNO+) experiment [O’Keefe et al., 2011], are sensitive to neutrinos with energies above 0.2 MeV including solar neutrinos produced in the CNO cycles. The Borexino and SNO+ neutrino experiments could effectively determine the primordial abundances of carbon and nitrogen with little dependence on the SSM. However, a correct interpretation of the expected CNO neutrino flux requires precise knowledge of the reaction rates of the CNO cycles, which is determined by the $^{14}\text{N}(p, \gamma)^{15}\text{O}$ reaction. It is estimated that the CNO neutrino flux in the low-metallicity scenario is approximately 30 – 35% lower than in the high-metallicity scenario. These upcoming neutrino measurements, along with a precise de-
Figure 1.5: Solar neutrino flux as a result of nuclear reactions in the pp chain (black lines) and CNO cycles (blue dashed lines) (reproduced from [Bahcall et al., 2005]).
termination of the $^{14}\text{N}(p, \gamma)^{15}\text{O}$ reaction rate, represent a series of recent advances that aim to resolve the solar abundance problem.

1.1.4 Globular Cluster Ages

Globular clusters (GCs) are extremely compact (diameters $\approx 100$ ly), spherically symmetric collections of stars, and the oldest GCs (12 to 15 Gy) represent the oldest structures in the galaxy [Zeilik and Gregory, 1998]. Nearly 200 have been documented in the Milky Way and can be found at great distances (out to $\approx 300,000$ ly) from the central plane of our Galaxy in what is referred to as the “galactic halo” [Zeilik and Gregory, 1998]. Globular clusters are very luminous and dense as they typically contain between $10^5$ to $10^6$ stars. Stellar spectra for the halo GCs indicate a very low metal abundance ($Z \leq 0.001$) and therefore are indicative of Population II stars. The stars in a GC are also formed at approximately the same instance relative to the age of the cluster and share the same initial composition [Zeilik and Gregory, 1998]. There are some stars that appear to be formed from separate star-formation events, but this simple picture underlies our basic understanding of most clusters. The major difference between stars in GCs are their initial masses, which in the end, causes the heaviest stars to evolve the fastest while the lightest stars evolve the slowest. A Hubble Space Telescope image of the globular cluster NGC 6397 is shown in Figure 1.6.

The age of a globular cluster can be estimated by examining a Hertzsprung-Russell (H-R) diagram. H-R diagrams are a tool which astrophysicists use to visualize stellar evolution. Stars are plotted as single points and the locations of each are determined by their absolute visual magnitude and spectral type which is equivalent to luminosity versus temperature. A simulated time sequence of a GC containing 100 stars plotted on an H-R diagram is shown in Figure 1.7. The y-axis is a measure of the luminosity in units of solar luminosity and the x-axis is the effective surface temperature in Kelvin, shown increasing toward the origin (right to left). The top H-R diagram represents the zero-age main sequence (ZAMS), the phase at which a star first gets all its energy from hydrogen fusion reactions, before it has converted any substantial amount of its hydrogen to helium. The entire phase of core hydrogen burning is called the main-sequence (MS) phase and is the longest stage in a stars life. The heat from
proton fusion keep these stars in hydrostatic equilibrium and most of the interior transports energy by radiation, while only the outer region of the star’s envelope is convective.

The H-R diagram in the middle of Figure 1.7 represents 10 My from ZAMS when the red giant branch (RGB) begins to form. After the hydrogen fuel is used up in the core, thermonuclear reactions no longer take place there but instead continue in a shell surrounding the core where fresh hydrogen still exists. Meanwhile, the core begins to contract which heats the layer of burning hydrogen, causing the temperature and associated nuclear energy generation rate to increase. The shell of burning hydrogen heats up the surrounding envelope of the star, and causes it to expand. As the radius of the star begins to increase, its surface temperature decreases, which in turn, increases the opacity because H$^-$ ions begin to form. Eventually convection carries most of the energy outward in the envelope. The luminosity will then increase drastically and this causes the star to move up the RGB branch.

The bottom H-R diagram in Figure 1.7 represents 110 My from ZAMS and a distinct knee can be seen in the main sequence referred to as the main-sequence turnoff (MSTO). Larger
mass stars have a higher effective temperature and therefore will leave the main sequence sooner than lower mass stars. For this reason, the MSTO will be seen to decrease in luminosity and temperature as the cluster ages. The turnoff is the hottest position on the main sequence and marks the point in a star’s evolution where hydrogen is exhausted in the core. As a star approaches the MSTO, energy generation switches from the \( pp \) chain to CN cycle reactions and thus the luminosity is mostly regulated by the \( ^{14}\text{N}(p, \gamma)^{15}\text{O} \) reaction rate [Rolfs and Rodney, 1988]. Subsequent stellar evolutionary stages, while equally fascinating, will not be discussed here as they are less relevant to the subject of globular cluster age estimation.

The observed turnoff points of clusters give their approximate age when compared to constant-time lines, or “isochrones”, from theoretical models. An isochrone is a snapshot in the life of a cluster as it represents the locations of the ends of evolutionary tracks for stars to that particular time since formation. The H-R diagram of the globular cluster NGC 6397 is shown in Figure 1.8. Isochrones of 13, 14, and 15 Gy are fit to the main-sequence turnoff region using the \( ^{14}\text{N}(p, \gamma)^{15}\text{O} \) reaction rate measured at the underground accelerator facility LUNA at Gran Sasso, Italy [Formicola et al., 2004]. The age of globular cluster NGC 6397 is determined to be 14 ± 1 Gy which is nearly 1.0 Gy older than previously thought, a result of the revised \( ^{14}\text{N}(p, \gamma)^{15}\text{O} \) reaction rate. Increased ages of the oldest GCs have a significant impact in cosmology as well. Age estimates of the Milky Way halo GCs provide a strict lower limit on the age of our Galaxy and also a consistency check of the age of the Universe.

Estimating the age of globular clusters using the isochrone fitting method is one way to get the ages of GCs and it is for the most part, the best understood. There are also techniques based on the location of the horizontal branch (HB) in H-R diagrams or the difference from the HB to the MSTO, but these methods have higher systematic uncertainty [Champagne, 2013]. Sources of uncertainty in the isochrone fitting procedure, in order of importance, are the distance scale (16%), oxygen abundance (7%), treatment of convection within stars (5%), helium abundance (3%), \( ^{14}\text{N}(p, \gamma)^{15}\text{O} \) reaction rate (3%), and helium diffusion (3%) [Chaboyer et al., 1996]. It should be noted however, that the uncertainty of the \( ^{14}\text{N}(p, \gamma)^{15}\text{O} \) reaction rate is more likely closer to 9% since the revised reaction rate is approximately half the value once thought [Bertone, 2010].
Figure 1.7: Simulated time sequence of a globular cluster containing 100 stars. The top, middle, and bottom HR diagrams represent: ZAMS, about 10 My from ZAMS, and about 110 My from ZAMS respectively [Scharein, 2000] (adapted from [Bertone, 2010]).
Figure 1.8: H-R diagram of the globular cluster NGC 6397 (reproduced from [Imbriani et al., 2004]). Isochrones of 13, 14, and 15 Gy (solid lines) are fit to the main-sequence turnoff region.
1.2 Nuclear Reactions

Nuclear reaction theory describes the interactions between nucleons in terms of two fundamental forces, the electromagnetic and strong nuclear force. The electromagnetic force, or Coulomb interaction between charged particles, is very well understood, whereas the strong nuclear force involved is extremely complex and not precisely known. The nucleon-nucleon interaction exhibits a repulsive force at short distances, yet at distances close to the size of the nucleus \((\approx 1 \, \text{fm} = 10^{-15} \, \text{m})\), the fundamental strong interaction exhibits an attractive force. Calculating every interaction between nucleons becomes computationally impractical after just a few nucleons, and therefore approximations using effective potentials are necessary. For the most part, the approximations are customized to a specific energy of a reaction and are not generalized to all nuclear interactions. The quantum-mechanical approximations are beyond the scope of this work; however, the next few sections outline some fundamentals of nuclear reactions and how the specific models apply to the \(^{14}\text{N}(p, \gamma)^{15}\text{O}\) reaction.

The nuclear structure influencing the reaction mechanism can be best understood through a physical quantity known as the cross section. The cross section, \(\sigma\), is a measure of the probability that a nuclear interaction occurs between the incident particles and target nuclei, defined as

\[
\sigma = \frac{\text{number of interactions per time}}{\text{number of incident particles per time} \times \text{number of target nuclei per area}} \quad (1.7)
\]

The cross section can be expressed in measurable laboratory quantities as

\[
\sigma = \frac{N_R/t}{(N_b/t)(N_t/A)} \quad (1.8)
\]

where \(N_R/t\) is the number of interactions per unit time, \(N_b/t\), is the number of incident particles per time or beam current, and \(N_t/A\) is the number of target nuclei per area covered by the beam. The traditional units of the nuclear cross section is the barn, \(b\), which is defined as

\[
1 \, b \equiv 10^{-24} \, \text{cm}^2 \quad (1.9)
\]
and is simply an interaction area between the incident particle and target nucleus. Measurable cross sections are typically greater than $10^{-9}$ b. The discussion of the cross section in this work is focused on radiative capture, or the capture of a particle by a nucleus, inducing the emission of electromagnetic radiation as a $\gamma$ ray.

In general, radiative-capture cross sections at low energies exhibit two distinct characteristics. The cross section has an overall energy dependence described by a smooth, exponential increase with beam energy. Superimposed on the smooth background are spikes in the cross section which rapidly change by many orders of magnitude, called resonances. The smoothly varying cross section is attributed to the Coulomb interaction of the incident and target particles and described as a single-step process known as direct capture. On the other hand, the resonant capture process corresponds to energies of distinct nuclear levels in the target nucleus and involves a two-step process of first forming a compound state, and the subsequent de-excitation of that state. A summary of both the direct capture and resonant capture mechanisms are described in the following sections.

Frequently, cross sections for radiative-capture reactions that take place in astrophysical environments are too weak to be measured in the laboratory. For example, at the interior temperature of the sun the $^{14}\text{N}(p, \gamma)^{15}\text{O}$ reaction has a cross section on the order of $10^{-20}$ b, which is nearly 10 orders of magnitude lower than what is typically measurable in the laboratory. Assuming a proton beam current of 1 mA and a detection efficiency of 1%, a statistically significant measurement of the cross section at energies of the nuclei in the sun’s core would require an experiment to run for nearly 100 My. In practice, the cross section is measured at the lowest reasonable energies that may require several weeks of run time, then extrapolated down to astrophysically relevant energies.

Extrapolating the cross section to low energies is ideally performed by distinguishing between the resonant and direct capture mechanisms. However, a distinction between the two processes is complicated in many cases since the reaction may proceed through either mechanism over the same energy range. Resonances in close proximity with one another on top of a relatively small direct capture contribution make disentangling the direct capture and resonant capture contributions especially problematic. Furthermore, the quantum mechanical
nature of each process allow resonances to interfere with other resonances or with direct capture. The $^{14}\text{N}(p, \gamma)^{15}\text{O}$ reaction exhibits all of the aforementioned complications and therefore, discrepancies exist in the extrapolations of the $^{14}\text{N}(p, \gamma)^{15}\text{O}$ cross section to low energies. The method for determining the nonresonant reaction rate for the $^{14}\text{N}(p, \gamma)^{15}\text{O}$ reaction is outlined later in this chapter.

1.2.1 Nonresonant Cross Sections

The nonresonant reaction mechanism is characterized by a single-step process where a proton is captured by the target nucleus as a $\gamma$ ray is emitted to form a bound state of a final nucleus. The interaction with the electromagnetic field allows for the transition of the proton from an initial scattering state to a final bound state without the formation of a compound nucleus. In this direct capture model, the incident proton interacts with the target nucleus as a single core and not with individual nucleons. Therefore, the direct capture reaction is a relatively quick process, on the order of $t \approx 10^{-22}$ s, in comparison to resonant capture, where the time needed to form a compound nucleus is on the order of $t \approx 10^{-17}$ s. Figure 1.9 is an illustration of the $^{14}\text{N}(p, \gamma)^{15}\text{O}$ direct capture reaction that can occur for all energies, $E_p$, of the incident proton. In each transition, a $\gamma$ ray of energy $E_\gamma = Q + E_p - E_x$ is emitted to form a bound state of energy $E_x$, in the final nucleus. The $Q$-value is defined as the difference in the masses before and after the reaction, and thus in the case of the $^{14}\text{N}(p, \gamma)^{15}\text{O}$ reaction is $Q = 7.297$ MeV.

The direct capture cross section discussed here is focused on the E1 transition, which is typically the dominant contribution to the cross section. The cross section for an E1 transition of a scattering state of initial angular momentum $l_i$ to a bound state of final angular momentum $l_f$ is given by

$$
\sigma(\text{E1}) = 0.0716m_{01}^{3/2}\left(\frac{Z_0}{m_0} - \frac{Z_1}{m_1}\right)^2 \frac{E_p^3}{E_\gamma^{3/2}} \frac{(2J_f + 1)(2l_f + 1)}{(2J_0 + 1)(2l_0 + 1)(2J_1 + 1)(2l_1 + 1)} \times (l_i010|l_f0)^2 R_{l_i,l_f}^2 (10^{-6} \text{ b})
$$

(1.10)

where $m_{01}$ is the reduced mass of the target projectile system $m_{01} = m_0m_1/(m_0 + m_1)$; $Z_0$,
Figure 1.9: Illustration of the direct capture level scheme of the $^{14}$N$(p,\gamma)^{15}$O reaction.
$Z_1, m_0, m_1, j_0,$ and $j_1$ are the charges, masses (in amu), and spins of the projectile and target respectively; $J_f$ is the angular momentum of the final state; $E_p$ is the bombarding energy in the center-of-mass system (in MeV); and $E_\gamma$ is the energy of the $\gamma$-ray transition [Rolfs, 1973, Iliadis and Wiescher, 2004]. The radial integral, $R_{l_i l_f}$, is defined as

$$R_{l_i l_f} = \int_0^{\infty} u_c(r) O_{E1}(r) u_b(r) r^2 dr$$  

(1.11)

where $O_{E1}(r)$ is the radial part of the electric dipole operator $E1$, and $u_c, u_b$ are the continuum and bound state wave functions respectively [Rolfs, 1973, Iliadis and Wiescher, 2004]. The radial integral has to be evaluated numerically, and requires significant knowledge regarding the nature of the bound state wave functions and is beyond the scope of the current work.

In practice, the absolute normalization of the theoretical direct capture cross section can not be reliably calculated. The previous assumption regarding the single particle nature of the direct capture reaction is not entirely correct, whereas in actuality, only a fraction of the total wave function exists as a single particle state. Spectroscopic factors provide an empirical estimate of the fraction of the final state wave function that can be described by a single particle bound in a potential well. The experimental cross section is related to the theoretical cross section through

$$\sigma_{\text{exp}} = \sum_{l_i, l_f} C^2 S(l_f) \sigma_{\text{theo}}(l_i, l_f)$$  

(1.12)

where the sum is over all possible initial and final state orbital angular momenta, $l_i$ and $l_f$ respectively, and $C^2 S(l_f)$ is the spectroscopic factor. Thus, the direct capture cross section can be used to determine the spectroscopic factor. Alternatively, the $C^2 S(l_f)$ values can be measured independently in a stripping reaction such as $(d, n)$ or $(^3\text{He}, d)$. Comparing the spectroscopic factors measured in stripping reactions and from normalizing cross section data provides a validation check of the direct capture cross section calculation.
1.2.2 Resonant Cross Sections

In contrast to direct capture, the resonant capture process results in a strongly energy-dependent cross section that can vary by many orders of magnitude over a small energy range (10 keV). The radial integral in the resonant capture model has a major contribution from the nuclear interior for energies that are near the quasi-bound states of the nuclear potential and is a maximum when the internal wavefunction matches the external scattering wavefunction. This occurs when the energy of the entrance channel, $Q + E_r$, closely matches the energy of the excited state, $E_x$, in the compound nucleus and implies that the resonance energies are

$$E_r = E_x - Q \quad (1.13)$$

where $Q$ is the constant $Q$-value for the given nuclear reaction, for the $^{14}\text{N}(p, \gamma)^{15}\text{O}$ reaction $Q = 7.297$ MeV. The total width of a resonance, $\Gamma$, is given by the sum of the partial widths of all energetically allowed decay channels

$$\Gamma = \Gamma_p + \Gamma_\gamma + \cdots \quad (1.14)$$

which in the present study are only the proton and $\gamma$ channels. An illustration of the $E_{\text{lab}}^{\text{res}} = 278$ keV resonant capture reaction is shown in Figure 1.10 where the entrance channel, $^{14}\text{N} + p$, forms an excited state at 7.556 MeV in the $^{15}\text{O}$ compound nucleus at $E_{\text{cm}}^{\text{res}} = 259$ keV center-of-mass energy. The 7.556 MeV state decays into lower-lying states, $E_i$, with the emission of a $\gamma$ ray of energy $E_\gamma = 7.556$ MeV $- E_i$.

The resonant capture cross section discussed here is focused on a resonance which is both isolated and narrow. A resonance is considered isolated if the level density in the compound nucleus is small, such that neighboring resonances do not overlap significantly. Moreover, a resonance is called narrow if the partial widths are approximately constant over the total resonance width, which is typically less than a few keV [Iliadis, 2007]. The cross section of a narrow, isolated resonance such as the $E_{\text{cm}}^{\text{res}} = 259$ keV resonance in $^{14}\text{N}(p, \gamma)^{15}\text{O}$, is described
Figure 1.10: Illustration of the $E_r = 0.259$ keV resonant capture level scheme of the $^{14}\text{N}(p, \gamma)^{15}\text{O}$ reaction.
by the single-level Breit-Wigner formula

$$\sigma(E) = \frac{\lambda^2}{4\pi} \omega \frac{\Gamma_p \Gamma_\gamma}{(E_r - E)^2 + \Gamma^2/4}$$

(1.15)

where $E_r$ is the energy of the resonance, $\Gamma_p$ and $\Gamma_\gamma$ are the resonance partial widths of the entrance and exit channel respectively, and $\Gamma$ is the total resonance width. The de Broglie wavelength, $\lambda$, is expressed as

$$\lambda = \frac{h}{(2m_{01}E_r)^{1/2}}$$

(1.16)

where $m_{01}$ is the reduced mass and $\omega$ is a statistical factor. The statistical factor is defined as

$$\omega = \frac{(2J + 1)(1 + \delta_{01})}{(2j_0 + 1)(2j_1 + 1)}$$

(1.17)

where $J$ is the angular momentum of the resonance, $j_0$ and $j_1$ are the spins of the target and projectile, and the factor $(1 + \delta_{01})$ is included because the cross section is increased by a factor of 2 in the case of identical particles in the entrance channel [Rolfs and Rodney, 1988].

The cross section of astrophysically relevant reactions at low energy, is often dominated by the tails of higher energy resonances since the direct capture contribution is negligible in comparison. Precise calculations of the resonant contribution far from $E_r$ require accurate values of the partial widths and resonance energy in the Breit-Wigner formula. The resonance energy can be measured with the use of high resolution charged-particle accelerators; however, the partial widths for many resonances remain unknown and typically contribute the majority of the uncertainty in calculations of the cross section.

1.2.3 Astrophysical S-factor and Reaction Rates

The total cross section from a previous $^{14}\text{N}(p, \gamma)^{15}\text{O}$ reaction measurement [Runkle et al., 2005] is shown in the left panel of Figure 1.11. Plotted in the right panel is what is known as the “astrophysical $S$-factor”, which is related to the cross section through

$$\sigma(E) = \frac{1}{E} \exp(-2\pi\eta) S(E)$$

(1.18)
where \( \exp(-2\pi \eta) \) is the Gamow factor and \( \eta \) is the Sommerfeld parameter. The Sommerfeld parameter is defined as

\[
\eta = \frac{Z_0 Z_1 e^2}{\hbar \nu}
\]  
(1.19)

where \( \nu \) is the velocity of the projectile. The exponent of the Gamow factor can be calculated numerically as

\[
2\pi \eta = (31.29) Z_0 Z_1 \left(\frac{m_{01}}{E}\right)^{1/2}
\]  
(1.20)

where \( m_{01} \) is the reduced mass of the target projectile system in amu, and \( E \) is the center-of-mass energy in keV [Rolfs and Rodney, 1988].

Figure 1.11: Total cross section and corresponding \( S \)-factor data for the \(^{14}\text{N}(p, \gamma)^{15}\text{O}\) reaction [Champagne, 2013]. The cross section, shown in the left panel, falls off rapidly at energies below the Coulomb barrier, whereas the \( S \)-factor, shown in the right panel, is a smoothly varying function which remains nearly flat with beam energy. The solar energy window is illustrated by the yellow band at low energy for a stellar temperature of \( T_\odot = 15 \times 10^6 \) K.

The expression of the \( S \)-factor in Equation 1.18 removes both the \( 1/E \) dependence of the nuclear cross section arising from the square of the de Broglie wavelength, and the Coulomb barrier transmission probability, which is contained in the Gamow factor. Therefore, to first order the \( S \)-factor contains all the strictly nuclear effects and varies far less with energy compared to the cross section as clearly shown in Figure 1.11. The transformation of the
cross section to the $S$-factor is used to extrapolate a fit of the data to astrophysical energies where direct measurements are prohibited by the Coulomb barrier. The significance and location of this astrophysical energy region is discussed in depth later in this section.

The following discussion is a treatment of nonresonant reaction rates for a nondegenerate, nonrelativistic, stellar environment in thermal equilibrium. The reaction rate per particle pair, $\langle \sigma v \rangle$, for a particle-induced reaction is given by

$$\langle \sigma v \rangle = \left( \frac{8}{\pi m_{01}} \right)^{1/2} \frac{1}{(kT)^{3/2}} \int_0^{\infty} \sigma(E) E \exp \left( -\frac{E}{kT} \right) dE$$

(1.21)

where the relative velocity of the nuclei are described by a Maxwell-Boltzmann velocity distribution

$$\phi(E) \propto E \exp \left( -\frac{E}{kT} \right)$$

(1.22)

and the Boltzmann constant is given by $k = 8.6173 \times 10^{-8}$ keV/K. The reaction rate per particle pair can also be written in terms of the astrophysical $S$-factor by substituting Equation 1.18 in Equation 1.21

$$\langle \sigma v \rangle = \left( \frac{8}{\pi m_{01}} \right)^{1/2} \frac{1}{(kT)^{3/2}} N_A \int_0^{\infty} S(E) \exp \left( -\frac{E}{kT} - 2\pi \eta \right) dE$$

(1.23)

In practice, the number of reactions per unit volume per time is evaluated at each temperature of interest for a stellar environment. The product of the number of particle pairs and the reaction rate per particle pair, $N_A \langle \sigma v \rangle$, in units of cm$^3$mol$^{-1}$s$^{-1}$, is given by

$$N_A \langle \sigma v \rangle = \left( \frac{8}{\pi m_{01}} \right)^{1/2} \frac{N_A}{(kT)^{3/2}} \int_0^{\infty} S(E) \exp \left[ -\frac{E}{kT} - 2\pi \eta \right] dE$$

(1.24)

where $N_A$ is Avogadro’s number. Typically, the $S(E)$ factor is nearly a constant over the narrow energy window around the effective burning energy for a given stellar temperature so Equation 1.24 simplifies to

$$N_A \langle \sigma v \rangle = \left( \frac{8}{\pi m_{01}} \right)^{1/2} \frac{N_A}{(kT)^{3/2}} S(E_0) \int_0^{\infty} \exp \left[ -\frac{E}{kT} - 2\pi \eta \right] dE$$

(1.25)
where the temperature $T$, energy $E$, and masses are in units of Kelvin, keV, and amu respectively.

The integrand in Equation 1.25 approaches zero at low energies as a result of the Gamow factor, $\exp(-2\pi\eta)$, but also approaches zero at higher energies on account of the Maxwell-Boltzmann distribution, $\exp(-E/kT)$. For this reason, the dominant contribution to the integral will arise from energies where the product of the exponential factors is near its maximum value. Figure 1.12 illustrates the energy dependence of the integrand in Equation 1.25. The maximum value energy $E_0$ can be found by taking the first derivative of the integrand with respect to $E$,

$$E_0 = \left[ \left( \frac{\pi \hbar}{\hbar} \right)^2 \left( e^2 Z_0 Z_1 \right)^2 \left( \frac{m_{01}}{2} \right) \left( kT \right)^2 \right]^{1/3}$$

(1.26)

Numerically, the energy $E_0$ is calculated using

$$E_0 = 1.22 \left( Z_0^2 Z_1^2 m_{01} T_6^2 \right)^{1/3} \text{ (keV)}$$

(1.27)

Approximating the Gamow peak as a Gaussian function, the $1/e$ width, $\Delta$, of the energy window is given by

$$\Delta = \frac{4}{\sqrt{3}} (E_0 kT)^{1/2} = 0.749 \left( Z_0^2 Z_1^2 m_{01} T_6^2 \right)^{1/6} \text{ (keV)}$$

(1.28)

which defines the effective width of the Gamow peak.

As an example, the effective mean energy for the $^{14}\text{N}(p, \gamma)^{15}\text{O}$ reaction at a stellar temperature of $T_6 = 15$ is calculated as $E_0 = 26.5$ keV, which is the effective burning energy for the sun. For reactions involving nuclei of increasing nuclear charge, the effective mean energy for thermonuclear fusion is shifted toward higher energy. Therefore, reactions with relatively high Coulomb barriers do not contribute much to the energy generation in a star, while reactions with the lowest Coulomb barriers will proceed most rapidly and make up the majority of energy production. After the nuclei with the smallest Coulomb barrier are consumed, the star will contract gravitationally until the temperature rises to a point where reactions with the next lowest Coulomb barrier can proceed, stabilizing the star against further gravitational
Figure 1.12: The convolution of the Maxwell-Boltzmann distribution and the quantum mechanical tunneling probability results in a peak near $E_0$ known as the Gamow peak (adapted from [Rolfs and Rodney, 1988]). The Gamow peak illustrates the narrow energy or temperature range over which the majority of nuclear reactions occur in a star.

contraction [Rolfs and Rodney, 1988].

Frequently, it is sufficient to calculate the nonresonant reaction rate in terms of an effective astrophysical $S$-factor that is only dependent upon the temperature and extrapolated $S$-factor at zero energy, $S(0)$ [Fowler et al., 1967]. If the $S$-factor is described by a slowly varying function of energy instead of a constant as assumed in Equation 1.25, it can be expanded in a Taylor series at $E = 0$ as

$$S(E) \approx S(0) + S'(0)E + \frac{1}{2}S''(0)E^2 + \cdots$$

(1.29)

where $S'(0)$ and $S''(0)$ are first and second derivatives of the $S$-factor with respect to energy and are obtained from fits to experimental data. An example of one such model used to fit $^{14}\text{N}(p,\gamma)^{15}\text{O}$ $S$-factor data is described in the following section. As a result of the Taylor
series expansion, an analytic expression for the reaction rate is

\[ N_A \langle \sigma v \rangle = \left( \frac{4}{3} \right)^{3/2} \frac{h}{\pi m_0 Z_0 Z_1 e^2} S_{\text{eff}} \tau^2 e^{-\tau} \] (1.30)

where the effective \( S \)-factor, \( S_{\text{eff}} \) \cite{Fowler et al., 1967}, is given by

\[ S_{\text{eff}}(E_0) = S(0) \left[ 1 + \frac{5}{12\tau} + \frac{S'(0)}{S(0)} \left( E_0 + \frac{35}{36} kT \right) + \frac{1}{2} \frac{S''(0)}{S(0)} \left( E_0^2 + \frac{89}{36} E_0 kT \right) \right] \] (1.31)

The dimensionless parameter \( \tau \) found in both Equations 1.30, and 1.31 is defined as

\[ \tau = \frac{3E_0}{kT} = 42.46 \left( \frac{Z_0^2 Z_1^2 m_0}{T_6} \right)^{1/3} \] (1.32)

1.3 Previous Measurements

Since a direct measurement of the total \( ^{14}\text{N}(p, \gamma)^{15}\text{O} \) \( S \)-factor at astrophysical energies is out of reach experimentally, the \( ^{14}\text{N}(p, \gamma)^{15}\text{O} \) reaction rate is determined by the extrapolated total \( S \)-factor at zero energy. In order to produce an accurate extrapolation of the total \( S \)-factor, each transition of the \( ^{14}\text{N}(p, \gamma)^{15}\text{O} \) reaction must be fit and extrapolated individually so that its distinct energy dependence is properly taken into account. Precision measurements of the ground-state \( S \)-factor have been performed since the 1980’s but the landscape of the low-energy extrapolation is still quite uncertain. In the last 25 years, the extrapolated \( S(0) \) value for the ground state has been revised by a factor of 5 and presently, the recommended \( S(0) \) value has a relative uncertainty of nearly 20% \cite{Adelberger et al., 2011}. The ground-state transition contributes approximately 15 – 30% to the total \( S(0) \) which makes it the second-largest component following the 6.79 MeV transition. The low-energy ground-state \( S \)-factor is especially complicated as a result of interference effects that make the extrapolation highly dependent upon the nuclear reaction model. The \( E_r^{\text{cm}} = 259 \) keV resonance resides on top of an interference “dip” created by a subthreshold resonance at \( E_r^{\text{cm}} = -504 \) keV and higher energy resonances at \( E_r^{\text{cm}} = 987 \) and 2187 keV, as shown in Figure 1.13.

When the tails of two or more resonances with the same spin and parity overlay, their am-
plitudes may interfere with one another. The total cross section of two interfering resonances with individual cross sections $\sigma_1$ and $\sigma_2$ is given by

$$\sigma(E) = \sigma_1(E) + \sigma_2(E) \pm 2\sqrt{\sigma_1(E)\sigma_2(E)} \cos(\delta_1 - \delta_2)$$

(1.33)

and the phase shifts, $\delta_i$, are defined as

$$\delta_i = \arctan\left(\frac{\Gamma_i}{2(E - E_{r,i})}\right)$$

(1.34)

where the resonance energy and width are given as $E_{r,i}$ and $\Gamma_i$ respectively [Rolfs and Rodney, 1975]. The reaction rate for the interfering resonances is calculated by substituting Equation 1.33 in Equation 1.21 and numerically integrating. In the case of the ground-state transition in the $^{14}$N$(p, \gamma)^{15}$O reaction, the tails of resonances at $E_{r,i}^{\text{cm}} = -504, 259, 987,$ and $2187$ keV extend into the low-energy region of the $S$-factor curve and interfere with each other and with the direct capture contribution. As a result of the complicated landscape, a model known as “R-matrix theory” [Lane and Thomas, 1958, Breit, 1959] is frequently used to fit the data and extrapolate the $S$-factor to zero energy.

R-matrix theory is a phenomenological model that requires empirical data to describe resonant capture cross sections and does not make predictions from first principles. R-matrix can be described as a boundary value theory, where a boundary separates an internal region, which represents the interior of the nucleus, from the external world around it. Only the Coulomb potential is present in the external region, so the interaction behavior is well known. In contrast, the internal region contains all the unknowns, which are the parameters of the characteristic resonances of the system. The observables, such as the cross section and $S$-factor, are calculated by matching the logarithmic derivative of the wave functions at the boundary.

A summary of published R-matrix fits to measured ground-state transition data is shown in Figure 1.13. Data from Schröder [Schröder et al., 1987], Runkle [Runkle et al., 2005], Imbriani [Imbriani et al., 2005], and Marta [Marta et al., 2008] are indicated and the R-matrix
Figure 1.13: A comparison of published R-matrix fits to ground-state $S$-factor data (adapted from [Bertone, 2010]).
fits were performed on various subsets of data. The fit of Angulo [Angulo and Descouvemont, 2001] considered only the data from Schröder whereas the fit of Imbriani included the Schröder data along with their measured results [Formicola et al., 2004]. The R-matrix fit to the Runkle data omitted the Schröder results except for the parameters of the higher energy resonances, which were obtained from the Angulo fit. A 2009 review [Costantini et al., 2009] by LUNA considered all the ground-state transition data at the time including the Marta data and performed a re-normalization to the strength of the $E_{\text{cm}} = 259$ keV resonance. The $(p, p)$ constrained fit [Bertone, 2010] is a multi-channel R-matrix fit that included $^{14}\text{N}(p, p)^{14}\text{N}$ scattering data with the global $^{14}\text{N}(p, \gamma)^{15}\text{O}$ ground-state data set excluding the Marta points. Fitting the measured $S$-factor data using R-matrix is computationally intensive to perform and beyond the scope of this study. An outline of the specific project goals for this work is detailed in the last section of this chapter.

1.3.1 Coincidence Summing Considerations

At the energies of interest in the present study, the nuclear level populated in $^{15}\text{O}$ by proton capture can de-excite via a single $\gamma$ ray directly to the ground state or by the sequential emission of two $\gamma$ rays in a cascade as illustrated in Figures 1.9 and 1.10. In some instances, the two coincident $\gamma$ rays from the 5.18, 6.18, or 6.79 MeV transition can interact simultaneously with the detector resulting in a summed energy signal. This effect, known as “coincidence summing”, creates spurious events in the ground-state peak and a deficit in the cascades. In this case, coincidence summing corrections must be applied to the experimental data in order to extract the total number of decays from the number of detected $\gamma$ rays in the primary, secondary, and ground-state full-energy peaks. Summing corrections can be particularly severe in the measurement of the $^{14}\text{N}(p, \gamma)^{15}\text{O}$ ground-state transition as a result of its relatively small branching ratio, which is only $1 - 2\%$ over the energy region of interest. Measured branching ratios for transitions to the ground state, 5.18, 5.24, 6.18, and 6.79 MeV states at the $E_{\text{cm}} = 259$ keV resonance are listed in Table 1.1 [Marta et al., 2011]. Subsequent decays from the intermediate states in $^{15}\text{O}$ have a branching ratio of 100% to the ground state.
<table>
<thead>
<tr>
<th>Transition</th>
<th>Branching (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ground State</td>
<td>1.49 ± 0.04</td>
</tr>
<tr>
<td>5.18 MeV</td>
<td>17.3 ± 0.2</td>
</tr>
<tr>
<td>5.24 MeV</td>
<td>0.15 ± 0.03</td>
</tr>
<tr>
<td>6.18 MeV</td>
<td>58.3 ± 0.4</td>
</tr>
<tr>
<td>6.79 MeV</td>
<td>22.6 ± 0.3</td>
</tr>
</tbody>
</table>

Table 1.1: Tabulation of branching ratios for the decay of the $E_{cm}^{r} = 259$ keV resonance [Marta et al., 2011].

The procedure for making coincidence summing corrections in the case of the $^{14}$N($p, \gamma$)$^{15}$O reaction, can be demonstrated by a simple example involving only three nuclear levels. Figure 1.14 illustrates the effect of coincidence summing on the intensities of the primary, secondary, and ground-state full-energy peaks in the $^{15}$O decay scheme. In this scenario, the 7.556 MeV state populated in the $E_{cm}^{r} = 259$ keV resonance either decays directly to the ground state, via emission of $\gamma_0$ with a branching ratio of $B_0$, or to the 6.18 MeV state, via emission of $\gamma_1$ with a branching ratio of $B_1$. The 6.18 MeV state subsequently decays to the ground state 100% of the time via emission of $\gamma_2$.

Ignoring angular correlation effects between the primary and secondary $\gamma$ rays, the number of detected primary $\gamma$ rays in the full-energy peak, $N_1$, can be calculated using

$$N_1 = N B_1 \eta_1^P - N B_1 \eta_1^P \eta_2^T$$
$$= N B_1 \eta_1^P (1 - \eta_2^T) \quad (1.35)$$

where $\eta_1^P$ is the full-energy peak efficiency of the primary $\gamma$ ray, $\eta_2^T$ is the total-efficiency of the secondary $\gamma$ ray, and $N$ is the total number of decays. The intensity of the primary peak in the energy spectrum is reduced by the factor $N B_1 \eta_1^P \eta_2^T$, which is the probability that the full-energy of the primary and some measurable amount of energy of the secondary is detected simultaneously. This effect is referred to as “summing-out” and shown in Figure 1.14 by the dashed lines at the primary and secondary full-energy peaks. Similarly, the number of
Figure 1.14: Coincidence summing of $\gamma$ rays in the $^{14}\text{N}(p, \gamma)^{15}\text{O}$ reaction. The left panel is a simplified level diagram of the $E_{cm} = 259$ keV resonance in $^{14}\text{N}(p, \gamma)^{15}\text{O}$. The energy spectrum in the right panel illustrates the effect of coincidence summing on the intensities of the primary, secondary, and ground-state full-energy peaks.
detected secondary $\gamma$ rays in the full-energy peak, $N_2$, is equal to

$$N_2 = NB_1\eta_2^P - NB_1\eta_2^P \eta_1^T$$

$$= NB_1\eta_2^P (1 - \eta_1^T)$$ (1.36)

where $\eta_2^P$ is the full-energy peak efficiency of the secondary $\gamma$ ray, and $\eta_1^T$ is the total-efficiency of the primary $\gamma$ ray.

Alternatively, the number of detected full-energy peak ground-state events, $N_0$, is related to the total number of decays by

$$N_0 = NB_0\eta_0^P + NB_1\eta_1^P \eta_2^P$$ (1.37)

which is increased by the amount $NB_1\eta_1^P \eta_2^P$, the probability that the full-energy of both $\gamma_1$ and $\gamma_2$ are detected. This effect is referred to as “summing-in” and shown in Figure 1.14 by the dashed lines at the full-energy peak of the ground state.

Finally, the total number of decays can be calculated from the intensities of the primary or secondary $\gamma$ ray full-energy peaks. Rearranging Equations 1.35 or 1.36 and solving for $N$ gives

$$N = \frac{N_1}{B_1\eta_1^P (1 - \eta_2^T)} = \frac{N_2}{B_1\eta_2^P (1 - \eta_1^T)}$$ (1.38)

and properly accounts for coincidence summing.

Coincidence summing corrections in the $^{14}$N($p, \gamma$)$^{15}$O reaction become increasingly complicated if all transitions in the decay scheme are considered and angular correlations of the primary and secondary $\gamma$ rays are taken into account. Moreover, uncertainty in the branching ratios of $^{15}$O levels in direct capture also add to the complexity of the summing corrections.

To avoid coincidence summing corrections in the present low-energy $S$-factor measurement, the current work will utilize a cylindrical, segmented NaI(Tl) detector array. Because of the segmentation and the distance between each detector and the target, the probability for $\gamma$ rays summing-in to the ground-state peak is negligible.
1.4 The Project Goals

The focus of the present study was to characterize a NaI(Tl) detector array and measure the low-energy $S$-factor of the $^{14}\text{N}(p, \gamma)^{15}\text{O}$ reaction. Of particular interest, was the ground-state $S$-factor residing in the low-energy interference dip between $E_{\text{cm}} = 150 - 180$ keV. The published data in this interference region have relatively large uncertainty as a result of low counting statistics and coincidence summing corrections as shown in Figure 1.13. The primary goal of this work was to measure the ground-state transition without the need for coincidence summing corrections in an attempt to better constrain future R-matrix fits that extrapolate the $S$-factor to zero energy. Additionally, the 5.18, 6.18, and 6.79 MeV $S$-factors were measured without additional cost to the experiment and with higher counting statistics since those transitions are stronger than the ground-state transition in the region of interest.

A secondary goal was to utilize the position-sensitivity of the $\gamma$-ray detector to measure the angular correlations of the primary and secondary $\gamma$ rays emitted in transitions to the 5.18, 6.18, and 6.79 MeV states. The direct capture angular correlation data could improve coincidence sum corrections in future $^{14}\text{N}(p, \gamma)^{15}\text{O}$ experiments that utilize detectors arranged in a close counting geometry.

Over the next several chapters, the apparatus, procedure, data analysis techniques, and the experimental results of the project, will be presented in detail. Chapter 2 and Chapter 3 will describe the experimental setup including the accelerators, $\gamma$-ray detector, and the fabrication of nitrogen targets. Chapter 4 and Chapter 5 will present the results of the $^{14}\text{N}(p, \gamma)^{15}\text{O}$ $S$-factor study and the $\gamma$ ray angular correlation measurements respectively. Lastly, Chapter 6 will conclude with a summary of the current work and considerations for future low-energy $S$-factor measurements.
CHAPTER 2: ACCELERATOR AND DETECTOR

2.1 The Laboratory for Experimental Nuclear Astrophysics

The $^{14}\text{N}(p, \gamma)^{15}\text{O}$ reaction experiments were performed in the Laboratory for Experimental Nuclear Astrophysics (LENA) at the Triangle Universities Nuclear Laboratory (TUNL). LENA is a dedicated nuclear astrophysics laboratory equipped with two charged particle accelerators and multiple $\gamma$-ray detector systems. A schematic drawing of the layout of LENA is shown in Figure 2.1 with accelerators, major beamline components, analyzing magnet, and the “APEX” $\gamma$-ray detector labeled. The JN Van de Graaff accelerator is capable of accelerating protons with energies up to 1 MeV with beam current intensities between 100 – 150 $\mu$A on target. The Electron Cyclotron Resonance (ECR) ion source resides on a 200 kV platform and can produce high intensity proton beams (> 1 mA) up to 200 keV. The data in the present work were recorded using the APEX $\gamma$-ray detector while the proton beam was provided by the JN Van de Graaff accelerator.

2.2 JN Van de Graaff Accelerator

The HVEC JN Van de Graaff accelerator has been modified from the original High Voltage model JN to accommodate a high-output ion source and associated radio frequency (RF) supply. Figure 2.2 is a photograph of the long bottle ion source installed in the LENA JN surrounded by permanent magnets. A hydrogen plasma was formed by an RF oscillator capacitively coupled to the bottle and intensified by an external magnetic field. Hydrogen gas was admitted into the quartz ion source bottle via a controlled gas leak, and the electrons in the gas were excited into oscillation by the RF electric field. The plasma strikes when the electrons have acquired enough kinetic energy through collisions with neutral gas particles,
to ionize the gas. The axial magnetic field produced by permanent magnets confined and positioned the plasma. Proton beam was extracted by an electric potential that ranged between $0 - 5$ kV, applied between the probe tip and the ion source base.

The beam output was optimized by control of the source gas pressure and oscillator loading. Optimal tuning parameter settings for striking a hydrogen plasma in the JN Van de Graaff accelerator and obtaining maximum beam output are shown in Table 2.1. The total beam current measured on the first few shorted planes of the acceleration column reached 2.2 mA while tuning the ion source, which was performed with the ion source open to air and a grounding rod attached to the acceleration column. During the experiment, the ion source was operated at high voltage inside a pressurized vessel filled with a mixture of carbon dioxide and nitrogen gas that electrically insulated the terminal.

The extracted proton beam is focused and steered through a set of optical slits before entering the analyzing magnet. The energy of the proton beam is selected by a feedback circuit that measures the beam current on horizontal slits at the exit of the magnet, and adjusts the terminal voltage as necessary to balance the current. The analyzing magnet
Figure 2.2: Photograph of a hydrogen plasma inside the JN Van de Graaff ion source bottle. The probe tip is visible protruding into the end of the source bottle near the left side of the image.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Setting</th>
</tr>
</thead>
<tbody>
<tr>
<td>Twin-lead length</td>
<td>15.0 cm</td>
</tr>
<tr>
<td>Tuning capacitor position</td>
<td>All the way out</td>
</tr>
<tr>
<td>Plate line clip position</td>
<td>1.25 inches from base</td>
</tr>
<tr>
<td>Rear bottle clip position</td>
<td>0.875 inches from magnets</td>
</tr>
<tr>
<td>JN gas pressure</td>
<td>$2.3 \times 10^{-6}$ Torr</td>
</tr>
<tr>
<td>Extraction voltage</td>
<td>5.0 kV</td>
</tr>
<tr>
<td>Grid current</td>
<td>4.0 mA</td>
</tr>
<tr>
<td>Plate current</td>
<td>250 mA</td>
</tr>
<tr>
<td>Total beam current</td>
<td>2.2 mA</td>
</tr>
</tbody>
</table>

Table 2.1: Typical tuning parameter settings for striking a hydrogen plasma in the JN Van de Graaff accelerator. The tank was open to air while tuning for maximum beam current and the plasma was struck with the charging belt instead of an external power supply. The total beam current was measured on the first few shorted planes of the acceleration column.
calibration is determined through several measurements of well-known resonance reactions. The reactions used to calibrate the analyzing magnet along with their natural widths are shown in Table 2.2. The calibrated magnetic field, \( B \), for a desired energy, \( E \), is determined by

\[
B = \frac{k}{q} \left( 2mc^2E + E^2 \right)^{1/2}
\]

(2.1)

where \( q \) and \( m \) are the charge and mass of the accelerated ion and \( k \) is the calibration constant [Rolfs and Rodney, 1988]. The energy selected proton beam leaving the magnet travels through a second set of quadrupole magnets and beam steerers that focus and position the beam on the center of the target. The energy resolution of the proton beam on target is typically \( 1 - 2 \) keV full width at half maximum (FWHM). The beam current is measured directly from an electrically isolated target chamber (i.e., a Faraday cup) and will be discussed in the following section.

<table>
<thead>
<tr>
<th>Reaction</th>
<th>( E_{\text{lab}}^\gamma ) (keV)</th>
<th>( \Gamma ) (eV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( ^{18}\text{O}(p,\gamma)^{19}\text{F} )</td>
<td>150.82(9)</td>
<td>130(10)</td>
</tr>
<tr>
<td>( ^{27}\text{Al}(p,\gamma)^{28}\text{Si} )</td>
<td>202.8(9)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>326.97(5)</td>
<td>&lt; 38</td>
</tr>
<tr>
<td></td>
<td>405.44(10)</td>
<td>&lt; 42</td>
</tr>
<tr>
<td>( ^{26}\text{Mg}(p,\gamma)^{27}\text{Al} )</td>
<td>292.06(9)</td>
<td>&lt; 37</td>
</tr>
<tr>
<td></td>
<td>338.4(1)</td>
<td>&lt; 40</td>
</tr>
<tr>
<td></td>
<td>453.8(1)</td>
<td>&lt; 81</td>
</tr>
</tbody>
</table>

Table 2.2: Tabulation of resonances used for calibrating the LENA analyzing magnet [Iliadis, 2007].

2.3 Target Chamber

The target chamber at the end of the LENA beamline is shown in Figure 2.3. The beamstop target was directly water cooled and the target holder was electrically isolated to measure the accumulated charge of the proton beam. A 1.27 cm diameter beam collimator located before the target ensured that the proton beam spot did not extend beyond the
nitrogen implanted region of the target. The target chamber pressure was maintained below $5 \times 10^{-7}$ Torr and a liquid nitrogen cooled copper shroud, commonly referred to as a “cold trap”, positioned before the target prevented carbon and other contaminants from plating onto the surface of the target. The copper shroud is biased to -300 V in order to suppress the emission of secondary electrons from the target, thereby improving the measurement of the integrated beam current.

Figure 2.3: Schematic drawing of the LENA target chamber. The beam passes through a copper collimator and secondary electron suppression ring (-300 V bias) before bombarding a directly water-cooled beamstop target. The target holder is electrically isolated to measure the accumulated charge of the incident proton beam. The copper shroud is cooled to liquid-nitrogen (LN$_2$) temperatures to reduce the buildup of contaminants on the target.

### 2.4 APEX Trigger Detector

The APEX trigger detector is a cylindrical NaI(Tl) scintillator array, originally constructed for the ATLAS Positron Experiment (APEX) [Betts, 1989] and has been upgraded and re-assembled for use in low-energy nuclear experiments at LENA. This is one of the two 24-element position-sensitive NaI(Tl) detectors that served as pair spectrometers in the APEX experiment and is on loan from ANL for the present $^{14}$N($p, \gamma$)$^{15}$O reaction study. Figure 2.4 is a photograph of the assembled NaI(Tl) array located in the detector setup area of LENA.
Figure 2.4: Photograph of the assembled APEX detector on loan from Argonne National Laboratory (ANL).
The APEX detector is segmented into 24 position-sensitive bars of NaI(Tl) scintillator. The crystals are trapezoidal in cross section with dimensions $55.0 \times 6.0 \times 5.5\,(7.0)$ cm$^3$ ($L \times H \times W$). Each crystal is encapsulated in a 0.4 mm thick evacuated stainless steel container with quartz windows, 4.4 cm in diameter and 1.1 cm thick, permanently fixed at either end. A 5 cm diameter Hamamatsu R2490 photomultiplier tube (PMT) is optically coupled to each window using Saint-Gobain BC-630 silicone grease. The performance of the original PMTs was inadequate for the present study and consequently the tubes were replaced, as described in the following section. An exploded view drawing of one APEX detector segment is shown in Figure 2.5 with the NaI(Tl) crystal, quartz windows, PMTs and aluminum housings indicated.

Figure 2.5: Drawing of a NaI(Tl) segment with major components identified. The photomultiplier tubes (PMTs) are optically coupled to each window using Saint-Gobain BC-630 silicone grease.

The NaI(Tl) segments are held in place by attachment to two stainless steel rings. The assembled detector array is 85.0 cm long with an outer diameter of 56.7 cm and inner diameter of 42.8 cm. For a source located at its center, the array covers 75% of $4\pi$ with a 15 degree azimuthal angular resolution [Kaloskamis et al., 1993]. The entire array is encapsulated by a cylindrical lead shield 1.9 cm thick resting inside an aluminum cradle. The bottom corners of
the cradle were mounted with leveling jacks that provided the necessary height adjustments to center the NaI(Tl) array around the LENA target chamber. Plastic standoffs placed under the leveling jacks electrically insulate the APEX detector from the beamline, which reduced electronic noise in the PMTs.

2.4.1 Assembly

The Hamamatsu R2490 PMTs that were originally coupled to the NaI(Tl) segments were chosen specifically for their performance in a strong magnetic field. Testing of the PMTs for the APEX experiment showed they suffered only a 3% gain reduction in a 300 Gauss longitudinal field [Kaloskamis et al., 1993]. PMTs such as these, are designed with a fine mesh dynode structure and minimal distance between the photocathode and first dynode, are suitable for operation in strong magnetic fields, but typically at the expense of energy resolution. Initial testing performed at LENA of the NaI(Tl) scintillators and R2490 tubes with a $^{60}$Co source indicated unusually poor energy resolution and noisy PMT signals. The two signature $\gamma$-ray lines at 1173.2 keV and 1332.5 keV were indistinguishable in the $^{60}$Co reconstructed energy spectrum. Consequently, we replaced the original tubes with 32 Hamamatsu R580 tubes and 16 Photonis XP2012B tubes in new aluminum mounts. The new PMT housings have eliminated light leaks, while the spectroscopy grade tubes provide improved energy resolution. A specification comparison of the different PMT models is listed in Table 2.3. The new PMTs although smaller in diameter, have greater luminous sensitivity, less dark current, and the 1173.2 keV and 1332.5 keV peaks in the $^{60}$Co energy spectrum could now be differentiated. A photograph of the Hamamatsu R2490, R580, and the Photonis XP2012 with separate mu-metal shield and voltage divider is shown in Figure 2.6.

Prior to assembly, each detector segment was thoroughly cleaned and photographed to catalog the condition of the NaI(Tl) crystal. Several NaI(Tl) crystals showed signs of damage in the form of streaking and fogging around the quartz windows. These minor defects most likely occurred during transportation but also may be a result of their age. An example photograph of one crystal is shown in Figure 2.7. NaI(Tl) is hygroscopic and will deteriorate if exposed to air because of water absorption. It is possible that the stainless steel containers
<table>
<thead>
<tr>
<th></th>
<th>Hamamatsu R2490</th>
<th>Photonis XP2012</th>
<th>Hamamatsu R580</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diameter</td>
<td>2.0 inches</td>
<td>1.5 inches</td>
<td>1.5 inches</td>
</tr>
<tr>
<td>Number of stages</td>
<td>16</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>Quantum efficiency</td>
<td>22%</td>
<td>25%</td>
<td>27%</td>
</tr>
<tr>
<td>Luminous sensitivity</td>
<td>70 µA/lm</td>
<td>85 µA/lm</td>
<td>90 µA/lm</td>
</tr>
<tr>
<td>Radiant sensitivity at 420 nm</td>
<td>75 mA/W</td>
<td>85 mA/W</td>
<td>88 mA/W</td>
</tr>
<tr>
<td>Dark current</td>
<td>200 nA</td>
<td>1 nA</td>
<td>&lt;3 nA</td>
</tr>
<tr>
<td>Pulse rise time</td>
<td>2.7 ns</td>
<td>2.5 ns</td>
<td>2.7 ns</td>
</tr>
</tbody>
</table>

Table 2.3: Tabulation of photomultiplier tube specifications as listed by the manufacturer.

Figure 2.6: Photograph of the APEX detector photomultiplier tubes. Pictured top to bottom is the Hamamatsu R2490, Hamamatsu R580, and the Photonis XP2012 with separate mu-metal shield and voltage divider.
surrounding a few of the crystals are no longer airtight, exposing the crystals to moisture and resulting in the appearance of a yellow tint to the crystal. The catalog of NaI(Tl) crystal photographs is kept on file for future reference in the event we suspect a scintillator bar is behaving poorly and will permit future comparison if we suspect continuing degradation.

Figure 2.7: Photograph of a NaI(Tl) scintillation crystal as viewed through the quartz window on one side of the segment. The length of the NaI(Tl) crystal is 55.0 cm and the diameter of the quartz window is 4.4 cm.

After cleaning the quartz windows with ethanol, a dime-size amount of Saint-Gobain BC-630 silicone optical grease was placed on the PMT glass surface, which was then methodically pressed against the quartz window using constant pressure. Rubber O-rings within the PMT housing maintain positive pressure between the PMT and quartz window. The optical grease and quartz window provide a transparent interface between the PMT and the scintillator crystal, while the O-rings also shield the tube from light leaks. A photograph taken of the detector setup area in LENA during the assembly of the APEX detector is shown in Figure 2.8.

A record was kept of the serial numbers of the NaI(Tl) crystals and the locations of each photomultiplier tube in the APEX detector array. Table 2.4 identifies the serial numbers of the NaI(Tl) scintillator crystals and photomultiplier tubes in the assembled detector. Upstream
and downstream PMTs, relative to the beam direction, were matched to the same NaI(Tl) crystal according to their cathode blue sensitivity index printed on the PMT test ticket issued by the manufacturer. The detailed specifications from each test ticket are included in Appendix A. The blue sensitivity index, expressed in $\mu$A/ImF (“F” as in Filtered), is a measure of the photoelectric current generated from the photocathode with light from a tungsten filament lamp, transmitted through a blue filter. Blue sensitivity index is an essential parameter in PMT selection because NaI(Tl) crystal scintillates light in the blue region of the electromagnetic spectrum with maximum emission at 415 nm [Knoll, 2010].

A drawing of the final NaI(Tl) segment configuration in the APEX detector is shown in Figure 2.9. The LENA segment numbers in Figure 2.9 correspond with those listed in Table 2.4. The slightly shorter length of the Photonis tubes and voltage dividers were selected for the top and bottom quadrants of the detector array. The locations of these tubes were in close proximity to the liquid nitrogen dewar and target turbopump while APEX was on the LENA beamline. The detector moved on rails with stop blocks so that it could be positioned in a reproducible manner.
LENA NaI(Tl) NaI(Tl) NaI(Tl) PMT Upstream Downstream
Segment Serial Origin Sticker Model PMT Serial PMT Serial
ID Number Label Number Number Number Number
0 OU503 APPL. LAB 45 XP2012 106722 105621
1 OU504 APPL. LAB 46 XP2012 106804 107030
2 OT432 B. U. #2 27 XP2012 106356 106747
8 OT433 B. U. #2 14 R580 CE2911 CE2454
9 OT492 NS #2 16 R580 CE2944 CE2906
10 OT637 B. U. 17 R580 CE2945 CE2950
11 OT638 B. U. 18 R580 CE2957 CE2951
12 OT640 B. U. 20 R580 CE2931 CE2904
13 OT641 B. U. 21 R580 CE2960 CE2894
14 OT931 B. UTTS 15 R580 CE2926 CE2397
15 OT932 B. UTTS 28 R580 CE2876 CE2036
3 OT933 B. UTTS 29 XP2012 106527 105736
4 OT934 B. UTTS 30 XP2012 105488 105967
5 OT935 B. UTTS 31 XP2012 106818 106986
6 OU272 APPL. 32 XP2012 106529 106781
7 OU273 APPL. 33 XP2012 106247 107174
16 OU274 APPL. 34 R580 CE2503 CE2932
17 OU276 APPL. 36 R580 CE2895 CE2927
18 OU278 APPL. 38 R580 CE2930 CE2909
19 OU364 APPL. LAB 39 R580 CE2964 CE2956
20 OU365 APPL. LAB 40 R580 CE2902 CE2912
21 OU366 APPL. LAB 41 R580 CE2949 CE2965
22 OU500 APPL. LAB 42 R580 CE2941 CE2898
23 OU501 APPL. LAB 43 R580 CE2967 CE2948

Table 2.4: Configuration of NaI(Tl) scintillator crystals and photomultiplier tubes in the assembled APEX detector. See Figure 2.9 for the relative positions of each NaI(Tl) segment.
Figure 2.9: Drawing of the NaI(Tl) segment configuration in the APEX detector. The LENA segment ID number corresponds with those listed in Table 2.4.
2.4.2 Characterization

A unique feature of the APEX detector is the ability to read out both the position and energy of γ-ray interactions in the scintillator array. The position and energy are reconstructed from the signals produced by the PMTs coupled to both ends of the NaI(Tl) crystal. Diffusing the surfaces of the NaI(Tl) crystal by grinding causes the angles of reflection to be approximately independent of the angle of incidence, and thus results in exponential attenuation of the scintillation light. Because of the exponential attenuation of photons along the length of the segment, the position can be reconstructed by comparing the relative pulse heights from the PMTs [Kaloskamis et al., 1993].

Figure 2.10 is a schematic drawing of a single bar of the APEX array with simplified pulse amplitudes $A_1$ and $A_2$, from PMT 1 and 2 respectively. The linear position, $X$, at which the scintillation light occurs is measured from the center of the NaI(Tl) crystal of length $L$. The signal pulse generated by the PMT closest to the interaction point of the γ ray will have a greater amplitude than that of the more distant PMT.

![Schematic drawing of a NaI(Tl) segment with simplified pulse amplitudes from both photomultiplier tubes.](image)

The amplitude of the signal from PMT 1 can be expressed as

$$A_1 = \frac{E_\gamma P}{E_0} \exp \left[ -\mu \left( \frac{L}{2} + X \right) \right]$$  \hspace{1cm} (2.2)

where $E_\gamma$ is the energy deposited by the γ ray, $P$ is the quantum efficiency of the PMTs.
(assumed to be the same for each tube), \(E_0\) is the energy deposited per light photon created in scintillator, and \(\mu\) is the light attenuation coefficient. Similarly, for PMT 2,

\[
A_2 = \frac{E_\gamma P}{E_0} \exp[-\mu(L/2 - X)]
\]

(2.3)

Dividing PMT signal 2, Equation 2.3, by the signal in PMT 1, Equation 2.2, and solving for the position, \(X\), gives

\[
\frac{A_2}{A_1} = \frac{\exp[-\mu(L/2 - X)]}{\exp[-\mu(L/2 + X)]} = \exp(2\mu X)
\]

\[
\ln \frac{A_2}{A_1} = 2\mu X
\]

\[
X = \frac{1}{2\mu} \ln \frac{A_2}{A_1}
\]

(2.4)

It follows that the reconstructed position is proportional to the logarithm of the ratio of PMT signals and inversely proportional to the attenuation coefficient. The reconstructed energy can be obtained after multiplying Equation 2.2 by Equation 2.3 and taking the square root of both sides

\[
E_\gamma^2 = A_1 A_2 \left(\frac{E_0}{P}\right)^2 e^{\mu L}
\]

\[
E_\gamma = \sqrt{A_1 A_2} \frac{E_0}{P} e^{\mu L/2}
\]

(2.5)

Therefore, the reconstructed energy is proportional to the square root of the product of the PMT pulse amplitudes. Experimentally, when the energy of incident \(\gamma\) rays was compared to their reconstructed energy, it was determined that the reconstructed energy was also dependent upon the position of the source. The reconstructed energy was calibrated during data analysis by performing a linear energy calibration of each segment divided into 16 pixels, as will be outlined in Chapter 4.

Measurements of the position and energy resolution were achieved by the use of encapsulated \(\gamma\)-ray disk sources. A radial collimator was constructed to precisely control the position of a 0.90 \(\mu\)Ci \(^{137}\)Cs source along the position-sensitive axis of the detector. The radial colli-
mator was also utilized for calibrating the reconstructed positions of all 24 NaI(Tl) segments simultaneously with a 0.25 $\mu$Ci $^{60}$Co source. Figure 2.11 is an Autodesk Inventor drawing of the collimator, which is seated on rails to allow for unrestricted movement and quick calibration while the detector is positioned on the beamline in LENA.

![Figure 2.11: Drawing of the APEX radial collimator used for position calibration. The $\gamma$-ray source, shown in red, is sandwiched between 5.08 cm thick cylinders of lead inside a delrin container.](image)

The collimator is constructed of a 11.43 cm diameter aluminum pipe that houses a delrin container attached to the end of a 1.07 m aluminum rod. The aluminum rod has been etched in 0.5 cm increments to allow for precise positioning of the source. Inside the delrin container are two cylinders of lead, each 5.08 cm thick and 10.16 cm in diameter separated by a distance of 3 mm that radially collimate the $\gamma$-ray source. The dimensions of the lead collimators were chosen so that the size of the $\gamma$-ray image on the detectors would be smaller than their intrinsic position resolution. The delrin container can slip freely throughout the aluminum pipe allowing the collimator base to be locked in position while the source is inserted into the detector using the incremented rod and container assembly. A photograph of the radial
collimator and APEX detector array on the beamline is shown in Figure 2.12.

Figure 2.12: Photograph of APEX and the radial collimator on the beamline in LENA.

The radial collimator was carefully positioned so that the etched scale on the source rod matched the physical boundaries of the NaI(Tl) crystals. The collimator was accurately aligned with the scintillator crystals using a distance of 43.2 cm measured between pillow blocks. Figure 2.13 is a scaled drawing of the relative positions of APEX and the collimator. Typical energy and position resolutions of the NaI(Tl) segments determined with the $E_\gamma = 662$ keV line in $^{137}$Cs were 14% and 3.5 cm FWHM respectively. The resolution of the APEX detector segments were consistent with previously measured values [Perry et al., 2003, Kaloskamis et al., 1993].

Position calibration of each segment was performed by moving a 0.25 $\mu$Ci $^{60}$Co source inside the radial collimator along the symmetry axis of the APEX detector array. Reconstructed energy and position data were recorded at 16 positions every 3.5 cm along the length of the NaI(Tl) crystals. The attenuation coefficient used when calculating the reconstructed position (Equation 2.4) was initially set to 0.0046 mm$^{-1}$ which was the average value reported in previous measurements [Kaloskamis et al., 1993]. Data were acquired for 10 minutes at
Figure 2.13: Drawing of the relative position of APEX and radial collimator used to match the scaling of the collimator source rod with the coordinates of the NaI(Tl) crystals.

Each location of the $^{60}$Co source in addition to an equal duration background run after the source was removed from the radial collimator. A 200 keV wide software gate selected only the 1332.5 keV $\gamma$-ray line in the reconstructed energy histogram and the resulting positions of the gated $\gamma$-ray events were sorted into the reconstructed position histogram. A series of reconstructed position histograms for one APEX segment are shown in Figure 2.14. A Lorentzian function was fit to the position peak in the background subtracted reconstructed position histogram. The centroids of the Lorentz distributions were plotted against the collimated source positions for each NaI(Tl) crystal. Figure 2.15 is a plot of the reconstructed position versus source position for one APEX segment.

A linear fit of the reconstructed position versus source position data was performed for each segment of the APEX detector. The slope from the linear fit was used as a scaling factor for the attenuation coefficient. Multiplying the original value of the attenuation coefficient in Equation 2.4, 0.0046 mm$^{-1}$, by the slope, effectively calibrated the reconstructed position. The detector segments retained their position calibration throughout the duration of the
Figure 2.14: Reconstructed positions of $\gamma$ rays from a $^{60}$Co source after position calibration. Background subtracted data are shown for one NaI(Tl) segment as the source was moved in 3.5 cm increments along the position-sensitive axis of the detector.
NaI(Tl) Position (mm)
0 100 200 300 400 500
Reconstructed Position (mm)
0 50 100 150 200 250 300 350 400 450 500

Linear fit
Data

Slope = 0.76 ± 0.02
y-Intercept = 66 ± 6

Figure 2.15: Reconstructed position versus source position for one NaI(Tl) segment of APEX.

$^{14}$N$(p, \gamma)^{15}$O experiment which lasted 18 consecutive days. Calibrated attenuation coefficients of each NaI(Tl) crystal are listed in Table 2.5.

The reconstructed position data from the $^{60}$Co source were sorted once more, this time using the calibrated attenuation coefficients inserted in Equation 2.4. Again, the reconstructed position is fit with a Lorentzian function after background subtraction. The difference between the centroid from the Lorentzian fit and actual position of the $^{60}$Co source was plotted versus the source position and shown in Figure 2.16 for one NaI(Tl) scintillator bar. The value of $\mu$ can be treated as a constant for most of the central part of the NaI(Tl) crystal. Using the radial collimator, it was determined that there is a deviation of $\mu$ throughout the last 5 cm of either end of the crystal. The position dependence of $\mu$ is caused by corner effects in the crystal geometry as well as a result from scintillation events nearest to the PMTs not undergoing sufficient attenuation [Kaloskamis et al., 1993].
<table>
<thead>
<tr>
<th>LENA Segment ID</th>
<th>Attenuation Coefficient (mm⁻¹)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.00447</td>
</tr>
<tr>
<td>1</td>
<td>0.00454</td>
</tr>
<tr>
<td>2</td>
<td>0.00413</td>
</tr>
<tr>
<td>3</td>
<td>0.00454</td>
</tr>
<tr>
<td>4</td>
<td>0.00441</td>
</tr>
<tr>
<td>5</td>
<td>0.00465</td>
</tr>
<tr>
<td>6</td>
<td>0.00411</td>
</tr>
<tr>
<td>7</td>
<td>0.00425</td>
</tr>
<tr>
<td>8</td>
<td>0.00350</td>
</tr>
<tr>
<td>9</td>
<td>0.00359</td>
</tr>
<tr>
<td>10</td>
<td>0.00404</td>
</tr>
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<td>11</td>
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<tr>
<td>12</td>
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</tr>
<tr>
<td>13</td>
<td>0.00368</td>
</tr>
<tr>
<td>14</td>
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</tr>
<tr>
<td>17</td>
<td>0.00367</td>
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<tr>
<td>18</td>
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<tr>
<td>21</td>
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<tr>
<td>22</td>
<td>0.00380</td>
</tr>
<tr>
<td>23</td>
<td>0.00405</td>
</tr>
</tbody>
</table>

Table 2.5: Attenuation coefficients after the position calibration of each NaI(Tl) scintillator.
Figure 2.16: Difference between the reconstructed and actual position of a $^{60}$Co source along one NaI(Tl) scintillator bar.
2.4.3 Electronics Setup

The APEX detector readout was integrated into the existing LENA analog electronics setup. Signal processing of the 48 PMTs was accomplished through a series of Nuclear Instrumentation Modules (NIM) and VERSAmodule Eurocard (VME) bus modules. The electronics were powered in a mobile electronics rack and connected to a data acquisition (DAQ) computer that recorded the energy, position, and timing of detected $\gamma$-ray events. The Java-based nuclear physics data acquisition software package JAM [Swartz et al., 2001] was used to store the data and the events were replayed offline during analysis. JAM has a user-friendly Graphical User Interface (GUI) for recording and sorting multi-parameter event-based data into 1-d and 2-d histograms. The data sorting was executed by a custom sort routine written in the Java programming language.

A CAEN SY2527 multichannel high voltage (HV) power supply biased the APEX detector’s 48 PMTs to their operating voltages (approximately -1500 V). A $^{60}$Co $\gamma$-ray source inside the radial collimator was centered in the APEX detector to provide PMT pulses for gain matching. Gain matching each PMT was performed initially by adjusting the voltages on the HV power supply that biased the tubes on either end of a detector segment. The PMT signals were amplified by CAEN N568B spectroscopy amplifiers that have shaping and fast amplifier outputs for 16 inputs per module. The coarse and fine gain settings were used to more precisely gain match the PMTs and to set the energy range of interest in the acquired spectrum. The spectroscopy amplifier shaped the PMT signals with 1 $\mu$s shaping time that were passed to a CAEN V785 peak sensing analog-to-digital conversion (ADC) module. A schematic diagram of the APEX electronics setup is shown in Figure 2.17.

The fast output of the spectroscopy amplifier provided a fast amplification with a fixed gain factor of 20 for timing purposes. The fast out signal was sent to a CAEN V812 constant fraction discriminator (CFD) for precise discrimination timing. The CFDs were programmed to have a majority threshold of two, which imposed the condition that two channels (PMTs) must have coincident pulses to trigger the output of a gate that initialized data collection. The upstream and downstream PMT signal of each segment were fed into the same CFD.
module; therefore, operating in this configuration reduced the number of triggers and set a requirement that both PMTs of a segment must fire to record the energy and position of an γ-ray event. The CFD output signal was passed to a 500 ns passive delay box and fed into a CAEN V775 time-to-digital conversion (TDC) channel as a timing stop signal. The logic “OR” of discriminator outputs was fed into a gate and delay generator that produced a 2.0 µs wide gate, which was used as the common gate for the ADC and daisy chained to the TDC. The TDC was operated in common start mode for acquiring PMT self-timing peaks. The self-timing peaks were used for data reduction by imposing the condition that both PMTs of a segment must have fired during the detection of a γ ray.

The busy time of the VME crate was used to veto the master gate, thereby prohibiting the pileup of data while the VME-bus was busy. This was accomplished by passing the ADC and TDC busy signals to a level translator and Phillips Scientific 756 logic unit. The overlap of these pulses was used to block the gate and delay generator. The busy time was monitored in JAM by vetoing one of two 60 Hz clock signals read from a scaler in the VME crate.
Several features are apparent in the APEX energy spectrum as a result of the amplifier gain settings and CFD thresholds. A 2-d histogram of $\gamma$ ray reconstructed energy versus position at the $E_{cm}^{\gamma} = 259$ keV resonance in $^{14}\text{N}(p, \gamma)^{15}\text{O}$ is shown in Figure 2.18. The data in the top panel include $\gamma$-ray energies in the range between $3 - 15$ MeV and the bottom panel include those events between $0 - 3$ MeV plotted on a log scale. The missing events in the top panel starting from the ends of the array and moving toward the center at higher energies is a result of the 4 V maximum input cutoff of the ADC. The signature $\gamma$-ray lines between 5000 keV and 7500 keV from the $^{15}\text{O}$ decay scheme shown in Figure 1.10, extend across the useful length of the APEX detector array for the HV and gain settings used in the experiment. Approximately 135 mm on either end of the array is insensitive to $\gamma$ rays from the ground-state transition at approximately 7500 keV, and instead was used as an active veto shield. The noticeable shadow through the $^{15}\text{O}$ $\gamma$-ray lines in the vicinity of the center of the detector array is from scattering and absorption in the target chamber materials. Specifically, the tantalum target backing accounts for the majority of the removed $\gamma$ rays.

There are subtle consequences in the low energy spectrum that arise from the choice of gain settings. Low energy $\gamma$ rays that strike near the ends of the APEX detector produce relatively small amplitude pulses in the opposite PMTs that if lower than the CFD threshold, are cut out of the energy spectrum entirely. The situation is exacerbated by the exponential attenuation of scintillation light traversing the length of the NaI(Tl) crystal. The low energy threshold becomes more pronounced when the PMT voltages and amplifier settings are lowered. While it may at first seem trivial if the low energy events remain undetected, they actually play a critical role in discriminating between single $\gamma$-ray hits and multiple scatters in data analysis. It was for this reason, the upturn of the reconstructed energy in the bottom panel was kept below 511 keV. In the end, there is a clear trade-off when setting the PMT bias voltages and amplifier gains to take advantage of the high and low energy sides of the $\gamma$-ray spectrum. A summary of the PMT bias voltages, amplifier gains, and CFD settings used in the current study is included in Appendix B.
Figure 2.18: Reconstructed energies plotted against the reconstructed positions for detected γ rays at the $E_{\text{cm}}^{\text{res}} = 259$ keV resonance. The data in the top panel include γ-ray energies in the range between 3 – 15 MeV and the bottom panel include those events between 0 – 3 MeV plotted on a log scale.
2.4.4 Monte Carlo Simulation

The $\gamma$-ray detection efficiency of the APEX detector was simulated using the \texttt{GEANT4} software package \cite{Agostinelli:2003}. \texttt{GEANT4} (for GEometry ANd Tracking) is a software toolkit for the simulation of the passage of particles and photons through matter using Monte Carlo (MC) methods. Included in the simulation were the target chamber, beam pipe, APEX NaI(Tl) segments, and the lead shield inside the detector cradle. The \texttt{GEANT4} geometry used in the simulation is shown in Figure 2.19 with major components labeled. The section of the \texttt{DetectorConstruction.cc} code that builds the APEX detector geometry in \texttt{GEANT4} is included in Appendix C.

![Figure 2.19: GEANT4 geometry used in the Monte Carlo simulation of the APEX detector. Included in the simulation were the target, target chamber, copper shroud, surrounding beam pipe, NaI(Tl) crystal in stainless steel containers, quartz windows, and 1.9 cm thick lead shield wrapped inside an aluminum cradle.](image)
For comparison purposes, the absolute efficiency of the APEX and HPGe detectors was simulated using GEANT4. The absolute efficiency is defined as

\[ \eta_{\text{abs}} = \frac{\text{number of detected } \gamma \text{ rays}}{\text{total number of } \gamma \text{ rays emitted}} \] (2.6)

and is dependent on the detector properties and counting geometry (distance from the source) [Knoll, 2010]. Related to the absolute efficiency, the full-energy peak efficiency counts only those interactions that deposit the full energy of the incident radiation. An overlay of full-energy peak efficiencies for the APEX and HPGe detectors from a GEANT4 Monte Carlo simulation is shown in Figure 2.20 as a function of energy. It is important to note that the simulation of the APEX detector was performed in “add-back” mode. Add-back is a data analysis technique where the energy collected in different segments is added together to obtain the total energy for an incident \( \gamma \) ray that scatters from one NaI(Tl) crystal to one or more crystals in the array. This mode effectively transforms the individual segments in the detector array into a single detector. As a result, the APEX detector is 9 times more efficient than the LENA HPGe detector at detecting \( \gamma \) rays at 7.5 MeV, approximately the energy of the ground-state transition. Even though the full-energy peak efficiency is greatest when a detector is run in add-back mode, add-back consequently maximizes the coincidence summing. Since the \( ^{14}\text{N}(p, \gamma)^{15}\text{O} \) reaction has the potential for relatively high summing corrections, this analysis method was not explored in the current work for studying the ground-state transition.

The exact method of data analysis will be discussed in detail in Chapter 4 and distinguishes between single \( \gamma \)-ray hits and multiple detector scatters.
Figure 2.20: Overlay of full-energy peak efficiencies for the APEX and HPGe detectors from a GEANT4 Monte Carlo simulation as a function of energy. The APEX data points are from a simulation performed in add-back mode and the HPGe detector was positioned at 0 degrees in its closest counting geometry.
CHAPTER 3: TARGETS

3.1 Introduction

The nitrogen targets employed in this $^{14}\text{N}(p, \gamma)^{15}\text{O}$ reaction study were fabricated using two different methods: implanting nitrogen ions into tantalum, and gas nitriding of titanium. The tantalum (99.95% metals basis purity) and titanium (99% metals basis purity) were purchased in 0.5 mm thick, 20 cm $\times$ 20 cm sized sheets from Alfa Aesar. The tantalum and titanium foils produced 25 target backings each after meticulous machining by the UNC machine shop. After machining, each square backing measured 38 mm $\times$ 38 mm with a 3 mm diameter hole punched in one corner. An ideal beamstop target for this study should be free of contaminants, have uniform thickness, be stable under bombardment with high intensity (> 100 $\mu$A) proton beams, and also be inexpensive to fabricate. Ultimately, nitrogen ion implanted targets were used in this experiment as they best satisfied all of the above conditions. The aim of the following sections is to summarize the target backing preparation, the implantation and nitriding process, and report on the target composition results.

3.2 Target Backing Preparation

The backings are first prepared by removing impurities in the metal using techniques such as sanding, wet etching, and resistive heating in a vacuum. If the low-energy primary $\gamma$ rays are ignored, the signature $\gamma$ rays emitted from the $^{14}\text{N}(p, \gamma)^{15}\text{O}$ reaction have energies between 5 and 7.5 MeV. The background for $E_{\gamma} > 3.5$ MeV is dominated by cosmic rays; however, while studying a particularly weak cross section, the background can also include $\gamma$ rays from proton-induced capture reactions on impurities in the target or backing material. Depending on the incident proton energy, impurities such as carbon, oxygen, fluorine, and boron may
have cross sections for proton-induced capture reactions several orders of magnitude larger than the $^{14}\text{N}(p, \gamma)^{15}\text{O}$ reaction. The presence of small concentrations of these impurities could thus obscure the signature $\gamma$ rays of interest. The most troublesome contaminants that can interfere with the detection of $\gamma$ rays from the $^{14}\text{N}(p, \gamma)^{15}\text{O}$ reaction are listed in Table 3.1.

<table>
<thead>
<tr>
<th>Contaminant</th>
<th>Reaction</th>
<th>$Q$-value (keV)</th>
<th>$E_\gamma$ (keV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{13}\text{C}$</td>
<td>$^{13}\text{C}(p, \gamma)^{14}\text{N}$</td>
<td>7550.56</td>
<td>$E_p^{cm} + Q$-value</td>
</tr>
<tr>
<td>$^{18}\text{O}$</td>
<td>$^{18}\text{O}(p, \gamma)^{19}\text{F}$</td>
<td>7994.8(6)</td>
<td>3908, 4238, 8028</td>
</tr>
<tr>
<td>$^{19}\text{F}$</td>
<td>$^{19}\text{F}(p, \alpha\gamma)^{16}\text{O}$</td>
<td>8113.67(7)</td>
<td>6130</td>
</tr>
</tbody>
</table>

Table 3.1: Summary of proton-induced capture reactions for several contaminants with published $Q$-values and emitted $\gamma$-ray energies that contribute to $\gamma$-ray background \cite{Audi et al., 2003}.

Considerable care was put forth while handling and storing targets to ensure utmost cleanliness. The backings, and eventually nitrogen targets, were contained in a polycarbonate target box shown in Figure 3.1 that can hold a low vacuum for several days to a week. The vacuum was established by connecting an oil-free scroll pump to the port on the top of the box. The use of an evacuated target box served two purposes, the first being to keep the targets organized and easy to transport, while the second was to prohibit the formation of an oxide layer on the metal.

Figure 3.1: Picture of the polycarbonate target box used to store targets under vacuum.
3.2.1 Wet Etching

Wet etching removes the outermost layer of the backing material through a chemical reaction with acid. Etching will effectively remove oxide layers, machining oil, and other contaminants leaving behind a clean, untouched, target backing surface. The accepted recipe for wet etching tantalum consists of five parts 95% sulfuric acid (H$_2$SO$_4$), two parts 70% nitric acid (HNO$_3$), and two parts 50% hydrofluoric acid (HF) mixed together in that order [Vermilyea, 1953]. The wet etching procedure is carefully carried out under a fume hood while wearing personal protective equipment since HF is a highly corrosive acid. The HF acid mixture is very reactive with both glass and metal and therefore must be mixed and contained in a teflon beaker. Each backing was held during the etching procedure by a pair of teflon tongs that have been modified to hook securely into the punched hole in the corner of the backing. It was found that after 60 seconds of submersion in the acid, the tantalum backing thickness was reduced from 0.5 mm thick to 0.35 ± 0.05 mm thick and the oxide layer and machining blemishes were completely removed. After three backings are processed in the acid solution, the effectiveness of the wet chemical etchant is reduced to a level that no longer reacts with a fresh tantalum backing and must be replaced. As a final step, the backing material was washed with 200 proof ethanol and left to air-dry.

Wet etching titanium was attempted with 36% hydrochloric acid (HCl) at room temperature and of elevated temperatures. Submerging titanium backings in HCl for 16 hours would reduce the thickness from 0.5 mm to approximately 0.48 mm. The etching process was accelerated at 175 degrees F but in both cases the final thickness was the same. The etchant also left a noticeably darker oxide layer on the surface of the target which could only be removed by bead blasting or sanding the surface. Bead blasting was attempted using glass (SiO$_2$) beads inside a re-circulating blast cabinet. In subsequent target fabrication steps, it was found that bead blasting negatively affected the gas nitriding process. It is likely that the titanium surface was coated with a silicon residue during abrasive blasting. A bead blasted titanium backing did not nitride as uniformly or efficiently as an untreated backing and even had a tendency to warp during the resistive heating process outlined in the following section.
A simple alternative to abrasive blasting was accomplished by hand sanding the surface. Hand sanding was most effectively performed by progressing through increasingly finer grit paper until finishing with 400 grit sandpaper. The finished titanium had a silver-white, lustrous appearance and was finally wiped with 200 proof ethanol to remove any debris on the surface left behind after sanding.

### 3.2.2 Resistive Heating

The etched backings were transferred to a high vacuum (10^{-7} Torr), oil-free evaporator system at LENA where they were subjected to resistive heating to drive out any remaining contaminants. The amount of ^{19}\text{F} contamination on tantalum backings was significantly reduced by resistively heating the tantalum in a vacuum [Longland et al., 2009]. A single tantalum backing was clamped between two water-cooled, copper electrodes placed inside the bell jar of the evaporator. The bell jar was evacuated to 1 \times 10^{-7} Torr by a cryogenic vacuum pump and a high current was passed through the backing between the copper electrodes. The current was increased until the backing would glow uniformly bright orange indicating the metal was heated to several thousand degrees Celsius. The current setting used was approximately 250 – 300 Amps for tantalum and 150 – 200 Amps for titanium. The pressure inside the bell jar began to rise as contaminants were released from the surface of the target backing. After sufficient time at high current, approximately 30 minutes to an hour, the gas pressure inside the bell jar would return to the normal operating range of 1 \times 10^{-7} – 1 \times 10^{-6} Torr. After the vacuum pressure stabilized, the resistive heating procedure was complete. The current was slowly turned down and after sufficient time to cool, the clean target backing was placed in the target box and kept under vacuum.

### 3.3 Implanted Targets

Ion implanted targets are best suited for nuclear reaction studies in which isotopically pure substances and stability under high intensity beam is required. ^{14}\text{N} ions are mass separated from ^{15}\text{N} by the implanter’s 90 degree analyzing magnet before being accelerated and directed onto the target backing. Removing ^{15}\text{N} from the ion beam is advantageous because of the
high yield of the $^{15}\text{N}(p, \alpha\gamma)^{12}\text{C}$ contaminant reaction which emits a characteristic 4.43 MeV $\gamma$ ray. The acceleration voltage determines the range at which the ions are implanted in the backing and consequently the thickness of the target. Typically, implanted targets are sufficiently stable under bombardment with high intensity proton beam and can be directly water cooled inside the target chamber.

### 3.3.1 Eaton NV-3206 Ion Implanter

The prepared tantalum backings were implanted with nitrogen using an Eaton NV-3206 ion implanter with a modified end station at the University of North Carolina. The Eaton ion implanter shown in Figure 3.2, can produce nitrogen ion beams as well as beams from other gas and solid sources with an energy range between 20-200 keV. Positive ions are produced by a low-pressure nitrogen gas discharge inside the ion source. A mixture of nitrogen ions and other species are extracted with a 20 kV power supply and pass through a 90 degree analyzing magnet. The magnetic field forces ions of different species to follow trajectories with varying radii of curvature, and a resolving slit located at twice the radius from the end of the magnet, selects the isotope of interest for acceleration. The magnetic field, $B$, is set by adjusting the current through the analyzing magnet. The required strength of the magnetic field to select the ion beam of interest is determined through the relation

$$B = \frac{1}{r_0} (2mV/q)^{1/2}$$

where $r_0 = 19$ cm is the radius of the analyzing magnet, $m$ is the mass of the ion, $q$ is the charge of the ion, and $V = 20$ keV is the potential difference through which the ion is accelerated before entering the magnet. In the present study, a $^2\text{N}^+$ ion beam is selected by adjusting the magnet current to provide a magnetic field of 5675 Gauss. The quoted mass selectivity of the analyzing magnet in the Eaton NV-3206 ion implanter is $1/100$ with a mass range up to 125 amu. The ability to select only a certain species of ions to direct on the backing typically guarantees a target with a very high level of purity.
The nitrogen ions that successfully pass through the resolving slit are injected into the acceleration column as shown in Figure 3.3. After the ions reach the set acceleration voltage, they pass through the remainder of the implanter components including quadrupole magnets, beam steerers, beam scanner, beam collimator, liquid nitrogen cooled copper shroud, and finally bombarding the tantalum target backing. The quadrupole magnets and beam steerers allowed precise control of the beam optics. The $N_2^+$ beam was raster scanned across the surface of the tantalum backing and a beam scanner was utilized to monitor the beam profile during implantation. The 2.54 cm diameter beam collimator ensured that the $N_2^+$ ion implantation region was reproducible and will not extend beyond the usable area of the tantalum backing. The target holder was directly water cooled and electrically isolated so that the incident $N_2^+$ ion dose could be measured. The target chamber pressure was maintained below $5 \times 10^{-7}$ Torr and a liquid nitrogen cooled cold trap, positioned before the target prevented carbon and other contaminants from plating onto the surface of the tantalum backing. Secondary electron suppression was achieved through a -300 V bias on the copper cold trap thereby improving the accuracy of dose monitoring. The ion implanter target chamber was
modeled after the chamber used in LENA which made the geometry very similar to the one discussed in Chapter 2.

![Figure 3.3: Schematic of the ion implantation system, with major components labeled including the ion source, analyzing magnet, acceleration column, liquid nitrogen cooled cold trap, and tantalum target backing.](image)

3.3.2 Dose calculations

The required dose of ions to produce a saturated target can be calculated from the desired stoichiometry and the number of backing atoms in the implantation volume. The stoichiometry is defined as the ratio of active target nuclei to the number of nuclei that do not participate in the reaction of interest. The volume of the implantation region, $V$, is defined by

$$V = \pi r^2 d$$  \hspace{1cm} (3.2)
where $d$ is the depth of implanted nitrogen ions and $r$ is the radius of the ion beam, set by the beam collimator. The depth at which nitrogen ions penetrate in the tantalum backing was calculated using the srim2013 computer application [Ziegler and Biersack, 2013]. The total number of backing atoms, $N_b$, in an implanted volume is given by

$$N_b = \frac{\rho_b V N_A}{A_b}$$  \hspace{1cm} (3.3)

where $\rho_b$ is the mass density of the backing, $A_b$ is the atomic mass of the backing atoms, and $N_A$ is Avogadro’s number. The total number of required target atoms, $N_t$, to produce a target of a given stoichiometry, $n/m$ is therefore

$$N_t = \frac{n}{m} N_b$$  \hspace{1cm} (3.4)

where the expected stoichiometry of $n/m = 1.5$ was used. The stoichiometry will be verified experimentally and the details of that analysis will be presented later in this chapter. Lastly, the necessary accumulated charge, $Q$, can be calculated using

$$Q = \frac{N_t}{\eta} \left(1.602 \times 10^{-19} \text{C}\right)$$  \hspace{1cm} (3.5)

where $\eta$ is the implanting to sputtering efficiency. To ensure nitrogen saturation in the target, $\eta = 0.25$ was assumed as a conservative estimate of the implantation efficiency. Since the species of the ion beam in this case is diatomic nitrogen, $Q$ is multiplied by two to account for the two atoms implanted for every ion registered by the current integrator. For the 10 keV thick targets used in the present study, saturation is achieved for implanting $\mathrm{N}_2^+$ into tantalum with a dose of 0.5 C.

### 3.3.3 Implantation of $^{14}\text{N}$

The typical implantation time to reach saturation was about 5 hours with an average beam current of $30 - 40 \ \mu\text{A}$, an incident dose of $30 \ \mu\text{g/cm}^2$. Typical $\mathrm{N}_2^+$ implantation run parameters for producing 10 keV thick targets measured at the $E_{\text{cm}} = 259 \ \text{keV}$ resonance are
listed in Table 3.2. In total, 40 nitrogen targets were successfully implanted using the UNC ion implanter over the span of approximately 3 months.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Setting</th>
</tr>
</thead>
<tbody>
<tr>
<td>Implantation energy</td>
<td>40.0 keV</td>
</tr>
<tr>
<td>Beamline vacuum</td>
<td>$6 \times 10^{-7}$ Torr</td>
</tr>
<tr>
<td>Chamber vacuum</td>
<td>$4 \times 10^{-7}$ Torr</td>
</tr>
<tr>
<td>Filament current</td>
<td>6.00 rotations</td>
</tr>
<tr>
<td>Magnet current</td>
<td>2.02 rotations</td>
</tr>
<tr>
<td>Beam centering Y</td>
<td>9 o’clock</td>
</tr>
<tr>
<td>Beam centering X</td>
<td>2 o’clock</td>
</tr>
<tr>
<td>Scan amplitude Y</td>
<td>8 o’clock</td>
</tr>
<tr>
<td>Scan amplitude X</td>
<td>8 o’clock</td>
</tr>
<tr>
<td>Focus X</td>
<td>2 o’clock</td>
</tr>
<tr>
<td>Focus Y</td>
<td>2 o’clock</td>
</tr>
<tr>
<td>Discharge</td>
<td>4.0 A</td>
</tr>
<tr>
<td>Extraction</td>
<td>5.0 mA</td>
</tr>
<tr>
<td>Collimator current</td>
<td>100 $\mu$A</td>
</tr>
<tr>
<td>Suppressor current</td>
<td>20 $\mu$A</td>
</tr>
<tr>
<td>Avg. beam current</td>
<td>40 $\mu$A</td>
</tr>
<tr>
<td>Accumulated charge</td>
<td>0.50 C</td>
</tr>
</tbody>
</table>

Table 3.2: Typical tuning parameter settings for implanting $\text{N}_2^+$ ions into tantalum using the UNC ion implanter.

### 3.4 Gas Nitrided Targets

A second fabrication technique known as gas nitriding, was explored for a suitable alternative to implanted targets. Titanium nitride targets were commonly used in previous $^{14}\text{N}(p, \gamma)^{15}\text{O}$ reaction studies [Strieder et al., 2003, Schröder et al., 1987, Rolfs et al., 1973]. The goal was to fabricate relatively thin TiN targets and compare the nitrogen yield with implanted TaN targets. Gas nitriding of titanium is a relatively simple process and could be conveniently performed after resistively heating the backing inside the evaporator system. A high vacuum ($10^{-7}$ Torr) was established in the bell jar and the cryogenic vacuum pump was closed as ultra high purity (99.9997%), research grade nitrogen was leaked through a high
purity gas regulator. The regulator was closed and the previous steps were repeated a total of five times in order to purge contaminants from the regulator and gas line. After purging, the high vacuum was established a final time and the current between the copper electrodes was slowly increased. The current was fixed near 120 Amps which brought the titanium backing to a dull red heat, approximately 800 degrees C [Gulbransen and Andrew, 1949]. The ultra high purity nitrogen was slowly leaked into the closed system until the gas pressure in the bell jar reached 100 Torr. After 20 minutes of nitriding, the current was slowly decreased and the nitrogen gas was evacuated from the bell jar. After sufficient time to cool, the nitried backing was placed in the target box and kept under vacuum. Figure 3.4 is a photograph of a nitrogen implanted tantalum target on the left and a titanium nitride target on the right. The following sections will detail testing the target composition and thickness.

![Figure 3.4: Photograph of TaN (left) and TiN (right) targets with a millimeter scale shown for size reference.](image)

### 3.5 Target Composition

Determining the target stoichiometry used in nuclear reaction measurements is as important as measuring the accumulated beam charge or $\gamma$-ray yield. The stoichiometry was measured through Rutherford Backscattering Spectrometry (RBS) performed at the Triangle Universities Nuclear Laboratory (TUNL) tandem lab. The RBS measurement utilized a Lab-
VIEW controlled, semi-automatic target system [Attayek et al., 2012], and the experimental setup is shown in Figure 3.5. The nitrogen targets were mounted on an aluminum target wheel connected to two stepper motors. The stepper motors moved the target wheel both vertically and rotated it around its central axis. A 3 mm × 3 mm collimated $^4\text{He}^+$ ion beam allowed for precise measurements on bare tantalum and nitrogen implanted regions on the target. The surface normal vector of the target wheel was set at an angle of 22.5 degrees with respect to the incident $^4\text{He}^+$ ion beam. A silicon charged particle detector was positioned approximately 24.4 cm away from the target wheel at an angle of 160 degrees with respect to the beam direction. A 1.0 mm × 9.5 mm aperture was mounted on the face of the charged particle detector to limit the count rate of backscattered $\alpha$ particles.

A collection of used and unused nitrogen implanted targets were mounted on the aluminum target wheel for stoichiometry measurements. It was of course expected, and later confirmed that unused targets would reproduce the published stoichiometry Ta$_2$N$_3$. However, it was unclear what effect the proton beam from the ECR and JN had on target composition over time. Figure 3.6 is a copy of the photograph that was converted into coordinates used by the custom LabVIEW computer applications that controlled the stepper motors. The collimated $^4\text{He}^+$ ion beam spot permitted measurements inside regions of what appeared as bald spots on targets that resulted from over-focusing the proton beam from the JN accelerator. The result of RBS performed directly on the bald spots of used targets confirmed that they reproduced the expected stoichiometry of Ta$_2$N$_3$. We concluded the beam spots altered the surface of the target only and most likely did not penetrate deeply enough in the nitrogen region to affect the stoichiometry. Table 3.3 lists the target descriptions and a summary of the accumulated dose history measured with Rutherford backscattering. Targets are listed by their position relative to the Au calibration sample at the 12 o’clock position and moving clockwise around the target wheel.
Figure 3.5: Schematic of the RBS experimental setup, with major components labeled including the collimators, target wheel, and silicon detector.
Figure 3.6: Photograph of the RBS target wheel used to measure the stoichiometry of various nitrogen implanted targets. See Table 3.3 for a description and summary of the history of each target. Targets are listed by their position relative to the Au calibration sample at the 12 o’clock position and moving clockwise. The target in the 1 o’clock position seems to show the most damage from JN beam; however, its stoichiometry was consistent with neighboring targets that had negligible discoloration from proton bombardment.
Table 3.3: Summary of nitrogen implanted targets analyzed with Rutherford backscattering. See Figure 3.6 for a photograph of the target wheel.

<table>
<thead>
<tr>
<th>Wheel Position</th>
<th>Target</th>
<th>Dose</th>
<th>Accelerator</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 o’clock</td>
<td>Ta$_2$N$_3$</td>
<td>$E_p = 214$ keV, 8.6 C</td>
<td>JN Van de Graaff</td>
</tr>
<tr>
<td>3 o’clock</td>
<td>Ta$_2$N$_3$</td>
<td>$E_p = 214$ keV, 8.7 C</td>
<td>JN Van de Graaff</td>
</tr>
<tr>
<td>5 o’clock</td>
<td>Ta$_2$N$_3$</td>
<td>$E_p = 236$ keV, 5.5 C</td>
<td>JN Van de Graaff</td>
</tr>
<tr>
<td>6 o’clock</td>
<td>Ta$_2$N$_3$</td>
<td>$E_p = 214$ keV, 8.0 C</td>
<td>ECR ion source</td>
</tr>
<tr>
<td>7 o’clock</td>
<td>Ta$_2$N$_3$</td>
<td>$E_p = 214$ keV, 5.0 C</td>
<td>ECR ion source</td>
</tr>
<tr>
<td>9 o’clock</td>
<td>Ta$_2$N$_3$</td>
<td>0.0 C</td>
<td>—</td>
</tr>
<tr>
<td>11 o’clock</td>
<td>Ta$_2$N$_3$</td>
<td>0.0 C</td>
<td>—</td>
</tr>
</tbody>
</table>

The backscattering yield from the Ta and Ta$_m$N$_n$ samples was extracted from the plateau heights ($H_{Ta}$ and $H_{Ta}^{Ta_mN_n}$) in the silicon detector energy spectra shown in Figure 3.7. The stoichiometry was calculated using

$$\frac{n}{m} = \frac{\epsilon_{Ta}}{\epsilon_{N}} \left( \frac{H_{Ta}}{H_{Ta}^{Ta_mN_n}} - 1 \right)$$

(3.6)

where $\epsilon_{Ta}$ and $\epsilon_{N}$ are the stopping powers of 2.0 MeV $^4$He$^+$ ions in tantalum and nitrogen respectively [Riihonen and Keinonen, 1977]. The stopping powers used in Equation 3.6 were calculated from the SRIM2013 computer application [Ziegler and Biersack, 2013]. The average stoichiometry at the centers of targets used in the $^{14}$N($p, \gamma$)$^{15}$O experiment was $n/m = 1.5 \pm 0.1$ (Ta$_2$N$_3$) which is consistent with published values [Runkle et al., 2005, Keinonen and Anttila, 1979]. Since Rutherford backscattering performed at the centers of target samples with noticeable beam spots also reproduced the expected stoichiometry, the data collected from runs on those targets were included in the final analysis.

### 3.6 Target Profiles

Target profiles or yield curves were used to measure the thickness of the active target region and were performed periodically throughout the $^{14}$N($p, \gamma$)$^{15}$O experiment to monitor the condition of each target. Yield curves of the $E_r^{cm} = 259$ keV resonance in $^{14}$N($p, \gamma$)$^{15}$O were
Figure 3.7: Silicon detector RBS energy spectra for bare tantalum and nitrogen implanted regions on a sample target used in the $^{14}$N($p, \gamma$)$^{15}$O direct capture experiment.
measured using the JN Van de Graaff proton accelerator at LENA. A yield curve of a nitrogen target is performed by increasing the proton beam energy in steps over the $E_{\text{cm}} = 259$ keV ($E_{\text{lab}} = 278$ keV) resonance and recording the relative yield of $\gamma$ rays as a function of energy. The measured yield was determined by the intensity of $\gamma$ rays detected from the primary transition to the 6.18 MeV state in $^{15}$O per $\mu$C accumulated on target. The slope of the relatively steep rise at the resonance energy is determined by the convolution of beam resolution and resonance width followed by a constant plateau. The published width of the $E_{\text{cm}} = 259$ keV resonance is $0.99 \pm 0.10$ keV [Ajzenberg-Selove, 1991]. Measured yield curves before and after 6.5 C of proton beam are shown in Figure 3.8. The thickness of the nitrogen region in units of energy, was determined by the FWHM of the yield curve. The flatness of the plateau is a reflection of the target uniformity over the active target region. Typically, the target thickness and composition remained stable over an accumulated dose of 8 C of proton beam delivered by either the JN Van de Graaff or ECR ion source.

Figure 3.8: Measured yield curves of the $E_{\text{lab}} = 278$ keV resonance on a Ta$_2$N$_3$ target before and after accumulating 6.5 C of proton beam from the JN Van de Graaff accelerator.
Yield curves were also used as a relative comparison of active nuclei deposited on a target backing between the two different target fabrication techniques. The titanium nitriding process resulted in a reduced nitrogen yield compared to a nitrogen implanted tantalum target as shown in the yield curve overlay in Figure 3.9. Not only was the yield lower for the TiN target, but there was also a high energy tail that extended deeper into the target backing material. The thickness of the nitrogen layer in the TiN target was determined by the temperature of the titanium backing, the gas pressure, and the length of time spent in the pure nitrogen environment. The parameters of fabricating the gas nitried target were more difficult to control than those of the nitrogen implanted targets and resulted in a nonuniform distribution of nitrogen over the target thickness. A nonuniform active target region is less desirable as it will introduce greater uncertainty in the determination of the effective beam energy. The calculation of the effective beam energy will be discussed in detail in Chapter 4. It was for these reasons the experiment proceeded with nitrogen implanted tantalum targets and not titanium nitride targets.

![Figure 3.9: Relative comparison between measured yield curves of the $E_{\text{lab}} = 278$ keV resonance for a Ta$_2$N$_3$ and TiN target.](image)
CHAPTER 4: MEASURING THE $^{14}$N$(p, \gamma)^{15}$O S-FACTOR

4.1 Experimental Procedures

Measuring the $^{14}$N$(p, \gamma)^{15}$O direct capture $S$-factor required proton beams with energies $E_p = 300$ keV and lower, stable nitrogen targets, and a high-efficiency $\gamma$-ray detector. The LENA facility is equipped with two charged particle accelerators, a 200 kV ECR ion source and 1 MV JN Van de Graaff accelerator. The addition of the NaI(Tl) APEX detector array helped to make LENA an ideal facility for performing the low-energy direct capture study. The LENA high-purity germanium detector (HPGe) was also utilized for target profiling throughout the experiment in an effort to monitor the composition of the nitrogen implanted targets. A photograph of the $\gamma$-ray detection setup located at the target end of the beamline is shown in Figure 4.1.

The APEX detector was positioned around the target chamber with the center of the NaI(Tl) array aligned with the face of the nitrogen implanted target. A lead wall $244 \times 244 \times 0.6$ cm (H × W × THK) that stood in front of the detector area was constructed to shield APEX from x-rays produced while running the ECR source. A HPGe detector located at 0 degrees relative to the incident beam was arranged in close geometry to the face of the target chamber for target yield curves measurements. During the direct capture S-factor study, the HPGe detector was pulled back from the target chamber so it would not interfere with the APEX detector.

During the experiment, $\gamma$-ray events were sorted in the JAM data acquisition software package, and written to a text file that contained the APEX segment number, timing, position, and energy information for each event. The text file was converted into a compressed binary format with $\gamma$-ray event information stored as separate branches in a tree structure.
and analyzed using ROOT [Brun and Rademakers, 1997], an object-oriented data analysis framework. The ROOT software was essential for analyzing the data recorded during this experiment which totaled more than 1 terabyte of disk space.

4.1.1 Energy Calibration

The position calibration procedure outlined in Chapter 2 made it convenient to divide the NaI(Tl) scintillators in 16 intervals along the length of each segment. The position intervals, referred to as “pixels”, were created by placing software gates in the calibrated reconstructed position histograms. A drawing of the APEX detector array divided into pixels is shown in Figure 4.2. A 0.25 $\mu$Ci $^{60}$Co source inside the radial $\gamma$-ray collimator was positioned at the center of each pixel during the calibration of the detector array. The pixels closest to the PMTs on either end of the NaI(Tl) crystal, measured 3.0 cm long. The remaining 14 pixels were chosen to be 3.5 cm long which matched the FWHM of the position resolution.

The energy calibration of the APEX detector was accomplished in two steps, the first
Figure 4.2: Drawing of the APEX detector array divided into 16 pixels per segment for calibration. Energy calibration of the 384 pixels was automated through a ROOT macro.
was to calibrate the detector using the 1460.8 keV and 2614.5 keV γ-ray lines from the $^{40}$K and $^{208}$Tl natural background radiation. Previously, it was discovered that the reconstructed energy of incident γ rays was dependent upon their relative positions in the NaI(Tl) segment. Therefore, each of the 16 pixels were individually calibrated and their reconstructed energy spectra were summed together. A ROOT macro was written that automated the procedure by finding the two signature background peaks in each of the 384 pixels and fitting the 1460.8 keV and 2614.5 keV γ-ray lines to their corresponding channels in the reconstructed energy spectrum.

The second step in the energy calibration procedure was to expand the calibration to higher γ-ray energies using the $E_{cm}^r = 259$ keV resonance in the $^{14}$N($p, \gamma$)$^{15}$O reaction. In addition to the background lines from $^{40}$K and $^{208}$Tl, also included in the calibration were the $^{14}$N($p, \gamma$)$^{15}$O primary and secondary γ-ray transitions, single-escape (SE) peaks, and the characteristic 511 keV positron annihilation radiation from the $^{15}$O $\beta^+$ decay. A tabulation of all γ-ray lines used in the APEX detector energy calibration are listed in Table 4.1.

<table>
<thead>
<tr>
<th>Calibration Source</th>
<th>$E_\gamma$ (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{15}$O $\beta^+$ decay</td>
<td>0.511</td>
</tr>
<tr>
<td>7.56 MeV → 6.79 MeV</td>
<td>0.764</td>
</tr>
<tr>
<td>7.56 MeV → 6.18 MeV</td>
<td>1.38</td>
</tr>
<tr>
<td>7.56 MeV → 5.18 MeV</td>
<td>2.38</td>
</tr>
<tr>
<td>5.18 MeV → 0 (SE)</td>
<td>4.67</td>
</tr>
<tr>
<td>5.18 MeV → 0</td>
<td>5.18</td>
</tr>
<tr>
<td>6.18 MeV → 0 (SE)</td>
<td>5.67</td>
</tr>
<tr>
<td>6.18 MeV → 0</td>
<td>6.18</td>
</tr>
<tr>
<td>6.79 MeV → 0</td>
<td>6.79</td>
</tr>
<tr>
<td>7.56 MeV → 0</td>
<td>7.56</td>
</tr>
</tbody>
</table>

Table 4.1: Tabulation of γ-ray lines used for calibrating the APEX detector array. Primary and secondary γ-ray transitions and single-escape (SE) peaks from the $E_{cm}^r = 259$ keV resonance in $^{14}$N($p, \gamma$)$^{15}$O were included in the calibration.

The γ-ray decay scheme of the $E_{cm}^r = 259$ keV resonance in $^{14}$N($p, \gamma$)$^{15}$O was simulated using GEANT4 [Agostinelli et al., 2003] and compared with experimental data recorded on
the resonance. Post-processing of the GEANT4 output was performed to replicate the energy resolution of the APEX detector using a Gaussian smearing function. A randomly-sampled Gaussian distribution of the form

\[ f(x) = e^{-x^2/2\sigma^2} \]  

\[ \sigma = m \text{(Energy)} + b \]  

with the slope, \( m \), and intercept, \( b \), of 0.015 and 40.0 keV respectively was used to smear the spectrum. The simple form of the linear calibration of \( \sigma \) matched the resolution of the APEX detector’s reconstructed energy histogram. The added detector resolution to the \( E_r^{\text{cm}} = 259 \) keV resonance Monte Carlo simulation, gave precise centroid values for calibrating the peaks in the APEX energy spectrum. A second-order polynomial was fit to the experimental data and the resulting fit coefficients were used in the energy calibration. This procedure resulted in an accurately calibrated reconstructed energy spectrum between 0.5 – 7.5 MeV for data analysis.

### 4.1.2 Detector Efficiency

The \( \gamma \)-ray detection efficiency of the APEX detector was determined from the \( E_r^{\text{cm}} = 259 \) keV resonance in \(^{14}\text{N}(p, \gamma)^{15}\text{O} \). The efficiency can be calculated from the plateau height of an infinitely thick target yield curve, in this case, a 20 keV thick nitrogen implanted target. The \( E_r^{\text{cm}} = 259 \) keV resonance has a natural width of 0.99 ± 0.10 keV [Ajzenberg-Selove, 1991]. For a 20 keV thick nitrogen target, \( \Delta E/\Gamma \approx 20 \) and therefore the maximum yield at the plateau corresponds to > 95% of the maximum yield for an infinitely thick target [Iliadis, 2007]. In the limit of an infinitely thick target, the yield of a Breit-Wigner resonant cross section is written as

\[ Y_{\text{max}} = \frac{\lambda^2 \omega \gamma}{2 \epsilon_r} \]  

assuming that the stopping power is constant over the resonance width [Iliadis, 2007]. The subscript \( r \) indicates that the corresponding quantity evaluated at the resonance energy \( E_r \).
The effective stopping power, $\epsilon_r$, of a target consisting of a compound, is defined as

$$\epsilon_r = \epsilon_N + \frac{n_{Ta}}{n_N} \epsilon_{Ta} \quad (4.4)$$

where $n_{Ta}$ and $n_N$ are the number of backing and target nuclei per square centimeter respectively. The target stoichiometry, Ta$_2$N$_3$, was determined through an RBS measurement outlined in Chapter 3 and the de Broglie wavelength, $\lambda$, is given by Equation 1.16 in Chapter 1. The total yield is experimentally given by

$$Y = \frac{N}{N_b B \eta W} \quad (4.5)$$

where $N$ is the total number of detected $\gamma$-rays, $N_b$ is the total number of incident projectiles, $B$ is the branching ratio, $\eta$ is the efficiency, and $W$ is the angular distribution. Substituting Equation 4.3 into Equation 4.5 and solving for the efficiency, $\eta$, gives

$$\eta = \frac{2 \epsilon_r N_{\text{max}}}{\lambda^2 N_b B W \omega_{\gamma}} \quad (4.6)$$

The total efficiency of the APEX detector was measured for the ground-state transition and the secondary $\gamma$ rays emitted on top of the $E_{cm} = 259$ keV resonance. The experimental yield, $N_{\text{max}}$, was measured using a fraction fitting analysis technique in ROOT and the specifics of this procedure will be outlined in the following sections. Software cuts were made to the data in order to identify single and coincident $\gamma$-ray events (hereafter, multiplicity 1 and 2) as described in more detail below. The efficiencies for multiplicity 1 and 2 cuts over the selected energy range $4.0 - 8.0$ MeV are shown in Figure 4.3 and have a relative uncertainty of approximately 10%. The uncertainty in the reported efficiency was attributed to a combination of both statistical (1%) and systematic sources. The systematic uncertainty was composed of the total charge (2.5%), target composition (6%), branching ratio (6.7%) and resonance strength (4.6%) [Marta et al., 2011]. Results from a GEANT4 Monte Carlo simulation are shown for comparison and agree within the uncertainty of the experimentally determined total efficiencies.
Figure 4.3: APEX total efficiency for multiplicity 1 and 2 cuts over the selected energy range 4.0 MeV to 8.0 MeV. Results from a GEANT4 Monte Carlo simulation are shown for comparison.

4.2 Run Time Summary

The $^{14}\text{N}(p, \gamma)^{15}\text{O}$ experiment can be separated into two phases: an attempt to measure the ground-state direct capture at $E_p = 198$ keV using the ECR ion source, followed by a run at higher energies $E_p = 214, 236, 257$ keV using the JN Van de Graaff accelerator (corresponding to effective center-of-mass energies $E_{\text{eff}} = 180, 195, 216, \text{ and } 235$ keV respectively). The first phase spanned 14 days while the second lasted 18 consecutive days, during which time the JN ran 24 hrs per day except for two maintenance breaks. It was discovered during the JN phase of the experiment that the RF oscillator tubes in the source were susceptible to breakdown and therefore had to be replaced twice. Nearly 220 hrs of direct capture data were recorded using the APEX detector combined with approximately 125 hours of background data. A total of 44 yield curves were performed with the HPGe detector on 24 nitrogen implanted targets throughout the experiment. Table 4.2 summarizes the total charge, time, and average beam current during the $^{14}\text{N}(p, \gamma)^{15}\text{O}$ S-factor measurement at LENA.
<table>
<thead>
<tr>
<th>$E_{\text{eff}}$ (MeV)</th>
<th>LENA Proton Accelerator</th>
<th>Charge (C)</th>
<th>Time (hrs)</th>
<th>Avg. Beam Current (µA)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.235</td>
<td>JN</td>
<td>6.23</td>
<td>24.5</td>
<td>71</td>
</tr>
<tr>
<td>0.216</td>
<td>JN</td>
<td>20.58</td>
<td>62.4</td>
<td>92</td>
</tr>
<tr>
<td>0.195</td>
<td>JN</td>
<td>38.50</td>
<td>103.5</td>
<td>103</td>
</tr>
<tr>
<td>0.180</td>
<td>ECR</td>
<td>51.09</td>
<td>27.7</td>
<td>512</td>
</tr>
</tbody>
</table>

Table 4.2: Run time summary for the low energy S-factor measurement of the $^{14}\text{N}(p, \gamma)^{15}\text{O}$ reaction at LENA.

### 4.3 Data Reduction

The data collected by the APEX detector were reduced primarily by placing software gates on the relative timing between PMT pulses. The average timing, $T_{\text{avg}}$ between PMTs was calculated for each segment and output to a new histogram in JAM. The average timing between PMTs for a NaI(Tl) segment is

$$T_{\text{avg}} = \frac{T_{\text{DC1}} + T_{\text{DC2}}}{2}$$  \hspace{1cm} (4.7)

where $T_{\text{DC1}}$ and $T_{\text{DC2}}$ are the timing values recorded by the TDC for PMT 1 and PMT 2 respectively. A software gate was placed around the peak in the average timing histogram that corresponded to when both PMT 1 and PMT 2 fired simultaneously in what is commonly referred to as a “self-timing peak”. The typical width of the software gate in the average timing histogram was 100 channels wide, which was equivalent to approximately 30 ns. A $^{60}\text{Co}$ source was used to verify the timing resolution of the APEX detector. The $^{60}\text{Co}$ source was placed on either end of a NaI(Tl) scintillator bar and produced a 10 ns timing difference between PMTs. The 30 ns gate width ensured that events corresponding to $\gamma$-rays striking near the ends of the detectors would be included in the gate.

The multiplicity is simply the number of scintillators that detected an amount of ionizing radiation above the CFD threshold in the signal processing electronics. A drawing of example multiplicity 1 and 2 events are illustrated in Figure 4.4 with arrows indicating the $\gamma$ rays.
emitted from a nuclear reaction at the center of the detector array. If two $\gamma$ rays happen to strike the same NaI(Tl) segment, the energies of each are summed together and the event will be recorded as multiplicity 1.

![Multiplicity 1 and Multiplicity 2](image)

Figure 4.4: Drawing of multiplicity 1 and 2 $\gamma$-ray events detected by the APEX detector in a cross-sectional view.

The data were reduced further by discriminating coincident $\gamma$-ray events based on their multiplicity, which is the number of simultaneous segments with a self-timing peak for a $\gamma$-ray event. The $^{14}\text{N}(p, \gamma)^{15}\text{O}$ decay scheme, shown in Figures 1.9 and 1.10, is a combination of the ground-state transition which emits a single $\gamma$ ray, and transitions to the 5.18, 6.18, and 6.79 MeV states which emit two $\gamma$ rays in a cascade. Discriminating between multiplicity 1 and 2 events effectively suppressed or enhanced the ground-state or secondary $\gamma$-ray peaks in the reconstructed energy spectrum.

The result of imposing multiplicity 1 and 2 cuts on the $^{14}\text{N}(p, \gamma)^{15}\text{O}$ $E_{\gamma}^{\text{cm}} = 259$ keV resonance data are shown in Figure 4.5. The intensities of the peaks in the energy spectrum change depending on which multiplicity cut was selected. The multiplicity 1 condition permitted the detection of the ground-state transition at $E_{\gamma} = 7556$ keV but also enhanced the background lines at 1460.8 keV ($^{40}\text{K}$) and 2614.5 keV ($^{208}\text{TI}$). Applying a multiplicity 2 cut enhanced the primary and secondary $\gamma$ rays from transitions to the 5.18, 6.18, and 6.79 MeV states, and completely suppressed the ground-state and background peaks. This result will form the basis of the data analysis technique and is described in detail in the following section.
Figure 4.5: Reconstructed energy spectra of $E_{\text{cm}} = 259$ keV resonance data with imposed multiplicity 1 and 2 cuts. Primary and secondary $\gamma$ rays for transitions to the 5.18, 6.18, 6.79 MeV, and ground state are indicated as well as single-escape peaks.
Another use for discriminating γ-ray events based on multiplicity is for vetoing cosmic-ray induced background. Cosmic rays interacting with air molecules in the upper atmosphere produce protons, electrons, pions, neutrons, and muons. The protons, electrons, and pions are easily absorbed by the exterior structure of the laboratory, while neutrons and muons are very penetrating particles. The muon-induced background dominates the energy region of interest between $4.0 - 8.0$ MeV; therefore, the suppression of this background is especially important. Cosmic-ray muons that strike the detector, shielding, or structure near the detector setup such as the concrete walls or ground, can produce γ-ray showers that deposit energy in more than one neighboring NaI(Tl) crystal. By applying a multiplicity 1 cut in the data analysis, γ-ray showers are effectively vetoed from the energy spectrum.

4.4 Analysis and Results

The data from the $^{14}\text{N}(p, \gamma)^{15}\text{O}$ direct capture $S$-factor measurement were analyzed offline using the ROOT software package. Multiplicity 1 and 2 cuts were applied during data reduction and the data sets were saved in two separate ROOT trees. Sorting by multiplicity number allowed for two simultaneous experiments to be performed. The multiplicity 2 data were used to measure the $S$-factor for transitions to the 5.18, 6.18, and 6.79 MeV states and the multiplicity 1 data were used to measure the ground-state transition. A measurement of the angular correlation of the two-step γ-ray cascades was also attempted and is discussed in Chapter 5. Obtaining the net counts for each transition was challenging because of the relatively poor energy resolution of the APEX detector array, which was 14% for the $E_\gamma = 662$ keV line in $^{137}\text{Cs}$ . The net counts of γ rays detected for each transition was extracted from fitting GEANT4 simulated energy spectra to the data, as outlined in the following sections.

4.4.1 Fraction Fitter

In order to measure the yield of each transition in the $^{14}\text{N}(p, \gamma)^{15}\text{O}$ reaction, the composition of the direct capture energy spectrum was estimated with a Monte Carlo simulation. A GEANT4 simulation of the direct capture γ-ray cascades provided template Monte
Carlo predictions that were used in a maximum likelihood fit. The *TFractionFitter* class in ROOT estimated the fractional contributions of each Monte Carlo prediction to the total energy spectrum. *TFractionFitter* calls the subroutine hmcml [Barlow and Beeston, 1993] for fitting the data histogram with a binned maximum likelihood approach. hmcml uses MINUIT [James and Roos, 1975], a numerical minimization code, to perform the log-likelihood fit and return a set of fractions with uncertainties for the GEANT4 templates.

The advantage of implementing a maximum likelihood fit with *TFractionFitter* is that it takes into account both data and Monte Carlo statistical uncertainties. Since the data are binned in a histogram, the number of counts in many bins tend to be relatively small, making a chi-squared minimization inappropriate [Barlow and Beeston, 1993]. The fit is performed using Poisson statistics; however, the Monte Carlo predictions are also varied within statistics, leading to additional contributions to the overall likelihood. In order for the fit to be properly carried out, two requirements must be fulfilled: (1) the total number of events in each template is large enough that its Poisson uncertainty can be neglected; and (2) the number of events in each bin of the data is much smaller than the total number of events in each template. If these conditions are not fulfilled, *T FractionFitter* can produce biased fit uncertainties [Nappi, 2009].

The procedure for extracting the net counts for each transition in the $^{14}\text{N}(p, \gamma)^{15}\text{O}$ reaction was relatively straight forward. First, the yields of the 5.18, 6.18, and 6.79 MeV transitions were obtained by fitting the multiplicity 2 data. *T FractionFitter* performed the fit using Monte Carlo templates from GEANT4 and background data obtained with the APEX detector. Typically, an energy range between 3.0 − 8.0 MeV was selected in the fit to highlight the signature $^{15}\text{O}$ lines and also to exclude $\gamma$ rays that may have originated from contaminant reactions at higher and lower energies. Fraction fit results of the $E_{cm} = 259$ keV resonance data for multiplicity 2 events are shown in Figure 4.6. As mentioned above, these data were used to determine the detection efficiency of APEX. The background fraction was constrained to match the run time while the fractions of the 5.18, 6.18, and 6.79 MeV transitions were left as free parameters in the fit.
Figure 4.6: Maximum likelihood fit of multiplicity 2 \( E_{\text{cm}} = 259 \) keV resonance data. The fractions of the 5.18, 6.18, and 6.79 MeV transitions were left as free parameters in the fit.

Next, the multiplicity 1 data was fit using \textit{TFractionFitter} to determine the yield of the ground-state transition. The fractions of the 5.18, 6.18, and 6.79 MeV transitions were constrained by the results of the multiplicity 2 fit while the ground-state fraction was left as a free parameter. Constraining the fractions of the \( \gamma \)-ray cascades improved the statistical uncertainty of the ground-state fit result. Fraction fit results of the \( E_{\text{cm}} = 259 \) keV resonance data for multiplicity 1 events are shown in Figure 4.7. The total efficiency for detecting the ground state transition was determined by the result of the multiplicity 1 fit and the 5.18, 6.18, and 6.79 MeV transitions by the multiplicity 2 fit. In the following sections, the results of the direct capture data will be addressed using the aforementioned \textit{TFractionFitter} analysis framework.
Figure 4.7: Maximum likelihood fit of multiplicity 1 $E_{\text{cm}} = 259$ keV resonance data. The 5.18, 6.18, and 6.79 MeV fractions were constrained by results from the fraction fit performed on the multiplicity 2 data. The ground-state fraction was left as a free parameter in the fit.
4.4.2 Effective Energy

In practice, the $\gamma$-ray yield was not measured at the beam energy but rather at an “effective energy” that takes into account the energy distribution of the protons traveling through the target. The experimentally observed reaction yield corresponded to the cross section, $\sigma(E)$, integrated over the thickness of the target, $\Delta E$. Even though nuclear reactions occur over the entire thickness of the target, the number of reaction products emitted will vary depending on the depth inside the target. Because the low-energy cross section of the $^{14}\text{N}(p, \gamma)^{15}\text{O}$ reaction is highly energy-dependent, this effect is exacerbated if thick targets ($>20$ keV) are used. There are several possible approaches to the energy deconvolution of charged-particle reaction measurements [Brune and Sayre, 2013]. The method adopted in this analysis was selected because it is a good approximation (to better than 6%) for ratios of $\sigma_1/\sigma_2 \leq 10$ [Rolfs and Rodney, 1988], which is indeed the case for 10 keV thick nitrogen targets in the energy range of interest. The mean effective beam energy, $E_{\text{eff}}$ is defined as the energy at which 50% of the total yield is obtained, equally dividing the cross section into two halves over the thickness of the target. The effective beam energy can be calculated from the expression

$$E_{\text{eff}} = E_0 - \Delta E + \Delta E \left\{ -\frac{\sigma_2}{\sigma_1 - \sigma_2} + \left[ \frac{\sigma_1^2 + \sigma_2^2}{2(\sigma_1 - \sigma_2)} \right]^{1/2} \right\} \quad (4.8)$$

if the cross section varies linearly between $\sigma_1 = \sigma(E_0)$ and $\sigma_2 = \sigma(E_0 - \Delta E)$. The target thickness was measured at the $E_{\text{cm}} = 259$ keV resonance and determined at the incident proton energy, $E_0$, using the relation

$$\Delta E_0 = \frac{\epsilon_0}{\epsilon_f} \Delta E_f \quad (4.9)$$

The effective stopping powers were calculated at the resonance and beam energies, $\epsilon_f$ and $\epsilon_0$ respectively, using the SRIM2013 computer application [Ziegler and Biersack, 2013]. The correction to the incident proton energy in Equation 4.8 resulted in approximately $4-5$ keV for the 10 keV thick nitrogen implanted targets used in this $^{14}\text{N}(p, \gamma)^{15}\text{O}$ reaction study.
4.4.3 Analysis of $E_{\text{eff}} = 180 \text{ keV}$ Data

At the onset of this project, the goal was to measure the ground-state direct capture contribution in the $^{14}\text{N}(p, \gamma)^{15}\text{O}$ reaction at energies between $E_{\text{cm}} = 150 - 180 \text{ keV}$. The first $S$-factor data point at $E_p = 198 \text{ keV}$ ($E_{\text{eff}} = 180 \text{ keV}$) was attempted with the ECR ion source, but backgrounds from contaminants in the target overwhelmed the yield of the ground-state transition. Figure 4.8 is an overlay of $E_{\text{eff}} = 180 \text{ keV}$ direct capture data with a scaled GEANT4 Monte Carlo simulation. The branching ratios used in the GEANT4 simulation were obtained from published $S$-factor values from previous $^{14}\text{N}(p, \gamma)^{15}\text{O}$ reaction studies [Runkle et al., 2005, Imbriani et al., 2005]. The net counts measured near 7.5 MeV in the APEX data is a factor of 100 times greater than the expected count rate. In an attempt to identify the contaminant $\gamma$-ray peak, the HPGe detector was employed to measure the energy spectrum at several energy points, $E_p = 198, 400, 434, \text{ and } 541 \text{ keV}$. The HPGe energy spectra confirmed that the contaminant $\gamma$ rays were emitted from the $^{13}\text{C}(p, \gamma)^{14}\text{N}$ direct capture reaction.

The energy resolution of the APEX detector array made it impossible to distinguish the contaminant peak from the ground-state peak. To remedy this problem, several steps were taken to remove possible contaminants from the LENA beamline. First, the target chamber was disassembled and each component abrasively sanded and cleaned thoroughly with acetone to remove surface layers of carbon. The copper shroud, electron suppression ring, and collimator had noticeable carbon buildup as they were all subject to direct bombardment by proton beam. After scrubbing the beamline components, the target chamber was re-assembled and optically aligned with the center of the slits located near the beam exit of the analyzing magnet. Aligning the target chamber with the beam axis ensured that the proton beam would be centered on target and pass freely through the center of the beamline components. Next, the vacuum system at the target end of the beamline was upgraded with a new turbomolecular pump and backed by an oil-free, scroll pump. The removal of the old turbomolecular pump and its mechanical backing pump eliminated the oil contaminated components and the new scroll pump minimized the chance of oil migration into the clean
Figure 4.8: Background subtracted energy spectrum of the $E_{\text{eff}} = 180$ keV direct capture data and a scaled GEANT4 Monte Carlo simulation. The discrepancy between the direct capture data and simulation near 7.5 MeV is attributed to proton capture on the contaminant $^{13}\text{C}$. 
target chamber.

The run plan was adjusted to higher energies to maximize the success of measuring the $^{14}\text{N}(p, \gamma)^{15}\text{O}$ S-factor using the APEX detector. The JN Van de Graaff accelerator was used exclusively to measure the S-factor at ($E_{\text{eff}} = 235$, 216, and 195 keV). Before the new run plan commenced, the direct capture energy spectrum was carefully studied at the proposed energy points using the HPGe detector. The first two data points at $E_{\text{eff}} = 235$, and 216 keV verified the absence of contaminant $\gamma$-ray lines among the signature $^{15}\text{O}$ peaks, and increased our confidence that the carbon contaminants were indeed removed. The results of the S-factor measurement at $E_{\text{eff}} = 235$, 216, and 195 keV is discussed in the following sections.

4.4.4 5.18, 6.18, and 6.79 MeV Transitions

Experimental S-factors for transitions to the 5.18, 6.18, and 6.79 MeV states were obtained from the observed $\gamma$-ray yields, target thicknesses, detector efficiencies, and beam charge collection at each energy. The maximum likelihood fits performed on the multiplicity 2 data at $E_{\text{eff}} = 235$, 216, and 195 keV are shown in Figures 4.9, 4.10, and 4.11 respectively. The signal to background ratio is greatest for the $E_{\text{eff}} = 235$ keV data but still $< 1$, and subsequently resulted in the lowest statistical uncertainty for the experimental yield of each transition. At $E_{\text{eff}} = 195$ keV, $\gamma$ rays the cosmic-ray background in the region of interest nearly obscured the $^{15}\text{O}$ lines in the energy spectrum. It was realized that the experiment could not successfully proceed to lower energies because of the level of background present in the APEX detector.

The number of $\gamma$ rays detected, $N$, was extracted from the APEX detector energy spectrum using $T\text{FractionFitter}$. The non-resonant cross section for each transition can then be calculated from the expression

$$\sigma (E_{\text{eff}}) = \frac{\epsilon_{\text{eff}} N}{\Delta E N_b \eta W}$$

(4.10)

since the effective stopping power, $\epsilon_{\text{eff}}$, is constant over the target thickness, $\Delta E$ [Iliadis, 2007]. $N_b$ is the total number of incident protons, measured as accumulated charge, and $\eta$ is the $\gamma$-ray detection efficiency. The detected secondary $\gamma$ rays have not shown an angular dependence
Figure 4.9: Maximum likelihood fit of multiplicity 2 data measured at $E_{\text{eff}} = 235$ keV. The top panel is plotted on a linear scale to show detail of the data, fraction fit result, and background contribution. The bottom panel is drawn using a log scale to illustrate the predicted fractions of the 5.18, 6.18, and 6.79 MeV transitions.
Figure 4.10: Maximum likelihood fit of multiplicity 2 data measured at $E_{\text{eff}} = 216$ keV. The top panel is plotted on a linear scale to show detail of the data, fraction fit result, and background contribution. The bottom panel is drawn using a log scale to illustrate the predicted fractions of the 5.18, 6.18, and 6.79 MeV transitions.
Figure 4.11: Maximum likelihood fit of multiplicity 2 data measured at $E_{\text{eff}} = 195$ keV. The top panel is plotted on a linear scale to show detail of the data, fraction fit result, and background contribution. The bottom panel is drawn using a log scale to illustrate the predicted fractions of the 5.18, 6.18, and 6.79 MeV transitions.
in previous $^{14}\text{N}(p, \gamma)^{15}\text{O}$ measurements [Schröder et al., 1987], thus the angular distribution, $W$, was assumed to be 1. In addition, the low-energy cross section is dominated by the $E_{\text{cm}}^{\text{res}} = 259$ keV resonance which primarily proceeds through $l = 0$ capture and consequently emits isotropic $\gamma$ rays [Runkle, 2003]. The primary $\gamma$ rays emitted in direct capture exhibit some angular dependence, specifically in the transition to the 6.79 MeV state [Schröder et al., 1987]; however, only the secondaries were used in the determination of the cross section.

The resulting $S$-factors for the 5.18, 6.18, and 6.79 MeV transitions in the $^{14}\text{N}(p, \gamma)^{15}\text{O}$ reaction are shown in Figure 4.12. The current work, labeled “APEX”, was compared with the results from previous $^{14}\text{N}(p, \gamma)^{15}\text{O}$ reaction experiments performed at LUNA [Imbriani et al., 2005] and LENA [Runkle et al., 2005]. The measured $S$-factors for the 5.18, and 6.18 MeV transitions were consistent with results from LUNA and LENA within their uncertainty. The $S$-factors measured for the 6.79 MeV transition were also consistent with the LENA data; however, they disagreed with those of LUNA at the 1-sigma level at the two lowest energies.

Since the focus of the current study was not necessarily to determine the $S$-factor for the 5.18, 6.18, and 6.79 MeV transitions, it was satisfying to achieve a successful measurement without added expense to the project. Ultimately, the observed $\gamma$-ray yields from the multiplicity 2 data were found to be essential for constraining the fraction fit of the ground-state transition and is detailed in the following section.

### 4.4.5 The Ground-state Transition

The ground-state $S$-factor was determined from the multiplicity 1 data using the observed $\gamma$-ray yields from the multiplicity 2 data as constraints in the fraction fit. Convergence of the fit was dependent on the constrained background and multiplicity 2 fractions. The maximum likelihood fits performed on the direct capture data at $E_{\text{eff}} = 235, 216, \text{and } 195$ keV are shown in Figures 4.13, 4.14, and 4.15 respectively. The fit window was adjusted to the energy range between $5.9 - 8.1$ MeV to provide a greater signal to background ratio. The relatively weak transition to the 5.18 MeV state was excluded from the fraction fit since it resided outside of the fit window. A sample ROOT analysis code for fitting the ground state in the $E_{\text{eff}} = 235$ keV direct capture data is included in Appendix D.
Figure 4.12: Astrophysical $S$-factor for the 5.18, 6.18, and 6.79 MeV transitions measured using the APEX detector in addition to data from previous experiments performed at LUNA [Imbriani et al., 2005] and LENA [Runkle et al., 2005].
Figure 4.13: Maximum likelihood fit of multiplicity 1 data measured at $E_{\text{eff}} = 235$ keV. The top panel is plotted on a linear scale to show detail of the data, fraction fit result, and background contribution. The bottom panel is drawn using a log scale to illustrate the predicted fractions of the ground-state transition.
Figure 4.14: Maximum likelihood fit of multiplicity 1 data measured at $E_{\text{eff}} = 216$ keV. The top panel is plotted on a linear scale to show detail of the data, fraction fit result, and background contribution. The bottom panel is drawn using a log scale to illustrate the predicted fractions of the ground-state transition.
Figure 4.15: Maximum likelihood fit of multiplicity 1 data measured at $E_{\text{eff}} = 195$ keV. The top panel is plotted on a linear scale to show detail of the data, fraction fit result, and background contribution. The bottom panel is drawn using a log scale to illustrate the predicted fractions of the ground-state transition.
The residuals for fraction fits performed on the $E_{\text{eff}} = 235, 216,$ and 195 keV multiplicity 1 data are shown in the top, middle, and bottom panels in Figure 4.16. The black points represent the difference between the data and the fraction fit result and the blue points represent the difference between the data and a fit composed only of the background contribution. The fraction fit residuals scatter around zero in a relatively tight band for the $E_{\text{eff}} = 235$ and 216 keV data, which supports the validity of the fraction fit model. The residuals for the $E_{\text{eff}} = 195$ keV data suggest that near the ground-state energy, 7.5 MeV, a distinction cannot be made between a fit composed strictly of background and a fit performed with $^{14}\text{N}(p, \gamma)^{15}\text{O}$ Monte Carlo templates. This conclusion is also supported by the relatively large uncertainties in the $E_{\text{eff}} = 195$ keV fraction fit results and is discussed in the following section.

Further examination of the energy spectra revealed that the multiplicity 1 requirement produced a smaller signal to background ratio than the multiplicity 2 data over the energy region of interest. The multiplicity 1 cut was more restrictive because it required 23 of the 24 scintillator bars were quiet, yet it produced a greater background fraction if the background consisted of multiplicity 1 events. The multiplicity 2 data showed a 25% reduction in background at 7.5 MeV over the multiplicity 1 data. Although $TFractionFitter$ was able to successfully converge, the statistical uncertainty of the ground-state fraction fit at $E_{\text{eff}} = 195$ keV was approximately 200%. As mentioned previously, the results of the fit demonstrated the useful energy range of the existing APEX experimental setup. In order to study the ground-state $S$-factor at energies lower than $E_{\text{eff}} = 195$ keV, higher intensity proton beam and improved shielding from cosmic-ray induced background would be necessary.

The ground-state $S$-factor results are shown in Figure 4.17. The data measured with the APEX detector were consistent with ground-state data from previous $^{14}\text{N}(p, \gamma)^{15}\text{O}$ experiments performed at LUNA [Imbriani et al., 2005] and LENA [Runkle et al., 2005]. Both the LUNA and LENA studies measured the low energy $S$-factor using HPGe detectors in close running geometry; therefore, summing corrections were incorporated into their analysis. The relative uncertainty of the APEX data is greater than that of LUNA, but closely matches the LENA data with the exception of the measurement at $E_{\text{eff}} = 195$ keV. The uncertainties associated with the APEX $S$-factor data are discussed in more detail in the following section.
Figure 4.16: Residuals for fraction fits to the $E_{\text{eff}} = 235$, 216, and 195 keV multiplicity 1 data are shown in the top, middle, and bottom panels respectively. The black points represent the difference between the data and the fraction fit result and the blue points represent the difference between the data and a fit composed only of the background contribution.
Figure 4.17: Astrophysical $S$-factor for the ground-state transition measured using the APEX detector in addition to data from previous experiments performed at LUNA [Imbriani et al., 2005] and LENA [Runkle et al., 2005].

4.5 Summary and Discussion

In summary, the $S$-factor was evaluated for transitions to the ground state, 5.18 MeV, 6.18 MeV, and 6.79 MeV states at $E_{\text{eff}} = 235$, 216, and 195 keV. The run plan had to be adjusted to slightly higher energies from the intended study of the interference dip between $E_{\text{cm}} = 150 - 180$ keV owing to the presence of the $^{13}\text{C}(p, \gamma)^{14}\text{N}$ contaminant reaction. While the current study did not probe energies as low as those performed in past experiments, the APEX results add confidence to the $S$-factor values reported in sum-corrected measurements [Runkle et al., 2005, Imbriani et al., 2005]. The segmentation and geometry of the APEX detector permitted a measurement of the $^{13}\text{C}(p, \gamma)^{14}\text{N}$ ground-state $S$-factor with negligible summing corrections. Furthermore, the minor summing contributions present in the ground-state peak were included in Monte Carlo templates that were fit to the APEX data using a maximum likelihood fraction fit.

The fraction fit analysis technique employed to extract single peaks from a composite
spectrum proved to be a successful strategy for a NaI(Tl) scintillator array that had an energy resolution of 14%. The majority of the relative uncertainty in the ground-state $S$-factor was due to the low counting statistics of the detected signal. The limiting factor in this experiment was the ratio of the $^{14}\text{N}(p, \gamma)^{15}\text{O}$ reaction yield to the cosmic-ray induced background. Unfortunately, the results for the ground-state transition presented in the current work will not further constrain an R-matrix fit to the $S$-factor data. However, the comparable error bars in the ground-state $S$-factor with previous studies [Runkle et al., 2005] highlights the current work as an alternative method for measuring weak nuclear cross sections near astrophysically relevant energies.

4.5.1 Relative Uncertainty

The relative uncertainty attributed to the measured astrophysical $S$-factor was a combination of both statistical and systematic sources. The statistical uncertainty in the observed $\gamma$-ray yield was derived from the results of the fraction fit and was an indicator of the strength of the maximum likelihood. Additionally, the statistical uncertainty was greatest for those $S$-factors measured at the lowest energy point, $E_{\text{eff}} = 195$ keV. This was a result of the sizable background fraction present in the fit window. The background fraction constituted 91%, 97%, and 99% of the multiplicity 1 data at $E_{\text{eff}} = 235, 216,$ and 195 keV respectively. A tabulation of the statistical uncertainties from the fraction fits to the direct capture data are listed in Table 4.3.

The overall systematic uncertainty in this study was minimized to 16% by careful procedure design and analysis of measured quantities used in the $S$-factor calculation. Sources of systematic uncertainty included the total charge, target thickness and composition, effective energy, and $\gamma$-ray detection efficiency. Table 4.4 lists the sources of systematic uncertainties that contributed to the measurement of the $S$-factor. The target thickness (10%) and detector efficiency (10%) were the most dominant sources of systematic uncertainty. In the end however, the systematics did not add a significant amount to the total uncertainty. The low counting statistics accounted for the majority of the $S$-factor uncertainty at nearly every data point.
### Table 4.3: Tabulation of the statistical uncertainties from fitting the direct capture data using TFractionFitter in ROOT.

<table>
<thead>
<tr>
<th>(E_{\text{eff}}) (MeV)</th>
<th>Transition</th>
<th>Net Counts (counts/Coulomb)</th>
<th>Statistical Uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.235</td>
<td>Ground state</td>
<td>168 ± 63</td>
<td>37.5%</td>
</tr>
<tr>
<td></td>
<td>5.18 MeV</td>
<td>1032 ± 99</td>
<td>9.6%</td>
</tr>
<tr>
<td></td>
<td>6.18 MeV</td>
<td>3745 ± 138</td>
<td>3.7%</td>
</tr>
<tr>
<td></td>
<td>6.79 MeV</td>
<td>1739 ± 123</td>
<td>7.1%</td>
</tr>
<tr>
<td>0.216</td>
<td>Ground state</td>
<td>41 ± 30</td>
<td>73.2%</td>
</tr>
<tr>
<td></td>
<td>5.18 MeV</td>
<td>264 ± 47</td>
<td>17.8%</td>
</tr>
<tr>
<td></td>
<td>6.18 MeV</td>
<td>897 ± 64</td>
<td>7.1%</td>
</tr>
<tr>
<td></td>
<td>6.79 MeV</td>
<td>405 ± 58</td>
<td>14.3%</td>
</tr>
<tr>
<td>0.195</td>
<td>Ground state</td>
<td>10 ± 21</td>
<td>210.0%</td>
</tr>
<tr>
<td></td>
<td>5.18 MeV</td>
<td>31 ± 31</td>
<td>100.0%</td>
</tr>
<tr>
<td></td>
<td>6.18 MeV</td>
<td>173 ± 43</td>
<td>24.9%</td>
</tr>
<tr>
<td></td>
<td>6.79 MeV</td>
<td>178 ± 40</td>
<td>22.5%</td>
</tr>
</tbody>
</table>

### Table 4.4: Tabulation of the systematic uncertainty that contributed to the \(^{14}\text{N}(p, \gamma)^{15}\text{O}\) S-factor uncertainty.

<table>
<thead>
<tr>
<th>Uncertainty Type</th>
<th>Uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total charge</td>
<td>3%</td>
</tr>
<tr>
<td>Target composition</td>
<td>6%</td>
</tr>
<tr>
<td>Target thickness</td>
<td>10%</td>
</tr>
<tr>
<td>Effective energy</td>
<td>3%</td>
</tr>
<tr>
<td>Efficiency</td>
<td>10%</td>
</tr>
<tr>
<td>Systematic Uncertainty</td>
<td>16%</td>
</tr>
</tbody>
</table>


4.5.2 Total $S$-factor

A summary of the measured astrophysical $S$-factors (in units of keV·b) for transitions to the ground, 5.18, 6.18, and 6.79 MeV states in the $^{14}\text{N}(p, \gamma)^{15}\text{O}$ reaction are listed in Table 4.5. The total $S$-factor at each energy was calculated by summing the results from the individual transitions and adding their uncertainties in quadrature. The relative uncertainty associated with the total $S$-factor at $E_{\text{eff}} = 235$, 216, and 195 keV were approximately 9%, 12%, and 20% respectively.

<table>
<thead>
<tr>
<th>Transition</th>
<th>$E_{\text{eff}}$ (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ground state</td>
<td>0.03 ± 0.06</td>
</tr>
<tr>
<td></td>
<td>0.07 ± 0.05</td>
</tr>
<tr>
<td></td>
<td>0.17 ± 0.07</td>
</tr>
<tr>
<td>5.18 MeV</td>
<td>0.16 ± 0.16</td>
</tr>
<tr>
<td></td>
<td>0.69 ± 0.16</td>
</tr>
<tr>
<td></td>
<td>1.6 ± 0.3</td>
</tr>
<tr>
<td>6.18 MeV</td>
<td>0.88 ± 0.25</td>
</tr>
<tr>
<td></td>
<td>2.4 ± 0.4</td>
</tr>
<tr>
<td></td>
<td>5.9 ± 0.9</td>
</tr>
<tr>
<td>6.79 MeV</td>
<td>0.94 ± 0.25</td>
</tr>
<tr>
<td></td>
<td>1.11 ± 0.25</td>
</tr>
<tr>
<td></td>
<td>2.9 ± 0.5</td>
</tr>
<tr>
<td>Total</td>
<td>2.0 ± 0.4</td>
</tr>
<tr>
<td></td>
<td>4.3 ± 0.5</td>
</tr>
<tr>
<td></td>
<td>11 ± 1</td>
</tr>
</tbody>
</table>

Table 4.5: Summary of astrophysical $S$-factors for the $^{14}\text{N}(p, \gamma)^{15}\text{O}$ reaction (in units of keV·b).

The total $S$-factor measured using the APEX detector is shown in Figure 4.18 in addition to data from previous experiments performed by Lamb and Hester [Lamb and Hester, 1957], Schröder [Schröder et al., 1987], LUNA [Imbriani et al., 2005, Bemmerer et al., 2006], and LENA [Runkle et al., 2005]. The data of Lamb and Hester were obtained in an activation measurement that included systematic uncertainties of 20% from the detection efficiency and 15% from the beam intensity [Lamb and Hester, 1957]. A significant background contribution from the $^{12}\text{C}(p, \gamma)^{13}\text{N}$ reaction was also reported that may have contributed to their slightly higher total $S$-factor values. The results of the APEX experiment at $E_{\text{eff}} = 235$, and 216 keV, rested nearly on top of the data from Schröder, LUNA, and LENA. The total $S$-factor measured at $E_{\text{eff}} = 195$ keV in the present study fell just below the published values from Schröder and LUNA, but was consistent with the results of the LENA measurement within uncertainty.
The results of the APEX $S$-factor measurement have not impacted the $^{14}\text{N}(p, \gamma)^{15}\text{O}$ reaction rate. The total $S$-factors were measured in an energy region that had already been investigated with relatively high precision. As mentioned in Chapter 1, stellar modelers often assume a constant $S$-factor equal to an extrapolated $S_{\text{tot}}(0)$ value determined from a fit to the data in Figure 4.18. In the end, the APEX data did not constrain the fit to the total $S$-factor and consequently did not change the extrapolation to $S_{\text{tot}}(0)$. However, the APEX results have supported the $S$-factor values reported in previous sum-corrected experiments and demonstrated the success of analyzing a composite energy spectrum from a segmented NaI(Tl) scintillator array for astrophysical $S$-factor measurements at low energies.

Figure 4.18: Total astrophysical $S$-factor measured using the APEX detector in addition to data from previous experiments performed by Lamb and Hester [Lamb and Hester, 1957], Schröder [Schröder et al., 1987], LUNA [Imbriani et al., 2005, Bemmerer et al., 2006], and LENA [Runkle et al., 2005].
CHAPTER 5: MEASURING ANGULAR CORRELATIONS

5.1 Experimental Procedures

The angular correlations of two-step $\gamma$ cascades in the $^{14}\text{N}(p, \gamma)^{15}\text{O}$ reaction were measured at the $E_{cm}^{\text{r}} = 259$ keV resonance and at the direct capture energies of $E_{\text{eff}} = 235, 216,$ and 195 keV. The relatively high detection efficiency, segmentation, and large solid angle coverage (75% of $4\pi$) of APEX, made it a suitable $\gamma$-ray detector for these measurements. The goal of the present study was to provide angular correlation data for the coincidence sum corrections in $^{14}\text{N}(p, \gamma)^{15}\text{O}$ direct capture experiments that utilized detectors arranged in a close counting geometry. A successful measurement would also highlight a supplementary application of the position sensing capability of the APEX NaI(Tl) array. Furthermore, the analysis could be performed on the previously sorted multiplicity 2 data from the $^{14}\text{N}(p, \gamma)^{15}\text{O}$ $S$-factor measurement and so additional run time was not required.

Figure 5.1 is a drawing of the APEX detector and angular correlation geometry in cylindrical coordinates. The vectors $\vec{r}_1$ and $\vec{r}_2$ represent the primary and secondary $\gamma$ rays emitted in the decay of the $^{15}\text{O}$ nucleus, and $\theta$ represents the angle between their correlated emission directions. The origin of the coordinate system was placed at the face of the nitrogen target that was exposed to the proton beam and corresponded with the center of the APEX detector. The $\gamma$-ray hit position, $z$, along the symmetry axis of APEX, was determined by the reconstructed position defined in Equation 2.4. The radius, $r$, was assumed to be a constant and measured from the detector center line to the center of the NaI(Tl) scintillator crystals. The azimuthal angle, $\phi$, was measured with respect to the segment positioned at the top of the array and spanned between adjacent scintillator bars in 15 degree increments.

The reconstructed vectors of the primary and secondary $\gamma$ rays were then used to calculate
Figure 5.1: Drawing of correlated $\gamma$ rays detected by APEX in a cross-sectional view. The $\gamma$-ray hit positions are recorded in cylindrical coordinates with respect to the origin located at the center of the detector.
the separation angle to determine their correlation. The dot product of vectors of \( \vec{r}_1 \) and \( \vec{r}_2 \) is defined as

\[
\vec{r}_1 \cdot \vec{r}_2 = \|\vec{r}_1\| \|\vec{r}_2\| \cos \theta
\]

(5.1)

and can be rearranged to solve for the angle, \( \theta \), between the two radiations

\[
\theta = \cos^{-1} \left( \frac{\vec{r}_1 \cdot \vec{r}_2}{\|\vec{r}_1\| \|\vec{r}_2\|} \right)
\]

(5.2)

The angle \( \theta \) was calculated for coincident \( \gamma \) rays emitted in transitions to the 5.18, 6.18, and 6.79 MeV states that had deposited their full-energy in the APEX detector. The results of the angular correlation measurement at the \( E_{cm}^{\gamma} = 259 \) keV resonance were more convincing than those at the direct capture energies and therefore were the primary focus of the current work.

The angular correlation function was determined for the 6.18 MeV transition at \( E_{\text{eff}} = 235 \) keV but unfortunately, the relatively low counting statistics of the direct capture yield and sizable background contribution, prohibited an accurate correlation measurement at lower energies. The method of analysis used in the resonance and direct capture data will be outlined in detail in the following sections.

### 5.2 Data Reduction

The data collected by the APEX detector were reduced using the procedure outlined in Chapter 4. Software gates placed on the relative timing between PMT pulses sorted events based on their multiplicity number. A multiplicity 2 requirement was imposed on the resonant capture and direct capture data in order to pick out the two-photon cascades for the angular correlation study. The events that corresponded to transitions to the 5.18, 6.18, and 6.79 MeV states were selected using a \( \gamma\gamma \)-coincidence technique. A 2-d histogram containing the energies of coincident \( \gamma \)-ray events at the \( E_{cm}^{\gamma} = 259 \) keV resonance is shown in the top panel of Figure 5.2. The energy deposited in one NaI(Tl) crystal is displayed on the vertical axis and the energy deposited in a corresponding pair scintillator bar on the horizontal axis. The superimposed elliptical energy gates were used to extract \( \gamma \)-ray events that matched the
full-energy peaks of the primaries and secondaries in the $^{14}\text{N}(p, \gamma)^{15}\text{O}$ $\gamma$-ray cascades. The elliptical shape was a result of the difference in detector energy resolution of the low energy primary and higher energy secondary $\gamma$ rays. The histogram in the bottom panel of Figure 5.2 is a projection of the resonance data and background defined by the elliptical energy gates imposed in the 2-d $\gamma\gamma$-coincidence histogram. The background data were normalized to the run time of the resonant capture data and sorted using the same multiplicity and energy cuts.

The most significant advantage of employing a $\gamma\gamma$-coincidence technique was the substantial reduction in the environmental background. Most background events occurred in only one segment at a time, and therefore were eliminated by demanding a coincidence between the timing signals of both scintillators. The environmental and Compton background were drastically cut further by the restrictive energy gate placed on the full-energy peaks of coincident $\gamma$ rays. The Compton scattering events from each of the 5.18, 6.18, and 6.79 MeV transitions were visible as horizontal and vertical lines extending from their respective full-energy peaks. Compared to the 1-d multiplicity 2 energy spectrum shown in Figure 4.5, the full-energy peaks of the $^{14}\text{N}(p, \gamma)^{15}\text{O}$ primaries and secondaries were easily discernible from the background using the $\gamma\gamma$-coincidence technique. The primary $\gamma$ ray lines that were located in an energy region dominated by room background ($E_\gamma < 3 \text{ MeV}$) could now be distinguished in the 1-d projection histogram with a signal-to-noise ratio (S/N) of approximately 1000:1. Because of their significantly reduced count rates, the direct-capture data had S/N of about 2:1 and nearly 1:1 for the lowest energy point.

5.3 Analysis and Results

A schematic level diagram of the $^{14}\text{N}(p, \gamma)^{15}\text{O}$ reaction proceeding through the $E_{cm}^{cm} = 259$ keV resonance is shown in Figure 5.3. Here the 7.556 MeV state in $^{15}\text{O}$ with spin and parity of $J^\pi = 1/2^+$, decays to an intermediate state, $J$, which in turn, decays to the ground state with spin and parity of $J^\pi = 1/2^-$. The multipolarities of the primary and secondary $\gamma$ rays are identified in Figure 5.3 by $L_1$ and $L_2$ respectively. The $j_1 \rightarrow J$ primary transitions in the $E_{cm}^{cm} = 259$ keV resonance proceeded via electric or magnetic dipole radiation.
Figure 5.2: Energy cuts used in the $E_{\text{cm}}^{\gamma} = 259$ keV resonance angular correlation measurement. The top panel is a 2-d $\gamma\gamma$-coincidence histogram of $\gamma$-ray energies detected using a multiplicity 2 condition. The energy deposited in one NaI(Tl) crystal is displayed on the vertical axis and the energy deposited in a corresponding pair scintillator bar on the horizontal axis. The histogram in the bottom panel is a projection of the resonance and background events defined by the elliptical energy gates imposed in the $\gamma\gamma$-coincidence histogram, plotted on a log scale.
\(L = 1\). The \(J \rightarrow j_2\) secondary transitions proceeded via pure or mixed radiations of different multipolarities and is discussed in the following section.

![Figure 5.3: Schematic level diagram of the \(^{14}\text{N}(p, \gamma)^{15}\text{O}\) reaction proceeding through the \(E_{\text{cm}} = 259\) keV resonance with spin and parity of \(J^\pi = 1/2^+\). The resonance \(\gamma\) ray decays to an intermediate state, \(J\), which in turn, decays to the \(^{15}\text{O}\) ground state with spin and parity of \(J^\pi = 1/2^-\). The multipolarities of the primary and secondary \(\gamma\) rays are identified by \(L_1\) and \(L_2\) respectively.](image)

The capture of unpolarized protons in the \(J^\pi = 1/2^+\) state uniformly populates the \(m = \pm 1/2\) magnetic substates of the resonance, and produces an isotropic radiation pattern. The correlation function in the case of the \(E_{\text{cm}} = 259\) keV resonance, is thus independent of the direction of the incident proton beam and the positions of the NaI(Tl) scintillators. A ROOT analysis code was written to calculate the angle of separation between the primary and secondary \(\gamma\) rays for each event using Equation 5.2. Since the correlation was independent of the direction of the primary \(\gamma\) ray, the analysis code took advantage of this subtle detail and sorted every coincident primary and secondary \(\gamma\) ray without the need for an angular distribution correction. The separation angles filled a histogram with a bin width of 15 degrees, chosen specifically to match the 15 degree azimuthal resolution of the APEX detector.

There were several experimental corrections that were necessary to perform before com-
paring the experimental data with the theoretical angular correlation functions. First, the background events were removed from the angular correlation data recorded by the APEX detector. The background was normalized to the run time and sorted according to the procedure presented above, and the remaining background events that resided inside $\gamma\gamma$-coincidence cuts were subtracted from the data. Next, the measured intensities of the experimental data were corrected for the finite solid angles subtended by the detector segments since the theoretical angular correlation applied only to an ideal detector of negligible size. If the measured correlation is

$$W_{\text{exp}}(\theta) = \sum_n a'_n P_n(\cos \theta)$$

(5.3)

and the ideal correlation function measured with a counter of negligible size is

$$W(\theta) = \sum_n a_n P_n(\cos \theta)$$

(5.4)

then $a_n$ and $a'_n$ are related by

$$a_n = a'_n / Q_n$$

(5.5)

where the attenuation factors, $Q_n$, depend on the detector geometry, and the $\gamma$-ray detection efficiency [Ferguson, 1965]. The $Q_n$ factors were determined for the APEX detector using a GEANT4 simulation of the $^{14}\text{N}(p, \gamma)^{15}\text{O}$ reaction decay scheme. Each branch of the $^{15}\text{O}$ decay scheme was simulated and compared to the experimental data analyzed with the same $\gamma\gamma$-coincidence cuts. In the end, the measured angular correlation was divided by the Monte Carlo simulation, which effectively corrected the angular correlation for detector effects without having to calculate the attenuation factors explicitly.

As a last correction, the separation angle measured in the laboratory was converted to the center-of-mass system. The laboratory and center-of-mass angle of the emitted photon in the $^{14}\text{N}(p, \gamma)^{15}\text{O}$ reaction is related by

$$\cos \theta = \frac{\cos \theta' + \beta}{1 + \beta \cos \theta'}$$

(5.6)
where $\theta$ is measured in the laboratory system and $\theta'$ in the center-of-mass system. The relativistic parameter, $\beta$, is defined as

$$\beta = \frac{\sqrt{E_p^{\text{lab}}(E_p^{\text{lab}} + 2m_p c^2)}}{m_N c^2 + m_p c^2 + E_p^{\text{lab}}}$$  \hspace{1cm} (5.7)$$

and $m_p$ and $m_N$ are the mass of the proton and $^{14}$N nucleus respectively [Iliadis, 2007]. In the present study, the low energy protons, $E_p^{\text{lab}}$, and relatively heavy target nuclei resulted in a negligible relativistic parameter, $\beta = 0.0014 - 0.0016$. Therefore, the angle of the emitted $\gamma$ rays in the laboratory and center-of-mass reference systems were within 0.1 degrees of each other and considered approximately the same ($\theta \approx \theta'$).

### 5.3.1 Resonant Capture Data

The angular correlation function was calculated for the 5.18, 6.18, and 6.79 MeV transitions at the $E_r^{\text{cm}} = 259$ keV resonance and compared to experimental data. In this section, all separation angles, $\theta$, refer to the center-of-mass system. The angular correlation between the two de-excitation $\gamma$ rays can be calculated using

$$W(\theta) = \sum_{n=0,2,...} F_n (L_1 j_1 J_1) F_n (L_2 j_2 J_2) P_n (\cos \theta)$$  \hspace{1cm} (5.8)$$

with $0 \leq n \leq \min(2L_1, 2L_2, 2J)$. The $F_n$ coefficients are expressed in terms of the Clebsch-Gordan $\left(L_1 L_1' - 1|n0\right)$ and Racah $\left(JJLL'; nj\right)$ vector coupling coefficients

$$F_n (LL'jJ) \equiv (-)^{j-J-1} \sqrt{(2L + 1)(2L' + 1)(2J + 1)} \left(L1L' - 1|n0\right) W (JJLL'; nj)$$  \hspace{1cm} (5.9)$$

where $j$ and $J$ are angular momenta of nuclear states and $L$ and $L'$, in this case, are the multipolarities of the primary and secondary $\gamma$ rays emitted in the decay [Biedenharn, 1960]. Numerical values for the $F_n (LjJ)$ can be found in published tables [Biedenharn and Rose, 1953] in addition to the mixed correlation coefficients $F_n (LL'jJ)$ for $L \neq L'$ [Appel, 1968].
The correlation function for the 5.18 MeV transition was calculated from the angular momentum sequence

\[ j_1(L_1)J(L_2)j_2 \rightarrow \frac{1}{2}(1)^1/2 \]

(5.10)

The spin and parity of the 5.18 MeV state was \( J = 1/2^+ \), so the allowed multipolarities for the primary and secondary \( \gamma \) rays were M1 and E1 respectively. The sum of the terms in Equation 5.8 reduced to \( n = 0 \) since \( 0 \leq n \leq \min(2L_1, 2L_2, 2J) \) where \( J = 1/2 \); therefore, the angular correlation function was given by

\[
W_{5.18}(\theta) = \sum_{n=0}^{\infty} F_n(L_1 \ j_1 \ J) F_n(L_2 \ j_2 \ J) P_n(\cos \theta)
\]

\[ = 1 \]  

(5.11)

and determined to be isotropic.

The correlation function for the 6.18 MeV transition was calculated from the angular momentum sequence

\[ j_1(L_1)J(L_2)j_2 \rightarrow \frac{1}{2}(1)^3/2 \]

(5.12)

where the spin and parity of the 6.18 MeV state was \( J = 3/2^- \). Conservation of angular momentum and parity demanded that the multipolarity of the primary \( \gamma \) ray be E1, while the secondary \( \gamma \) ray was allowed to be a mixture of M1 and E2. The total angular correlation was then given by the sum of the individual correlations, each weighted according to their probability,

\[
W(\theta) = W_L(\theta) + 2\delta W_{LL'}(\theta) + \delta^2 W_{L'}(\theta)
\]

(5.13)

where \( \delta \) was the \( \gamma \)-ray multipolarity mixing ratio, \( \delta^2 = \Gamma_{E2}/\Gamma_{M1} \). A \( \gamma \)-ray mixing ratio of \( \delta(E2/M1) = 0.125 \pm 0.007 \) has been tabulated for this transition [Ajzenberg-Selove, 1991].
Here the angular correlation function is given by

\[ W_{6.18}(\theta) = 1 + F_2(L_1 j_1 J) F_2(L_2 j_2 J) P_2(\cos \theta) \]
\[ + 2\delta \gamma F_2(L_1 j_1 J) F_2(L_2 L' j_2 J) P_2(\cos \theta) \]
\[ + \delta^2 \gamma F_2(L_1 j_1 J) F_2(L_2' j_2 J) P_2(\cos \theta) \]
\[ = 1 + F_2(1 \frac{1}{2} \frac{3}{2}) F_2(1 \frac{1}{2} \frac{3}{2}) P_2(\cos \theta) \]
\[ + 2\delta \gamma F_2(1 \frac{1}{2} \frac{3}{2}) F_2(1 2 \frac{1}{2} \frac{3}{2}) P_2(\cos \theta) \]
\[ + \delta^2 \gamma F_2(1 \frac{1}{2} \frac{3}{2}) F_2(2 \frac{1}{2} \frac{3}{2}) P_2(\cos \theta) \]
\[ = 1 + 0.138 P_2(\cos \theta) \] (5.14)

The correlation function for the 6.79 MeV transition was calculated from the angular momentum sequence

\[ j_1(L_1) j_2 \rightarrow \frac{1}{2}(1)^{3/2}(1)^{1/2} \] (5.15)

where the spin and parity of the 6.79 MeV state was \( J = 3/2^+ \). Conservation of angular momentum and parity demanded the multipolarity of the primary \( \gamma \) ray to be \( M1 \), while the secondary \( \gamma \) ray was allowed to be a mixture of \( E1 \) and \( M2 \). However, the \( \gamma \)-ray mixing ratio for the 6.79 MeV transition was reported as \( \delta(M2/E1) = 0.02 \pm 0.02 \) in previous studies [Ajzenberg-Selove, 1991] and thus, the contribution of the \( M2 \) component is negligible compared to \( E1 \). Therefore, only the \( E1 \) case needed to be considered for the secondary transition. The angular correlation function was given by

\[ W_{6.79}(\theta) = 1 + F_2(L_1 j_1 J) F_2(L_2 j_2 J) P_2(\cos \theta) \]
\[ = 1 + F_2(1 \frac{1}{2} \frac{3}{2}) F_2(1 \frac{1}{2} \frac{3}{2}) P_2(\cos \theta) \]
\[ = 1 + 0.250 P_2(\cos \theta) \] (5.16)

The results of the angular correlation measurement at the \( E_{cm} = 259 \) keV resonance are shown in Figures 5.4, 5.5, and 5.6 for the 5.18, 6.18, and 6.79 MeV transitions respectively. In each figure, the top panel displays the background subtracted data recorded by the APEX
detector, and a scaled Monte Carlo simulation of the same transition with an isotropic correlation \(W(\theta) = 1\). The environmental background detected in the \(\gamma\gamma\)-coincidence cuts is plotted on the same axes for comparison, but has negligible intensity as compared to the in-beam data. The histogram in the bottom panel in each figure was filled by dividing the APEX data by the isotropic geant4 simulation in order to correct the measured angular correlation for the detector geometry as outlined in the previous section. The data for the 5.18 MeV transition were fit with a line where the slope and intercept were kept as free parameters. The data for the 6.18, and 6.79 MeV transitions were fit with a function of the form \(W(\theta) = a_0 + a_2P_2(\cos \theta)\) where \(a_0\) and \(a_2\) were free parameters and \(P_2(\cos \theta)\) was the second-order Legendre polynomial.

A summary of the measured and predicted angular correlation functions at the \(E_{\text{cm}}^{\text{res}} = 259\) keV resonance are listed in Table 5.1. The extracted Legendre polynomial coefficients from fits to the 5.18 MeV and 6.18 MeV transition data, \(a_0(5.18) = 1.02 \pm 0.04\) and \(a_0(6.18) = 0.99 \pm 0.01\), \(a_2(6.18) = 0.12 \pm 0.02\) respectively, were consistent with theoretical values. Additionally, the Legendre polynomial coefficients from the fit to the 6.79 MeV transition, \(a_0(6.79) = 0.98 \pm 0.01\) and \(a_2(6.79) = 0.21 \pm 0.02\), were only slightly lower than the predicted coefficients, but still in close agreement.

### 5.3.2 Direct Capture Data

The angular correlation of \(\gamma\) rays emitted in direct capture at \(E_{\text{eff}} = 235, 216,\) and 195 keV was analyzed with the same \(\gamma\gamma\)-coincidence technique used in the resonant capture data, but several difficulties were encountered. The predicted angular correlation functions for direct capture have been calculated in previous radiative capture studies [Rolfs, 1973], but were not as straightforward to determine as those for resonant capture. Thus, the focus of the current work was to investigate the direct capture correlation purely from an experimental perspective. The environmental background presented the largest difficulty to overcome, as it obscured the correlation function for every transition except for the 6.18 MeV transition measured at \(E_{\text{eff}} = 235\) keV. The results of the angular correlation measurement for the 6.18 MeV transition are shown in Figure 5.7.
Figure 5.4: Angular correlation of $\gamma$ rays detected from the 5.18 MeV transition in the $E^\text{cm}_r = 259$ keV resonance data. The histogram in the bottom panel is filled by dividing the data by the isotropic Monte Carlo simulation, and the result is fit with a line.
Figure 5.5: Angular correlation of γ rays detected from the 6.18 MeV transition in the $E_{cm} = 259$ keV resonance data. The histogram in the bottom panel is filled by dividing the data by the isotropic Monte Carlo simulation, and the result is fit with the second-order Legendre polynomial.
Figure 5.6: Angular correlation of γ rays detected from the 6.79 MeV transition in the $E_{cm} = 259$ keV resonance data. The histogram in the bottom panel is filled by dividing the data by the isotropic Monte Carlo simulation, and the result is fit with the second-order Legendre polynomial.
Figure 5.7: Angular correlation of $\gamma$ rays detected from the 6.18 MeV transition in the $E_{\text{eff}} = 235$ keV direct capture data. The histogram in the bottom panel is filled by dividing the data by the isotropic Monte Carlo simulation, and the result is fit with the second-order Legendre polynomial.

$$W(\theta) = 1.0 - 0.2\cos^2\theta$$
The measured correlation function for the 6.18 MeV transition was successfully fit with a second-order Legendre polynomial and the result of the fit is shown in the bottom panel of Figure 5.7. The variations in the intensity of the correlation angle, \(\theta\), was a direct result of the low counting statistics and relatively poor outcome of the background subtraction. The unique shape of the background histogram in the top panel of the figure was a result of \(\gamma\) rays Compton scattering between nearest and next-to-nearest neighboring NaI(Tl) bars, a separation angle of \(\theta = 15\) and 30 degrees respectively. The angular correlation function was fit with coefficients of \(a_0(6.18) = 0.95 \pm 0.07\) and \(a_2(6.18) = -0.2 \pm 0.1\), indicating a peak at 90 degrees, which was 90 degrees out of phase with the 6.18 MeV measured correlation at the \(E_{\text{cm}} = 259\) keV resonance. The direct capture data for the 6.18 MeV transition suggests that the primary and secondary \(\gamma\) rays would be less likely to sum together at small angles which of course is favorable when employing detectors positioned in a close counting geometry.

5.4 Summary and Discussion

In summary, angular correlations were measured for the 5.18, 6.18, and 6.79 MeV transitions and compared to the theoretical angular correlation functions. The background was drastically reduced using a \(\gamma\gamma\)-coincidence technique which included both a multiplicity requirement and full-energy peak cuts in the energy spectra. The background contribution that remained after the software cuts was subtracted from the data. A detector geometry correction was applied to the measured intensities by simulating the \(^{14}\text{N}(p,\gamma)^{15}\text{O}\) reaction with isotropic correlations using GEANT4, and dividing the data by the results of the Monte Carlo simulation. The theoretical and experimental angular correlation functions measured at the \(E_{\text{cm}} = 259\) keV resonance are listed in Table 5.1. Overall, the correlation coefficients extracted from the Legendre polynomial fit were in good agreement with the theoretical values.

The success of the angular correlation measurement at the \(E_{\text{cm}} = 259\) keV resonance prompted the examination of the direct capture data using the same analysis framework. Unfortunately, the environmental background obscured the correlation function for nearly every transition except for the 6.18 MeV transition, which was measured at \(E_{\text{eff}} = 235\) keV. In
Table 5.1: Tabulation of theoretical and measured angular correlation functions for transitions to the 5.18 MeV, 6.18 MeV, and 6.79 MeV states at the $E_{cm}^m = 259$ keV resonance. Uncertainties are given in parentheses and refer to the last significant digit.

<table>
<thead>
<tr>
<th>Transition</th>
<th>Theoretical $W(\theta)$</th>
<th>Experimental $W(\theta)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.18 MeV</td>
<td>1</td>
<td>1.02(4)</td>
</tr>
<tr>
<td>6.18 MeV</td>
<td>$1 + 0.138P_2(\cos \theta)$</td>
<td>0.99(1) + $0.12(2)P_2(\cos \theta)$</td>
</tr>
<tr>
<td>6.79 MeV</td>
<td>$1 + 0.250P_2(\cos \theta)$</td>
<td>0.98(1) + $0.21(2)P_2(\cos \theta)$</td>
</tr>
</tbody>
</table>

an effort to determine the effect that this correlation function has on summing-in to the ground state, a Geant4 simulation was performed of the LENA HPGe detector in close geometry to the target. To increase the statistics of the Monte Carlo simulation, the primary $\gamma$ ray of the 6.18 MeV transition was aimed directly at the face of the HPGe detector while the secondary $\gamma$ ray was aimed according to the measured correlation function, $W(\theta) = 1 - 0.2P_2(\cos \theta)$. For comparison, an isotropic correlation function was also simulated for an identical detector and target geometry. The number of detected events from summing-in to the ground-state peak indicated an 8% decrease in coincidence summing for the measured correlation function compared to the isotropic case.

The Monte Carlo simulation revealed that while summing corrections would still be necessary for a single detector positioned in a close counting geometry, the corrections for the 6.18 MeV transition may be slightly reduced since the most probable angle of emission was measured at 90 degrees. The limiting factor in measuring the angular correlation of $\gamma$ rays using the APEX detector was the ratio of the signal to background, as was the case in the $S$-factor measurement. If the angular correlation measurement were to proceed to non-resonant reactions in future experiments, higher intensity proton beam and improved shielding from background would be necessary.
CHAPTER 6: CONCLUSION

While the physics goals of this project will require further work, we have demonstrated a new approach to doing low-energy measurements that, with refinements, has some unique capabilities. We have assembled and characterized an array of 24 NaI(Tl) scintillator detectors and demonstrated its effectiveness at measuring the astrophysical $S$-factor for the $^{14}\text{N}(p, \gamma)^{15}\text{O}$ reaction. The APEX detector array was outfitted with 48 new photomultiplier tubes and light-tight aluminum mounts. The detector readout was integrated into the existing LENA data acquisition system and powered inside a mobile electronics setup that recorded the energy, position, and timing of $\gamma$-ray events. A radial $\gamma$-ray source collimator was designed and constructed to calibrate the $\gamma$-ray hit positions in the APEX detector. A measurement of $\gamma$-ray angular correlations was also performed to highlight the position sensing capability of the NaI(Tl) array.

Considerable time and effort were put toward fabricating nitrogen targets and maximizing the beam current delivered by the accelerators. The Eaton NV-3206 ion implanter at UNC was repaired and made reliable for the production of dozens of nitrogen targets used in the $S$-factor and angular correlation studies. Implanted targets were chosen for the measurements over titanium nitride targets for their higher yield and uniformity. The 1 MV JN Van de Graaff accelerator was optimized for maximum proton beam on target in the energy range of $E_p = 200 – 300$ keV. The average beam current on target ranged between 100 – 150 $\mu$A and the accelerator tune was stable throughout the experiment.

Previous $^{14}\text{N}(p, \gamma)^{15}\text{O}$ experiments that utilized $\gamma$-ray detectors in close geometry to the target, resulted in considerable coincidence summing. We successfully measured the astrophysical $S$-factor for the $^{14}\text{N}(p, \gamma)^{15}\text{O}$ reaction using the APEX detector without the need for coincidence summing corrections. The $S$-factor was evaluated for transitions to the ground,
5.18, 6.18, and 6.79 MeV states at $E_{\text{eff}} = 235, 216,$ and 195 keV. While we did not probe energies as low as those performed in past experiments, our data add confidence to the $S$-factor values reported in previous coincidence sum-corrected measurements. The current work does not further constrain the R-matrix fit of the $S$-factor for the ground state transition, but it does highlight an alternative method for measuring weak nuclear cross sections near astrophysically relevant energies. The fraction fit analysis technique makes use of information that is usually ignored or treated as background, and thus has broader applications in $\gamma$-ray spectroscopy.

The limiting factor in this experiment was the ratio of the $^{14}\text{N}(p, \gamma)^{15}\text{O}$ reaction yield to the cosmic-ray induced background. The majority of the relative uncertainty in the ground-state astrophysical $S$-factor was a result of the low counting statistics of the detected signal. Increasing the signal to background ratio could be accomplished by improving the detector shielding, increasing the beam current, and using a nitrogen target with a higher yield. Shielding the APEX detector with 2.0 inch thick lead bricks would reduce the environmental background between 3.0 and 8.0 MeV by about 20%, but still would not be substantial enough for probing direct capture at lower energies. The next $^{14}\text{N}(p, \gamma)^{15}\text{O}$ experiment if performed in a mine deep underground, would achieve a drastically reduced background from cosmic-ray events otherwise unattainable in a surface level laboratory. Utilizing an electron cyclotron resonance ion source instead of a Van de Graaff accelerator would increase the beam intensity considerably. Maximum beam currents of 1.5 mA on target have been achieved with the 200 kV ECR ion source in LENA; however, the proton-induced capture reactions on contaminants in the target backing obscured the $^{14}\text{N}(p, \gamma)^{15}\text{O}$ signal. An alternative target system, such as a supersonic gas jet target, would eliminate the target backing and therefore achieve lower beam-induced background and decreased effective stopping power. The experimental hurdles mentioned above suggest that we have reached a limit with the current apparatus in the LENA laboratory for measuring the low-energy astrophysical $S$-factor of the $^{14}\text{N}(p, \gamma)^{15}\text{O}$ reaction.
# APPENDIX A: PMT SPECIFICATIONS

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Table A.1: Specifications for Photonis XP2012 photomultiplier tubes as reported on the test ticket issued by the manufacturer.
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Table A.2: Specifications for Hamamatsu R580 photomultiplier tubes as reported on the test ticket issued by the manufacturer.
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Table B.1: Settings for the CAEN high voltage supply, spectroscopy amplifiers, and CFD modules.
// The APEX Detector
void DetectorConstruction::BuildAPEX(){
  G4double SlabLength = 57.0*cm;
  G4double SlabHeight = 0.8*cm;
  G4double SlabWidth = 7.1*cm;

  G4double HalfSlabLength = SlabLength*0.5;
  G4double HalfSlabHeight = SlabHeight*0.5;
  G4double HalfSlabWidth = SlabWidth*0.5;

  G4double TrapLength = 55.0*cm;
  G4double TrapHeight = 6.0*cm;
  G4double TrapWidthShort = 5.5*cm;
  G4double TrapWidthLong = 7.0*cm;

  G4double HalfTrapLength = TrapLength*0.5;
  G4double HalfTrapHeight = TrapHeight*0.5;
  G4double HalfTrapWidthShort = TrapWidthShort*0.5;
  G4double HalfTrapWidthLong = TrapWidthLong*0.5;

  G4double QuartzDia = 4.4*cm;
  G4double QuartzWidth = 1.1*cm;

  G4double QuartzRad = QuartzDia*0.5;
  G4double QuartzThick = QuartzWidth*0.5;

  G4double Radius = 24.5*cm;
  G4double NaIsegRotate;

  G4double Theta = 0.0*deg;
  G4double Phi = 0.0*deg;
  G4double Alpha = 0.0*deg;

  G4ThreeVector zAxis(0,0,1.);
  G4ThreeVector SSteelSlab(0,3.44*cm,0);
  G4ThreeVector QuartzHoleL(0,0,-28.05*cm);
  G4ThreeVector QuartzHoleR(0,0,28.05*cm);
  G4ThreeVector APEXradius(0,-24.5*cm,0);
  G4ThreeVector AngCorradius(0,0,24.5*cm);
  G4RotationMatrix noRotate;

  //-----------------------------------------------------------
  // Scintillator Casing
  //-----------------------------------------------------------

  G4Box* scintcasing_box = new G4Box("SSteel Slab",HalfSlabWidth,HalfSlabHeight,HalfSlabLength);

  G4Tubs* cyl_hole = new G4Tubs("Cylindrical Hole",0.*mm,QuartzRad,QuartzThick,0,twopi);}
G4Trap* ssteelcasing_trap = new G4Trap("NaI Casing", HalfTrapLength+10.0*mm, Theta, Phi, ←
HalfTrapHeight + 0.4*mm, HalfTrapWidthShort + 0.4*mm, ←
HalfTrapWidthLong + 0.5*mm, Alpha, HalfTrapHeight + 0.4*mm, ←
HalfTrapWidthShort + 0.4*mm, HalfTrapWidthLong + 0.5*mm, Alpha);

G4Box* volume_box = new G4Box("Slab Volume", HalfSlabWidth, HalfSlabHeight, ←
HalfSlabLength + 2.0*mm);

G4Trap* volume_trap = new G4Trap("Trap Volume", HalfTrapLength+12.0*mm, Theta, Phi, ←
HalfTrapHeight + 0.4*mm, HalfTrapWidthShort + 0.4*mm, ←
HalfTrapWidthLong + 0.5*mm, Alpha, HalfTrapHeight + 0.4*mm, ←
HalfTrapWidthShort + 0.4*mm, HalfTrapWidthLong + 0.5*mm, Alpha);

// Subtract material for quartz windows
G4SubtractionSolid* ssteelcasing_trap_sub = new G4SubtractionSolid("Quartz Window Hole L", ssteelcasing_trap, ←
cyl_hole,&noRotate,QuartzHoleL);
G4SubtractionSolid* scintcasing_trap = new G4SubtractionSolid("Quartz Window Hole R", ←
ssteelcasing_trap_sub,cyl_hole,&noRotate,QuartzHoleR);

// Now join the SSteelSlab to the main NaIcasing
G4UnionSolid* scintcasing = new G4UnionSolid("Scintillator Casing", scintcasing_trap, ←
scintcasing_box,&noRotate,SSSteelSlab);

// Now join the SlabVolume to the main TrapVolume
G4UnionSolid* scintvolume = new G4UnionSolid("Scintillator Volume", volume_trap, volume_box,&←
noRotate,SSSteelSlab);

G4LogicalVolume* ScintCasing_log = new G4LogicalVolume(scintcasing, ssteel, "SSteel NaI Casing", 0, ←
0, 0);

G4LogicalVolume* ScintVolume_log = new G4LogicalVolume(scintvolume, Air, "Air Volume", 0, 0, 0);

// Place SSteel NaI Casing inside the Segment Volume
new G4PVPlacement(0,
  G4ThreeVector(), // at (0,0,0)
  ScintCasing_log, // Logical Volume
  "Segment Volume", // Name
  ScintVolume_log, // Mother Volume
  false, // no boolean operations
  0); // Copy number
for(G4int i=0;i<24;i++){
    NaIsegRotate=(i*(360/24))*deg;
    new G4PVPlacement(G4Transform3D(G4RotationMatrix(zAxis,−→NaIsegRotate),
        G4ThreeVector(Radius,std::sin(NaIsegRotate),−→Radius,std::cos(NaIsegRotate),0)),
        ScintVolume_log, // Logical Volume
        "APEX Segment ", // Name
        LENAALab_log, // Mother Volume
        false, // no boolean operations
        i); // Copy number
}

//—— Quartz Windows ——
G4Tubs* quartz_cyl
    = new G4Tubs("Quartz Cylinder",0.*mm,QuartzRad,QuartzThick,0,twopi)→

G4LogicalVolume* Quartz_log
    = new G4LogicalVolume(quartz_cyl, Quartz,"Quartz Window",0,0,0);
new G4PVPlacement(0, // no rotation
        QuartzHoleL, // at (0,0,−28.05*cm)
        Quartz_log, // Logical Volume
        "Quartz Window L", // Name
        ScintVolume_log, // Mother Volume
        false, // no boolean operations
        0); // Copy number
new G4PVPlacement(0, // no rotation
        QuartzHoleR, // at (0,0,28.05*cm)
        Quartz_log, // Logical Volume
        "Quartz Window R", // Name
        ScintVolume_log, // Mother Volume
        false, // no boolean operations
        1); // Copy number

//—— Scintillators ——
G4Trap* scintillator_trap
    = new G4Trap("NaI Crystal",HalfTrapLength,Theta,Phi,HalfTrapHeight,←
        HalfTrapWidthShort,HalfTrapWidthLong,Alpha,HalfTrapHeight,←
        HalfTrapWidthShort,HalfTrapWidthLong,Alpha);

// G4LogicalVolume* APEXseg_log
APEXNaI_log
    = new G4LogicalVolume(scintillator_trap, NaITl, "APEX NaI ←
        Scintillator", 0, 0, 0);

// G4VPhysicalVolume* scintillator_phys
new G4PVPlacement(0, // no rotation
        G4ThreeVector(), // at (0,0,0)
        APEXNaI_log, // Logical Volume
        "NaI Scintillator", // Name
        ScintCasing_log, // Mother Volume
        false, // no boolean operations
        0); // Copy number
// The scintillator volume is invisible
ScintVolume_log->SetVisAttributes(G4VisAttributes::Invisible);

// Scintillator casing is dark gray
G4VisAttributes* ssteelVisAtt = new G4VisAttributes(G4Colour(0.4, 0.4, 0.4));
ScintCasing_log->SetVisAttributes(ssteelVisAtt);

// NaI(Tl) scintillator is yellow
G4VisAttributes* detVisAtt = new G4VisAttributes(G4Colour(1.0, 1.0, 0.0));
APEXNaI_log->SetVisAttributes(detVisAtt);

// Quartz window is white
G4VisAttributes* quartzVisAtt = new G4VisAttributes(G4Colour(1.0, 1.0, 1.0));
Quartz_log->SetVisAttributes(quartzVisAtt);
APPENDIX D: ROOT ANALYSIS

Summary

Calls TFractionFitter class in ROOT to fit ground–state fraction and calculate S–factors for 235 keV direct capture data.

{
// Read ROOT files
TFile *inFile_mc_5181keV = new TFile("/usr/project/cygnus2/daigle/root/rootFiles/MC/235keV/sorted/5181keV_sorted.root","READ");
TFile *inFile_mc_6172keV = new TFile("/usr/project/cygnus2/daigle/root/rootFiles/MC/235keV/sorted/6172keV_sorted.root","READ");
TFile *inFile_mc_6792keV = new TFile("/usr/project/cygnus2/daigle/root/rootFiles/MC/235keV/sorted/6792keV_sorted.root","READ");
TFile *inFile_mc_7532keV = new TFile("/usr/project/cygnus2/daigle/root/rootFiles/MC/235keV/sorted/7532keV_sorted.root","READ");
TFile *inFile_data = new TFile("/usr/project/cygnus2/daigle/root/rootFiles/257keV/257keV_sorted.root","READ");
TFile *inFile_bkgrnd = new TFile("/usr/project/cygnus2/daigle/root/rootFiles/bkgrnd/bkgrnd_sorted.root","READ");

// Get histograms from ROOT files
TH1F *hmc_5181keV = (TH1F*)inFile_mc_5181keV->Get("hAPEXmult1Res");
TH1F *hmc_6172keV = (TH1F*)inFile_mc_6172keV->Get("hAPEXmult1Res");
TH1F *hmc_6792keV = (TH1F*)inFile_mc_6792keV->Get("hAPEXmult1Res");
TH1F *hmc_7532keV = (TH1F*)inFile_mc_7532keV->Get("hAPEXmult1Res");
TH1F *hdata = (TH1F*)inFile_data->Get("hAPEXmult1");
TH1F *hbkgrnd = (TH1F*)inFile_bkgrnd->Get("hAPEXmult1");

// Rebin histograms
Float_t Emin = 5900.0;
Float_t Emax = 8120.0;
Float_t Bin = 20.0;
Float_t Bmin = Emin/ BIN;
Float_t Bmax = Emax/ BIN;
hmc_5181keV->Rebin(Bin);
hmc_6172keV->Rebin(Bin);
hmc_6792keV->Rebin(Bin);
hmc_7532keV->Rebin(Bin);
hdata->Rebin(Bin);
hbkgrnd->Rebin(Bin);

// Scale histograms prior to fitting
Double_t scalefactor = hdata->Integral(Bmin,Bmax);
hmc_5181keV->Scale(scalefactor/hmc_5181keV->Integral(Bmin,Bmax));
hmc_6172keV->Scale(scalefactor/hmc_6172keV->Integral(Bmin,Bmax));
hmc_6792keV->Scale(scalefactor/hmc_6792keV->Integral(Bmin,Bmax));
hmc_7532keV->Scale(scalefactor/hmc_7532keV->Integral(Bmin,Bmax));
hbkgrnd->Scale(scalefactor/hbkgrnd->Integral(Bmin,Bmax));
}
// Fit data using TFractionFitter
TObjArray *data = new TObjArray(5);
data->Add(hmc_5181keV);
data->Add(hmc_6172keV);
data->Add(hmc_6792keV);
data->Add(hmc_7532keV);
data->Add(hbkgrnd);

TFractionFitter *fit = new TFractionFitter(hdata, data);

// Constrain background and multiplicity 2 transitions
fit->Constrain(1, 0.00000000, 0.000000001);
fit->Constrain(2, 0.03906600, 0.03906601);
fit->Constrain(3, 0.03774190, 0.03774191);
fit->Constrain(5, 0.91000000, 0.91000001);

fit->SetRangeX(Bmin, Bmax);
Int_t status = fit->Fit();
std::cout << "fit status: " << status << std::endl;

if (status == 0) {
  TH1F* result = (TH1F*) fit->GetPlot();
  TH1F* h5181 = (TH1F*) fit->GetMCPrediction(0);
  TH1F* h6172 = (TH1F*) fit->GetMCPrediction(1);
  TH1F* h6792 = (TH1F*) fit->GetMCPrediction(2);
  TH1F* h7532 = (TH1F*) fit->GetMCPrediction(3);
  TH1F* hbkgrnd = (TH1F*) fit->GetMCPrediction(4);
}

// Get fractions and corresponding uncertainties
Double_t par[5];
Double_t err[5];

for (int i=0;i<5;i++) {
  fit->GetResult(i, par[i], err[i]);
}

// Scale MC predictions by fractions
h5181->Scale(par[0]);
h6172->Scale(par[1]);
h6792->Scale(par[2]);
h7532->Scale(par[3]);
hbkgrnd->Scale(par[4]);

// Define APEX total efficiencies
Double_t e_5181 = 0.00000001;
Double_t e_6172 = 0.0150077;
Double_t e_6792 = 0.0301760;
Double_t e_7532 = 0.1240170;

Double_t Ue_5181 = 0.000000001;
Double_t Ue_6172 = 0.00115849;
Double_t Ue_6792 = 0.00235672;
Double_t Ue_7532 = 0.01267450;
// Calculate integrals
Double_t integralerr;
Double_t integral = hdata->IntegralAndError(Bmin, Bmax, integralerr, "")
;
Double_t i_5181 = par [0] * integral;
Double_t i_6172 = par [1] * integral;
Double_t i_6792 = par [2] * integral;
Double_t i_7532 = par [3] * integral;

Double_t Uxs_7532 = Double_t Uxs_6792
Double_t Uxs_6172 = Double_t Uxs_5181

Double_t xs_7532 = Double_t xs_6792
Double_t xs_6172 = Double_t xs_5181

Double_t sommerfeld = Double_t E_eff
Double_t Np = Double_t E_ta
Double_t E_n = Double_t E_n
Double_t Mt = Double_t Mt
Double_t Mp = Double_t Mp

Int_t Zt = Int_t Zt
Int_t Zp = Int_t Zp
Int_t Zt = 7;

Double_t E_df = 0.1 * dE;

Double_t Ec = 235.0; //keV
Double_t dE = 10550.0; //eV
Double_t Ue_df = 0.1 * dE;

Double_t E_n = 11.21e-15; Double_t UE_n = 0.05 * E_n;
Double_t E_ta = 32.59e-15; Double_t UE_ta = 0.05 * E_ta;
Double_t BCI = 6.229893; Double_t UBCI = 0.025 * BCI;
Double_t UNp = BCI / 1.60218e-19; Double_t UNp = 0.025 * Np;

Double_t E_eff = E_n + ((2.0 / 3.0) * E_ta);
Double_t UE_eff = pow((pow(UE_n, 2.0) + pow(UE_ta, 2.0)), 0.5);

Double_t sommerfeld = 31.291813 * Zp * Zt * pow(((1.0 / Ec) * ((Mp * Mt) / (Mp + Mt))

Double_t xs_7532 = ((E_eff * i_7532) / (Np * e_7532 * dE)) / 1.0e-24;
Double_t xs_6172 = ((E_eff * i_6172) / (Np * e_6172 * dE)) / 1.0e-24;
Double_t xs_6792 = ((E_eff * i_6792) / (Np * e_6792 * dE)) / 1.0e-24;
Double_t xs_7532 = ((E_eff * i_7532) / (Np * e_7532 * dE)) / 1.0e-24;

Double_t Uxs_5181 = xs_5181 * pow((pow(UE_eff / E_eff, 2.0) + pow(Ui_5181 / i_5181, 2.0), 0.5);
Double_t Uxs_6172 = xs_6172 * pow((pow(UE_eff / E_eff, 2.0) + pow(Ui_6172 / i_6172, 2.0), 0.5);
Double_t Uxs_6792 = xs_6792 * pow((pow(UE_eff / E_eff, 2.0) + pow(Ui_6792 / i_6792, 2.0), 0.5);
Double_t Uxs_7532 = xs_7532 * pow((pow(UE_eff / E_eff, 2.0) + pow(Ui_7532 / i_7532, 2.0), 0.5);
Double_t s_5181 = exp(sommerfeld)*xs_5181*Ecm;
Double_t s_6172 = exp(sommerfeld)*xs_6172*Ecm;
Double_t s_6792 = exp(sommerfeld)*xs_6792*Ecm;
Double_t s_7532 = exp(sommerfeld)*xs_7532*Ecm;

Double_t Us_5181 = s_5181*(Uxs_5181/xs_5181);
Double_t Us_6172 = s_6172*(Uxs_6172/xs_6172);
Double_t Us_6792 = s_6792*(Uxs_6792/xs_6792);
Double_t Us_7532 = s_7532*(Uxs_7532/xs_7532);

// Print results
printf("Ecm = 235 keV\n");
printf("Fit Range = %g keV to %g keV\n",Emin,Emax);
printf("Net Counts\n");
printf("5181 keV Integral = %g +/- %g\n",i_5181,Ui_5181);
printf("6172 keV Integral = %g +/- %g\n",i_6172,Ui_6172);
printf("6792 keV Integral = %g +/- %g\n",i_6792,Ui_6792);
printf("Grnd St. Integral = %g +/- %g\n",i_7532,Ui_7532);

printf("S-factor\n");
printf("5181 keV S-factor = %g +/- %g keV*b\n",s_5181,Us_5181);
printf("6172 keV S-factor = %g +/- %g keV*b\n",s_6172,Us_6172);
printf("6792 keV S-factor = %g +/- %g keV*b\n",s_6792,Us_6792);
printf("Grnd St. S-factor = %g +/- %g keV*b\n",s_7532,Us_7532);
REFERENCES


A. E. Champagne. private communication, 2013.


