\( ^{12} \text{(d, p)} ^{13} \text{C} \): DISTORTED WAVE BORN APPROXIMATION ANALYSIS OF PROTON ANGULAR DISTRIBUTIONS AND PROTON-GAMMA ANGULAR CORRELATIONS

by

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Date: Dec. 28, 1963

Approved:

Henry W. Newson, Supervisor

A dissertation submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy in the Department of Physics in the Graduate School of Arts and Sciences of Duke University

1963
ABSTRACT
(Physics)

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ABSTRACT

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ANGULAR CORRELATIONS

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DWBA fits have been made to existing $^{12}(d,p_{2,3})C^{13}$ proton angular
distribution and proton-gamma angular correlation data. The optical
model potentials used in the distorted wave calculations were found by
fitting $^{12}(d,d)C^{12}$ and $^{13}(p,p)C^{13}$ angular distributions measured at
the appropriate energies. The results of the DWBA analysis have also
been compared with the predictions of the Butler theory. Bombarding
energies of 2.8, 3.2, and 3.7 MeV were used during the $^{12}(d,d)C^{12}$
measurements. The $^{13}(p,p)C^{13}$ measurements were made at energies
of 1.365, 1.548, 1.734, 1.917, 2.195, and 2.378 MeV. In addition to the
elastic scattering angular distributions, $^{12}(d,d), (d,p_0),(d,p_1),(d,p_2),$
and $(d,p_3)$ yield curves were measured between 2.6 and 4.0 MeV at labor-
atory angles of $30^\circ$ and $90^\circ$.

The DWBA fits to the stripping angular distribution data are better
than the Butler fits. Absolute reduced widths of $0.21 \pm 0.05$ and $1.03 \pm 0.25$
single particle units were found for the transitions to the second and third
excited states of $C^{13}$ respectively. The DWBA fits to the angular corre-

(ii)
lation parameters reproduced the trend of the data much better than the Butler curves but are not in quantitative agreement with the data. The anomalies observed in the elastic deuteron and reaction product yield curves show considerable correlation which indicates that compound nucleus effects are present at the energies used in this work.
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Finally, I wish to express my gratitude to my wife, Rita, who has faithfully typed and corrected the manuscript of this dissertation.

All computations involved in this research except the optical model and DWBA calculations were performed at the Duke University Computing Laboratory which is supported in part by the National Science Foundation.

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David George Gerke
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CHAPTER I

INTRODUCTION

During the last twelve years, a large amount of spectroscopic information about nuclear level structure has been obtained by measuring the angular distributions of nucleons emitted in deuteron stripping reactions. The early plane wave deuteron stripping theories (Butler, 1951; Bhatia et al., 1952), which were widely used to determine the parities, possible spin values, and reduced widths of nuclear levels until recently, have now been superseded by the distorted wave Born approximation (DWBA) theory which takes into account both Coulomb and nuclear distortions of the incident deuteron and outgoing nucleon waves (Horowitz and Messiah, 1953; Tobocman, 1959; Buck and Hodgson, 1961; Austern, 1963). Detailed DWBA calculations require a knowledge of the nuclear optical potentials that generate the distorted waves and the aid of a high-speed electronic computer. The optical model parameters are usually obtained from analysis of deuteron-target nucleus and nucleon-residual nucleus elastic scattering at the appropriate energies. Several investigations of this type concerning direct interaction angular distributions have been reported (Tobocman, 1959; Buck and Hodgson, 1962; Hinds et al., 1962; Andrews et al., 1962; Smith and Ivash, 1962;
Smith and Ivash, 1963; Macefield et al., 1963; Daum, 1963). In general
the DWBA predictions have been found to be in much better agreement
with the absolute magnitude and structural details of measured angular
distributions than the predictions of the plane wave theories. However,
the DWBA predictions concerning deuteron interactions with light nuclei
at low bombarding energies are known to be of questionable accuracy
(Smith and Ivash, 1963). The presence of isolated resonances in com-
ound nucleus processes which are in competition with the direct inter-
actions is one of the main factors responsible for this lack of accuracy.

Satchler (1952; 1953; 1960) and others (Biedenharn et al., 1952;
Huby et al., 1958) have pointed out that the angular distribution of de-
excitation gamma rays measured in coincidence with stripped nucleons
should be valuable both as a spectroscopic tool and as a sensitive means
of investigating the reaction mechanism. At the present, the general
usefulness of these angular correlations as spectroscopic tools is still
an open question (Williamson, 1963) because distortion and/or compound
nucleus effects may attenuate the predicted anisotropies to such an ex-
tent that straightforward interpretation of experimental results is impos-
sible. When distortion and compound nucleus effects are not too large,
information such as level spins, channel spin mixing ratios, and gamma
ray multipolarities may be obtained.

Cox and Williamson (1957) made one of the first experimental
attempts to use the angular correlation between the gamma rays and
nucleons emitted in a stripping reaction as a source of nuclear level
structure information. In order to test the plane wave theory and assign
quantum numbers, they investigated the positive Q value Be⁹, Be¹⁰, and
Mg$^{24}$ (d, p$\gamma$) reactions between 2.5 and 4.0 MeV. The measured correlations were approximately independent of the deuteron energy and approximately symmetric about the axis of momentum transfer in agreement with the predictions of the plane wave theory, but they were greatly attenuated at 2.5 and 3.0 MeV. Distortion and/or compound nucleus effects were also evident in the proton angular distributions. The (d, p$\gamma$) investigations of other workers (Allen et al., 1956; Hill and Blair, 1958; Taylor, 1959) also show a certain measure of agreement with the plane wave theory although the anisotropies measured in the reaction plane are generally found to be smaller than those predicted by the plane wave theory. Cox and Williamson concluded that useful spectroscopic information can be extracted from angular correlations measured at low deuteron energies if regions in which compound nucleus effects are strong are detected and avoided.

This conclusion was utilized by Fletcher et al. (1962) who studied the angular correlation between the 3.68 and 3.85 MeV de-excitation gamma rays and the stripped protons produced in the C$^{12}$(d, p$_2\gamma$)C$^{13}$ and C$^{12}$(d, p$_3\gamma$)C$^{13}$ reactions. These p-$\gamma$ correlations were measured at deuteron energies of 2.8, 3.2, and 3.7 MeV. An energy-level diagram for the C$^{12}$(d, p$\gamma$) reaction appears in Figure 1. All of the information presented in this figure was known from previously published work (Ajzenberg-Selove and Lauritsen, 1959). This experiment was performed with the dual purpose of investigating the multipole mixtures of the de-excitation gamma rays and testing the general DWBA predictions of Huby, Refai, and Satchler (1958). These authors have shown that the DWBA assumptions result in relations between the angular correlation
Figure 1. Energy-Level Diagram.
REACTION ENERGY-LEVEL DIAGRAM
coefficients which are not dependent on the details of the distortion and can therefore be tested without resorting to complicated DWBA calculations. The measured angular correlations were in qualitative agreement with the general DWBA predictions. That is, the reaction plane symmetry axis was shifted away from the residual nucleus recoil direction, the reaction plane anisotropy was attenuated slightly with decreasing deuteron energy, and anisotropy was observed in the azimuthal plane. However, evaluation of the multipole mixing ratios suggested that an appreciable compound nucleus contribution may be present in the $C^{12}(d, p_2, 3')$ reactions at the bombarding energies which were used.

A number of other investigations in which direct interaction angular correlations were compared with the general DWBA predictions of Huby, Refai, and Satchler have been reported (Gorodetsky et al., 1960; Kuehner, Almqvist, and Bromley, 1960; Martin, Quisenberry, and Low, 1960; Read, Calvert, and Schork, 1961; Garg, Gale, and Calvert, 1961a; Green and Parkinson, 1962; Huang and Ritter, 1963), but very few attempts to compare measured correlation coefficients with detailed DWBA calculations have been published (Rook and Hodgson, 1961; Garg, Gale, and Calvert, 1961b; McDaniel et al., 1962). Although the accuracy of the DWBA when applied to $C^{12}$ at energies as low as 2.8 MeV is almost certain to be rather poor, it was decided to carry out detailed DWBA calculations and attempt a quantitative comparison with the existing $C^{12}(d, p_2, 3')$ angular distribution and angular correlation data. This decision was made for two reasons: (1) The necessary elastic scattering measurements could be made with the Duke Van de Graaff because the $C^{12}(d, p_2, 3')$ reactions have negative $Q$ values. (2) In spite of the fact
that the DWBA was not expected to be very accurate in this case, it was felt that, because of the scarcity of investigations of this type, a detailed comparison with the measured correlation coefficients would be of interest.
CHAPTER II

EXPERIMENTAL APPARATUS AND PROCEDURE

A. $^1{}^2{}^C(d,p_2,\gamma)C^{13}$ Angular Correlations

The $^1{}^2{}^C(d,p_2,\gamma)$ angular correlations were investigated at deuteron energies of 2.8, 3.2, and 3.7 MeV by Fletcher (1961). Both correlations were measured at several different proton angles in the reaction plane, and the $p_2-\gamma$ correlation was measured in the azimuthal plane. The apparatus and procedures used in this work have been discussed elsewhere (Fletcher, 1961; Fletcher et al., 1962).

B. $^1{}^2{}^C(d,p_2,\gamma)C^{13}$ Angular Distributions

The $^1{}^2{}^C(d,p_2,\gamma)$ angular distributions were also measured at bombarding energies of 2.8, 3.2, and 3.7 MeV. The deuteron beam from the Duke University four million volt Van de Graaff accelerator was analyzed and maintained homogeneous to better than 0.2% by a beam steering magnet. Natural carbon targets were made by collecting the smoke of burning camphor on a 0.1 mm Ni foil. The energy loss in these targets was roughly 30 keV for 3 MeV deuterons. The protons corresponding to the second and third excited states of $C^{13}$ were selected with a magnetic spectrometer (Meier et al., 1958) which could be rotated about the target position.
These protons were detected by a scintillation counter mounted at the exit of the spectrometer. The effective solid angle of the counter (4π x 10⁻³ steradian) was defined by the spectrometer entrance aperture. The details of the experimental apparatus and procedure employed in this work have been described previously (Gerke, 1961). Absolute differential cross sections were not measured in this study or in any of the other work discussed below. The data were converted from a relative scale to an absolute scale by using the results of other experimenters.

C. C₁²(d, d)C₁² Angular Distributions and the 12.7 cm Scattering Chamber

Angular distributions of deuterons elastically scattered from C₁² were measured between laboratory angles of 20° and 150° in steps of 2° to 10°. Bombarding energies of 2.8, 3.2, and 3.7 MeV were used. The scattering chamber was attached to the exit of the Duke University electrostatic analyzer (Warren et al., 1947). The electrostatic analyzer was calibrated to an accuracy of better than 10 keV by observing the threshold of the Li⁺(p, n)Be⁷ reaction (Marion, 1961). Self-supporting natural carbon foils approximately 10 keV thick for 1.881 MeV protons were made by spreading a thin, even film of a colloidal suspension of graphite in alcohol (Acheson Colloids Company) over a glass slide coated with an invisible layer of detergent. After the carbon film dried, it was floated off on water and picked up on a target ring (Weiss, 1962). All of the natural carbon targets used in the measurements discussed below were also made by this method. Carbon buildup on the foils used here was minimized by placing a small liquid nitrogen trap near the target position and by keeping the deuteron beam off the target when it was not
in use. The target, which could be positioned at any angle with respect to the deuterons beam direction, was used at an angle of $45^\circ$. The angular range between $20^\circ$ and $100^\circ$ was studied with the foil positioned so that the detected deuterons passed through it. The foil was then rotated through $90^\circ$ and the range $80^\circ$ to $150^\circ$ was studied. In this way the "forward angle" and "backward angle" data could be normalized to each other at several overlapping angles.

The cylindrical scattering chamber has an internal diameter of 12.7 cm and is 6.4 cm deep. Two beam collimating apertures 0.3 mm in diameter and 20.3 cm apart were used to produce a target spot with a diameter of about 1.4 mm. The scattering angle and the zero of the angular scale are believed to be accurate to $\pm 0.25^\circ$. The geometry of the chamber was checked by measuring the angular distributions of deuterons elastically scattered from thin gold targets. The scattered deuterons were detected with a surface barrier type semiconductor counter attached to a rotating arm. The solid angle of detection was about $2 \times 10^{-4}$ steradian, and the angular aperture subtended by the counter was $\pm 0.5^\circ$. A similarly collimated semiconductor detector was mounted at a fixed position in the chamber and was used as a monitor counter. The semiconductor detectors were purchased from the Oak Ridge Technical Enterprises Corporation. The charge collected by a Faraday cup attached to the exit port of the chamber was integrated and used as an independent monitor. Negative bias was applied to an insulated ring at the entrance of the Faraday cup to suppress secondary electron emission. After suitable amplification, the pulses from the movable counter were recorded by a Penco 100-channel pulse-height analyzer. Beam currents
of 0.05 μA or less were used so that the dead time corrections indicated by a meter on the analyzer never exceeded 1%. The overall energy resolution of the detection system was about 1.5%.

D. C\textsuperscript{13} (p, p)C\textsuperscript{13} Angular Distributions

Angular distributions of 1.365, 1.548, 1.734, 1.917, 2.195, and 2.378 MeV protons elastically scattered from C\textsuperscript{14} were measured at 5° intervals between laboratory angles of 10° and 165°. These bombarding energies were chosen so that the exit channel optical model parameters necessary for a DWBA analysis of the C\textsuperscript{12}(d, p\textsubscript{2,3}) reactions could be obtained from the data. The appropriate proton energies were determined by means of the formula

\[ E_{p_{2,3}} = \frac{E_d (M_T/M_d + M_d)}{1 + (M_p/M_R)} + Q_{2,3}. \]

In this expression \( E_{p_{2,3}} \) is the kinetic energy of the outgoing proton in the rest frame of the residual nucleus; \( E_d \) is the kinetic energy of the incident deuteron in the laboratory frame; \( M_d, M_p, M_T, \) and \( M_R \) are the masses of the deuteron, proton, target nucleus, and residual nucleus respectively; and \( Q_{2,3} \) is the Q value of the reaction. If \( E_d = 2.8, 3.2, \) and 3.7 MeV, then \( E_{p_2} = 1.548, 1.917, \) and 2.378 MeV and \( E_{p_3} = 1.365, 1.734, \) and 2.195 respectively. The momentum of the protons passing through the beam steering magnet was assumed to be proportional to the frequency given by a proton resonance flux meter. The flux meter was calibrated several times during the course of the experiment by means of the Li\textsuperscript{7}(p, n) threshold. The uncertainty in the bombarding
energies is about \( \pm 10 \) keV.

Since protons elastically scattered from different carbon isotopes could not be resolved, a large \( C^{12}(p, p)C^{12} \) background was present in the data obtained with the enriched targets. Each point of the \( C^{13}(p, p) \) angular distributions was corrected for this background by using the formula \( N(E, \theta) = N^1(E, \theta) - fN^2(E, \theta) \), where \( N(E, \theta) \) is the number of protons of incident energy \( E \) that would be scattered at an angle \( \theta \) by a pure \( C^{13} \) target, \( N^1(E, \theta) \) is the number of protons scattered by the \( C^{13} \) enriched target, \( f \) is the ratio of the number of \( C^{12} \) nuclei/cm\(^2\) in the enriched target to the number of \( C^{12} \) nuclei/cm\(^2\) in a natural carbon target, and \( N^2(E, \theta) \) is the number of protons scattered by the natural target. No correction was made for the 1.1\% \( C^{13} \) content of the natural carbon target. The quantity \( f \) was determined by comparing the \( C^{12}(d, p) \) \( C^{13} \) yields from the enriched and natural targets under identical experimental conditions. The comparison was made at a bombarding energy of 2.2 MeV and a laboratory angle of 75° because the ground state proton yields of McEllistrem et al. (1956) are almost flat in this region. The \( C^{13} \) content of the enriched targets was calculated to be 42\% by using the measured value of \( f \) and published absolute cross sections. This is in good agreement with the 41\% enrichment quoted by the manufacturer (Isomet Corporation). The energy loss of the incident protons in the enriched and natural targets was obtained by first measuring the threshold energy of the \( Li^7(p, n) \) reaction when the proton beam passed through one of the targets and then measuring the threshold energy with the target removed. The energy loss in the targets was taken to be the difference between these two threshold energies. This process gave values of 38
keV for the $C^{13}$ target and 11 keV for the $C^{12}$ target.

E. $C^{12}(d,d)$, $(d,p_0)$, $(d,p_1)$, $(d,p_2)$, and $(d,p_3)$ Yield Curves

The $C^{12} + d$ yield curves were measured over a range of bombarding energies from 2.6 to 4.0 MeV at laboratory angles of 30° and 90°. The data were taken at intervals of 20 keV except in the region between 2.7 and 2.8 MeV where 10 keV steps were used. The energy scale for this work was established to an accuracy of better than $\pm$ 10 keV by using a precision digital voltmeter in conjunction with the generating voltmeter of the Van de Graaff accelerator (Hollandsworth et al., 1963). This system was calibrated at the beginning and end of the yield curve study by measuring the threshold energy of the Li$^7(p,n)$ reaction. The energy loss method described above gave a thickness of 17 keV for this target. All ten of the yield curves studied were measured simultaneously.

The relative yields were determined by integrating the beam which passed through the target and was collected by a Faraday cup. The Faraday cup is estimated to intercept at least 99% of the transmitted beam. The basic current integrator has been described previously (Helmer and Hemmendinger, 1957). The modified model now in use is estimated to be accurate and stable to within 0.1% to a minimum current of 0.1 μA. Beam currents of 0.2 to 0.5 μA were used during the yield curve measurements.
F. The 30.5 cm Scattering Chamber

The scattering chamber described here was used during the $^13\text{C}(p,p)$ and $^12\text{C}(p,p)$ angular distribution and the $^12\text{C} + d$ yield curve measurements. It was designed after experience in the use of semiconductor detectors and a knowledge of the features desirable in a target chamber built for their use had been gained with the 12.7 cm chamber described above. Since all of these features were present in a scattering chamber built at Florida State University (Feldl et al., 1963), the basic design of this chamber was adopted. The general features of our chamber are shown in schematic form in Figure 2. Some of the details are shown photographically in Figures 3 through 7. The chamber has an internal diameter and depth of 30.5 cm and 7.6 cm respectively. Aluminum was used as much as possible in the construction of the chamber.

Two 30.2 cm diameter scattering tables support the semiconductor detectors and their collimating systems. The lower and upper scattering tables are shown in Figures 3 and 4 respectively. Each table is mounted on an 11.4 cm diameter hub which passes through an O-ring seal and is attached to a handle outside the chamber. In this manner the advantages of a large scattering table are achieved without the disadvantage of having to rotate a large area against a vacuum seal. The rotating assembly rides against and is held in position by Teflon bearings (1 in Figure 2). The bearings between the upper scattering table and the top of the chamber may also be seen in Figure 4. The bearing pressure is controlled by set screws attached to the exterior handles. One degree
Figure 2. Cross Section Drawing of Scattering Chamber.

A. Teflon insulators.
B. Tantalum-lined Faraday cup.
C. Electron suppressor ring.
D. Semiconductor detector mount.
E. Detector collimating system.
F. Lower scattering table.
G. Upper scattering table.
H. Four position target holder.
I. Teflon bearings.
J. Tantalum beam-scraping slits.
K. Tantalum beam-defining slits.
L. Removable, tantalum-lined beam collimating assembly.
M. Beam alignment quartz.
N. Ball joint for chamber alignment.
O. Handles for rotating the scattering tables.
P. Vacuum-tight coaxial cable connector.
Q. Target selector and angle indicator.
R. Target-rod clamp.
Figure 3. Interior of Scattering Chamber.
Figure 4. Top Lid Assembly.
Figure 5. Side View of Chamber Showing Angular Scales and Vernier Indexes.
Figure 6. Exploded View of Counter Mount.
Figure 7. Exterior View of Scattering Chamber and Associated Equipment.
graduations on the rims of the scattering tables together with the vernier
indexes secured to the top and bottom of the chamber enable the operator
to read the scattering angle to ± 0.1°. The readings are made through
two glass windows attached to the side of the chamber as shown in
Figure 5.

A detector mount can be placed in any one of six equally spaced
slots cut in each scattering table. Metric scales are placed along each
slot so that detectors may be set at reproducible distances from the
center of the chamber. Oak Ridge Technical Enterprises Corporation
(ORTEC) series SB semiconductor detectors were used during all of
the measurements made with this chamber. One of these detectors is
shown in Figure 6 along with its mount and collimator. ORTEC C-13
O-ring sealed connectors were used to provide electrical feed-through
to the detectors. These connectors are mounted on the bases of both of
the rotating hubs (P in Figure 2). This mounting position allows the
coaxial cables leading to the detectors to rotate with them and prevents
mechanical interference from the cables. These cables can be seen
in Figures 3 and 4. The vacuum-tight connectors can be seen in Figure 7.

The target mount shown in Figures 2 and 3 holds a maximum of
four targets. Any one of these targets can be selected by loosening the
target-rod clamp (R in Figure 2), removing the target-positioning pin
(Q in Figure 2), and moving the target rod up or down until the hole in
it which corresponds to the desired target is aligned with the hole for
the positioning pin. The targets can be set at any angle with respect to
the incident beam direction by rotating the target rod. The target angle
is read from a scale rigidly attached to the top of the chamber (Q in
Figure 2). The target mount is electrically isolated from the rest of the chamber so that it can be used for current integration when targets too thick for the beam to penetrate are used. The target mount can be removed from the chamber through an opening in the base of the lower rotating hub. This opening is uncovered by removing the aluminum piece which supports the bottom target-rod insulator. The target rod, target-rod clamp, target-angle scale, target-positioning pin, and top and bottom target-rod insulators can all be seen in Figure 7.

Before entering the reaction volume of the chamber, the incident beam passes through two tantalum slits 0.8 mm in diameter and 35.6 cm apart which defines its direction, diameter, and divergence. The beam which enters the chamber has an angular spread of approximately ± 0.1° and a diameter of about 1.2 mm at the target position. The beam-defining slits and the slightly larger beam-scraping slits (K and J in Figure 2) are mounted in a tantalum-lined stainless steel tube (L in Figure 2) which is held in the middle of the entrance tube by two circular supports. The whole beam-collimating assembly can be removed through the chamber. This allows the slits to be changed without having to disturb the alignment of the chamber.

A thin quartz plate with a hole in its center which is slightly larger than the diameter of the first beam-defining slit is mounted immediately in front of this slit (M in Figure 2). The hole in the quartz is at the center of rotation of a ball joint (N in Figure 2). One side of the ball joint is attached to the chamber. The other side is attached to a section of the beam pipe containing a glass window through which the quartz can be viewed. This viewing port can be seen in Figure 7 just to the left of
the ball joint. The chamber is aligned by centering a well-focused beam on the quartz and then pivoting the chamber about the center of the quartz until its axis coincides with that of the beam. The optimum position is determined by replacing the Faraday cup by a quartz plate and centering the beam passing through the chamber on this plate. The position of the chamber with respect to the center of rotation of the ball joint is changed in the vertical plane by using the three screw jacks on which the chamber is mounted. Motion in the horizontal plane is accomplished by rotating the plate to which the screw jacks are attached about a pivot fixed to the table supporting the chamber. The axis of rotation about this pivot coincides with the vertical axis of rotation of the ball joint. The screw jacks, the plate to which they are attached, and the supporting table are shown in Figure 7.

The tantalum-lined Faraday cup (B in Figure 2) is 4.4 cm in diameter and 15.2 cm long. It is supported by a Teflon disk which also provides electrical isolation from the rest of the chamber. Secondary electron emission is suppressed by an insulated ring (C in Figure 2) held at -1 kv with respect to the Faraday cup. The electronic integrator used to measure the charge collected by the Faraday cup was described in the preceding section.

A tantalum flap pivoted about an axis along its lower edge is mounted inside and near the back of the Faraday cup. A small piece of soft iron is attached to the upper front edge of this flap so that it may be raised or lowered from outside the chamber with a permanent magnet. The surface of the flap which faces the beam is coated with a thin layer of lithium. With this arrangement the calibration of the beam energy can be checked
at any time during the course of an experiment without breaking vacuum by raising the flap and measuring the energy at which the Li\(^7\)(\(p, n\)) threshold occurs. The tantalum flap and its support are not shown in Figure 2.

In order to reduce carbon buildup on the thin, self-supporting targets usually used in conjunction with the scattering chamber, a water-cooled 15 liter/second VacIon pump (Varian Associates) is used to maintain a pressure of about 4\(\times\)10\(^{-6}\) Torr in the chamber. The chamber volume is effectively isolated from the vacuum system of the Van de Graaff accelerator by placing a large, efficient liquid nitrogen trap in the beam tube near the ball joint and by limiting the connection between the two vacuum systems to the 0.8 mm aperture of the first beam-defining slit. Carbon buildup was checked during the C\(^{13}\)(\(p, p\)) and C\(^{12}\)(\(p, p\)) measurements by comparing the counting rate at a given scattering angle at the beginning and end of a run. Enough counts were accumulated in each case to make the statistical error less than 0.5%. No significant carbon buildup could be detected by this method which indicates that the VacIon pump can maintain a clean, hard vacuum in the chamber.

The geometry of the chamber was tested by measuring Au(\(p, p\))Au angular distributions from 5\(^\circ\) to 90\(^\circ\) on both sides of the nominal zero of the angular scale. The bombarding energy was 2.4 MeV. The scattered protons were detected with two semiconductor counters mounted on the upper scattering table. These measurements fixed the true zero of the angular scale of this table, showed that the scale could be read with an accuracy of \(\pm\) 0.1\(^\circ\), and verified the intersection of the beam axis with the axis of rotation of the detectors. With counting statistics on
the order of 2%, a maximum deviation from the Rutherford cross section of approximately 3% was observed at a scattering angle of 10°. The measured Au(p, p) cross section divided by cosec^4(θ/2) is shown in Figure 8. Curve a shows this ratio when the values indicated by the angular scale on the rim of the upper scattering table are assumed to represent the true scattering angles. Curve b shows the ratio when the values read from the angular scale are adjusted for the discrepancy between the nominal and true zeros of the scale. The true zero was found to be at -0.3° with respect to the nominal zero. The cosec^4(θ/2) dependence of the Rutherford cross section made it possible to determine the true zero with an accuracy of better than ±0.1°. Curves a and b are both normalized to unity at 30°. Curves c and d show the effects of assuming that the corrected angles used in b are in error by ±0.1°. The discrepancy, if any, between the true and nominal zeros of the angular scale on the lower scattering table was not measured. Since this table was only used to support a monitor counter, the nominal scattering angle was sufficiently accurate.

The performance of the scattering chamber and associated apparatus was also evaluated by remeasuring one of the C^{12}(d, p) angular distributions which had been measured with the magnetic spectrometer. The two sets of data coincided with experimental accuracy over the whole range from 10° to 150°.

The protons elastically scattered from C^{12} and C^{13} were detected with two semiconductor counters mounted 60° apart on the upper scattering table. One counter was used at scattering angles between 10° and 80° and the other between 70° and 165°. The former subtended an angular
Figure 8. $Au(p, p)Au$ Angular Distribution Measured to Check the Geometry of the Scattering Chamber.
Au(p,p)Au
$E_p = 2.4$ MeV
a Nominal Angular Scale
b Corrected Angular Scale
c Corrected Scale Assuming An Error Of +0.1° At Each Point
d Corrected Scale Assuming An Error Of -0.1° At Each Point

$\sigma_p / \sigma_R$

$\theta_{pp} (lab)$
aperture of $\pm 0.1^\circ$ while the latter subtended to $\pm 0.7^\circ$. The solid angles of detection were $9 \times 10^{-6}$ and $4 \times 10^{-4}$ steradian respectively. The monitor counter subtended an angular aperture of $\pm 0.7^\circ$. The charge collected by the Faraday cup was integrated to provide an independent monitor. The detector pulses were counted by scalers after passing through low-noise, charge-sensitive preamplifiers, non-overloading amplifiers, and single-channel analyzers. The scaler counting rate was kept at a reasonable level by varying the beam current from $0.01\mu A$ at small scattering angles to $0.2\mu A$ at large angles. The amplified pulses were also stored in a TMC 400-channel pulse-height analyzer so that peak positions and background contributions could be monitored.

The $^1C_{12} + d$ reaction products were simultaneously detected at $30^\circ$ and $90^\circ$ with two counters mounted on the upper scattering table. The pulses from the two counters were suitably amplified and stored in separate 200-channel blocks of the 400-channel analyzer memory. The counting rates in the two blocks were made comparable by adjusting the angular apertures subtended by the $30^\circ$ and $90^\circ$ counters. Values of $\pm 0.5^\circ$ and $\pm 0.7^\circ$ respectively were used. In order to obtain reasonable counting rates for the second and third excited state proton groups, a high counting rate for the elastically scattered deuterons had to be accepted. Because of this high counting rate, dead time corrections as large as 30% had to be made. The dead time corrections were determined by comparing the live time recorded by the analyzer with the real time recorded by a clock which was automatically turned on and off at the beginning and end of each data point. The pulses corresponding to the ground state protons were recorded by scalers so that dead time corrections are
negligible for this group. The depletion depths of the counters were adjusted so that elastically scattered deuterons and all proton groups except the ground state group \( Q_o = 2.719 \text{ MeV} \) produced pulses proportional to their energy. The peaks in the pulse-height spectra which corresponded to the different particles detected were positively identified by their kinematic variation with deuteron bombarding energy and scattering angle. A typical 30° pulse-height spectrum is shown in Figure 9. The prominent proton group due to the \( \text{H(d, p)D} \) reaction completely obscures the proton groups of interest at some bombarding energies and scattering angles. This unfortunate situation has been observed before in the laboratory (Gerke, 1961) and elsewhere (Sellschop, 1959; Gorodetsky et al., 1961). The full width at half-height of the peaks shown in Figure 9 is about 30 keV. The detection system was responsible for about 25 keV of this width. The over-all energy resolution of the detection system was about 1%. 
Figure 9. $^{12}C + d$ Pulse-Height Spectrum.
CHAPTER III

EXPERIMENTAL RESULTS AND ERRORS

A. Yield Curves

The $^{12}C + d$ reaction product yield curves are shown in Figures 10 through 14. Resonance structure similar to that seen in these yields has been reported previously (Bonner et al., 1956; McEllistrem et al., 1956; McEllistrem, 1958; Jeronymo et al., 1963; Ohlsen and Shamu, 1963). These resonances correspond to levels in $^{14}N$ at excitation energies between 12.5 and 13.7 MeV (Ajsenberg-Selove and Lauritsen, 1959).

The $^{12}C + d$ yield curves and angular distributions were converted from a relative scale to an absolute scale by normalizing to the absolute $^{12}C(d,d)$ cross section data of McEllistrem et al. (1956). The normalization was made at deuteron energies of 2.8 MeV ($\theta_{C.M.} = 95.9^\circ$) and 3.2 MeV ($\theta_{C.M.} = 90^\circ$). An uncertainty of 15% has been assigned to the process of reading the absolute cross sections from the published graphs. Sources of uncertainties which affected the $^{12}C + d$ yield curve measurements are scattering angle inaccuracies, finite detection apertures, bombarding energy fluctuations, changes in target thickness, counting statistics, inaccuracies in dead time corrections, incorrect current integration, and background subtraction. The rms sum of the uncertainties

(39)
Figure 10. $^{12}\text{C}(d,d)^{12}\text{C}$ Yield Curves at Laboratory Angles of $30^\circ$ and $90^\circ$. 
\[ C^{12} (d,d) C^{12} \]

- 30°
- 90°

\[ \frac{d\sigma}{d\Omega} \text{ (Lab) } \text{ mb/sr} \]

\[ E_d \text{ (MeV)} \]

\[ (\times 10) \]
Figure 11. $^{12}\text{C}(d,p)^{13}\text{C}$ Yield Curves at Laboratory Angles of $30^\circ$ and $90^\circ$. 
\( \text{C}^{12}(d, p_0) \text{C}^{13} \)

- 30°
- 90°

\[ \frac{d\sigma}{d\Omega} \text{ (Lab)} \text{ mb/sr} \]

\[ E_d \text{ (MeV)} \]

2.6 2.8 3.0 3.2 3.4 3.6 3.8 4.0
Figure 12. $C^{12}(d, p_1)C^{13}$ Yield Curves at Laboratory Angles of $30^\circ$ and $90^\circ$. 
$^{12}\text{C}(d, p_1)^{13}\text{C}$

- $30^\circ$
- $90^\circ$

$d\sigma/d\Omega$ (Lab) mb/sr vs. $E_d$ (MeV)
Figure 13. $^{12}\text{C}(d, p_2)^{13}\text{C}$ Yield Curves at Laboratory Angles of 30° and 90°.
\[
\text{C}^{12}(d,p_2)\text{C}^{13}
\]

\[
\begin{align*}
d\sigma/d\Omega \text{ (Lab) mb/sr} & \\
E_d \text{ (MeV)} & \\
2.6 & 2.8 & 3.0 & 3.2 & 3.4 & 3.6 & 3.8 & 4.0
\end{align*}
\]
Figure 14. $^{12}\text{C}(d,p)^{13}\text{C}$ Yield Curves at Laboratory Angles of $30^\circ$ and $90^\circ$. 
due to all of these sources except the last is less than 6%. All of the yield curves are subject to background subtraction uncertainties of 10 to 15% with the exception of certain parts of the 30° p₂ and p₃ curves. The contaminant peak corresponding to protons from the H(d, p)D reaction completely obscured the p₂ yield between 3.2 and 3.6 MeV and was responsible for background subtraction uncertainties as large as 20% between 3.1 and 3.2 MeV and between 3.6 and 3.7 MeV. The points on the 30° p₂ yield curve between 3.2 and 3.6 MeV were obtained with the Duke magnetic spectrometer and are subject to total rms uncertainties on the order of 10%. The p₃ yield is subject to background uncertainties as large as 20% between 3.7 and 3.8 MeV and was completely obscured between 3.8 and 4.0 MeV because of interference from the contaminant proton peak. The total rms uncertainty at any point of the yield curves is obtained by combining the background subtraction uncertainty with the rms sum (16.2%) of the uncertainties due to all other sources affecting relative yields (6%) and the absolute normalization uncertainty (15%).

B. Angular Distributions

1. $^{12}\text{C}(d, p, 3)C^{13}$. The $^{12}\text{C}(d, p, 3)$ angular distribution data are shown in the upper third of Figures 16, 17, and 18 along with plane wave and DWBA fits which are discussed in the next chapter. Sources of uncertainties which could affect any of the relative differential cross section $\sigma$ scattering angle measurements are scattering angle inaccuracies, finite detection apertures, bombarding energy fluctuations, changes in target thickness, counting statistics for both the movable
and monitor counters, inaccuracies in dead time corrections, and background subtraction. The rms sum of all of these uncertainties is on the order of 10% or less for the \((d, p_{2,3})\) data with the exception of a few points. Between laboratory angles of 35° and 45° at 2.8 MeV, 25° and 35° at 3.2 MeV, and 15° and 25° at 3.7 MeV the proton peak corresponding to \(H(d,p)D\) either partly or completely obscured one or both of the proton peaks corresponding to the groups which were being studied. The total relative uncertainty is as large as 15% in these regions. Fortunately, the variation of the energy of the contaminant protons with scattering angle was more rapid than that of the protons of interest. No more than two successive points were missed on any one angular distribution. The total relative uncertainties at several scattering angles are shown in Figures 16, 17, and 18. The total rms uncertainty is less than 17% at most scattering angles. The maximum value of this uncertainty is about 21% (15% due to relative uncertainties plus a 15% uncertainty due to absolute normalization).

2. \(C^{12}_{d,d}C^{12}\). The \(C^{12}_{d,d}\) angular distribution data are shown in the middle of Figures 16, 17, and 18. The optical model fits to these data are discussed in the following chapter. The rms sum of all uncertainties affecting these data, excluding those due to scattering angle inaccuracies and background subtraction, is less than 5%. An error of 0.25° in setting the scattering angle results in an uncertainty which varies from about 4.4% at 23° to about 1% at 90°. Background subtraction uncertainties are about 2% except at laboratory angles greater than 100° where they are as large as 30%. These large uncertainties are due
to the fact that proton peaks corresponding to the \( ^{12}\text{C}(d, p) \) reaction overlapped the peaks corresponding to the elastically scattered deuterons in this angular region. This overlap is also responsible for the gap between laboratory angles of 105° and 125° in the 3.7 MeV angular distribution. The \( \text{r.m.s.} \) sum of the relative uncertainties is less than the size of the data points in Figures 16, 17, and 18 except where shown. The 15% absolute normalization uncertainty must be combined with this sum to obtain the total \( \text{r.m.s.} \) uncertainty at each scattering angle.

3. \( ^{13}\text{C}(p, p)^{13}\text{C} \). The \( ^{13}\text{C}(p, p) \) angular distribution data are shown in the lower third of Figures 16, 17, and 18. The optical model fits to these data are discussed in the next chapter. As discussed in the preceding chapter, each point of the \( ^{13}\text{C}(p, p) \) angular distributions was obtained by subtracting the number of protons elastically scattered by the \( ^{12}\text{C} \) component of the enriched target (41% \( ^{13}\text{C} \)) from the total number of protons elastically scattered by this target. This means that the uncertainties in the \( ^{13}\text{C} \) data are a combination of those which affect the measurements made with the enriched and natural targets as well as those which affect the \( ^{12}\text{C}(p, p) \) subtraction. The \( \text{r.m.s.} \) sum of all of these uncertainties, excluding scattering angle inaccuracies, is less than 3.5% for the measurements made at bombarding energies of 1.365, 1.548, 1.917, 2.195, and 2.378 MeV. The measurements made at 1.734 MeV could not be used for reasons discussed below. An error of 0.1° in setting the scattering angle results in an uncertainty which varies from about 4% at 10° to about 0.4% at 90°. The maximum uncertainty from this source in the \( ^{13}\text{C} \) data is therefore about 7.8%. The \( \text{r.m.s.} \) sum of
the relative uncertainties is slightly larger than the size of the first two data points shown in Figures 16, 17, and 18 but is smaller than the remaining points so no error bars are shown. The 1.365 and 1.548 MeV angular distributions were converted into absolute units by using the data of Milne (1954). At 1.917, 2.195, and 2.378 MeV the conversion was made by using the work of Zipoy, Freier, and Famularo (1957).

At all energies except 1.548 MeV the absolute normalization uncertainty is estimated to be less than 6%. At 1.548 MeV the conversion uncertainty could be as large as 30% because there is a resonance in the $^\text{13}\text{C}(p, p)$ yield near this energy. The total uncertainty is less than 10% for most of the data. The maximum value of this uncertainty is about 31% (at 1.548 MeV and $10^6$).

The $^\text{13}\text{C}(p, p)$ angular distribution could not be extracted from the 1.734 MeV measurements because the $^\text{12}\text{C}(p, p)$ background subtraction process produced negative $^\text{13}\text{C}$ differential cross sections at some scattering angles. This is not very surprising because there is a large, sharp anomaly in the $^\text{12}\text{C}(p, p)$ yield near 1.734 MeV (Jackson et al., 1953; Jackson and Galonsky, 1953; Reich, Phillips, and Russell, 1956). A shift of $\pm 10$ keV in the nominal bombarding energy between the measurements made with the enriched and natural targets could easily make accurate $^\text{12}\text{C}(p, p)$ background subtraction impossible.

The $^\text{12}\text{C}(p, p)$ angular distribution data are shown in Figure 15.

The dotted lines were added for clarity and do not represent fits of any sort. The behavior of the 1.734 MeV angular distribution is due to the presence of resonances at 1.698 MeV ($T_{\text{C.M.}} = 55$ keV) and 1.748 MeV ($T_{\text{C.M.}} = 61$ keV) (Ajzenberg-Selove and Lauritsen, 1959). The rms sum
Figure 15. Center-of-Mass Angular Distributions of Protons Elastically Scattered from $^{12}$C at Various Laboratory Bombarding Energies.
$C^{12}(p, p)C^{12}$

$d\sigma / d\Omega$ (b/sr)

$E_p = 1.365$ MeV

$E_p = 1.548$ MeV

$E_p = 1.734$ MeV

$E_p = 1.917$ MeV

$E_p = 2.195$ MeV

$E_p = 2.378$ MeV

$\theta_{C.M.}$ (deg)
of all relative uncertainties, excluding scattering angle inaccuracies, is about 2%. An error of 0.1° in setting the scattering angle results in an uncertainty which varies from about 4% at 10° to about 0.4% at 90°. The rms sum of the relative uncertainties is 5% or less. Since this is smaller than the size of the data points in Figure 15, no error bars are shown. The 1.365 and 1.548 MeV angular distributions were converted into absolute units by using the data of Jackson et al. (1953). The 1.734, 1.917, 2.195, and 2.378 MeV data were converted by using the work of Reich et al. (1956). At all energies except 1.734 MeV the conversion uncertainty is estimated to be 5%. The conversion uncertainty is about 10% at 1.734 MeV because of the resonances near this energy. The total rms uncertainty is less than 8% at most scattering angles. The maximum value of this uncertainty is about 12% (at 1.734 MeV and 10°).
CHAPTER IV

DATA ANALYSIS

A. Procedure

The application of the DWBA formalism to deuteron stripping reactions is outlined in the appendix. It is shown there that the differential cross section for a (d, p) reaction can be written

\[
\frac{d\sigma}{d\Omega} = (T_{\ell}^{1/2}) (T_{\ell}^{1/2}) \left[ \frac{(2J_f + 1)(2J_i + 1)}{(2J_f + 1)(2J_i + 1)} \right] \sum_{\ell s j} S(\ell s j) \sigma_{\ell s j}(\theta)
\]

In this relation the Clebsch-Gordan coefficient represents the coupling of the isobaric spin of the initial nucleus to that of the captured neutron to give the isobaric spin of the final nucleus (Macfarlane and French, 1960; Daum, 1963); \(J_i\) is the ground state spin of the target nucleus; \(J_f\) is the spin of the state of the final nucleus excited in the stripping process; \(S(\ell s j)\) is the spectroscopic factor (analogous to the single particle reduced width of the (d, p) transition to the excited state of the final nucleus); \(J = \ell + s\) is the angular momentum transferred to the target nucleus by the captured neutron (\(\ell\) is the orbital angular momentum with which the neutron is captured and \(s\) is its spin); and \(\sigma_{\ell s j}(\theta)\) is a function which is proportional to the DWBA transition amplitude for a particular angular momentum transfer \(\ell s j\).

The function \(\sigma_{\ell s j}(\theta)\) was calculated by using the DWBA program

(57)
SALLY (Bassel, Drisko, and Satchler, 1962) and an IBM-7090 computer. This work was done at the Oak Ridge National Laboratory. The DWBA program assumes local optical potentials for the entrance and exit channels, uses the zero-range approximation, and neglects the effects of spin-orbit coupling on the elastic scattering in the entrance and exit channels. In addition to $\varphi_{lsj}(\theta)$, SALLY computes the $(d, p \gamma)$ angular correlation parameters defined by Huby et al. (1958) and differential cross sections for the incident and outgoing particles elastically scattered by the optical potentials which describe the entrance and exit channels.

The optical potentials appropriate to the $C^{12}_{12}(d, p_2, 3)$ reactions were determined by fitting the $C^{12}_{12}(d, d)$ and $C^{13}_{13}(p, p)$ data with an automatic parameter-search program which is briefly discussed in the next paragraph. The potential parameters which gave the best fits to the elastic scattering data were read into SALLY, and the corresponding stripping angular distributions and $p-\gamma$ correlation parameters were calculated. The radial integrations performed by the DWBA program were cut off at radii of 0, 1.87, 2.88, 3.89, and 5.00 fermi so that the effect of excluding contributions from the different regions of the nuclear interior could be studied. In an attempt to find better fits to the stripping data, the potential parameters which gave the best fits to the elastic scattering data were modified, and the new parameters were used in SALLY. Since the program computes the entrance and exit channel elastic scattering angular distributions corresponding to the input potentials, the effect of the modified parameters could be determined simultaneously for the $(d, p)$, $(d, d)$, and $(p, p)$ angular distributions.

The automatic parameter-search program used to analyze the
elastic scattering data was written by R. M. Drisko (unpublished). These calculations were made with one of the IBM-7090 computers at Oak Ridge. This program adjusts the optical potential parameters in such a way that the difference between the experimental and theoretical differential cross sections is minimized. A quantitative measure of this difference is obtained by evaluating

$$\chi^2(\lambda) = \sum_{i=1}^{N} \left( \frac{[\sigma_{\text{theor}}(\theta, \lambda) - \sigma_{\text{expt}}(\theta)]}{\Delta \sigma_{\text{expt}}(\theta)} \right)^2.$$ 

Here, $\lambda$ represents the set of optical model parameters which are automatically adjusted; $N$ is the number of experimental points; and $\Delta \sigma_{\text{expt}}(\theta)$ is the estimated experimental error. The search proceeds by examining $\chi^2$ along a "tracking path" in parameter space and not by examining the behavior of $\chi^2$ throughout a predetermined volume of parameter space. The parameters are normally varied until $\chi^2$ is less than or equal to a pre-set number or until a pre-set number of attempts to minimize $\chi^2$ have been made. A successful search results in a set of parameters for which $\chi^2$ has a local minimum. Several different local minima which give almost equally good fits to the elastic scattering data can usually be found (Perey and Perey, 1963). Unfortunately, the parameter sets which correspond to the various local minima give somewhat different results when used in DWBA calculations unless contributions from the nuclear interior are ignored (Drisko, Satchler, and Bassel, 1963).

The most general optical potential which can be used in conjunction with the parameter-search program is of the form
\[ U(r) = V_c(r) - V_d(r, r', a) - i [ W - 4a' W_d (d/dr) ] f(r, r', a') \]
\[ + \hat{\sigma} \cdot \hat{T} (\hbar/m_e c)^2 \left( V_{so} / r \right) (d/dr) f(r, r', a) \]

where \( r \) is the separation of the particles; \( V_c(r) \) is the Coulomb potential between a point-charge particle and a uniformly charged sphere of radius \( R_c = r_c A^{1/3} \) (\( A \) is the atomic mass of the target nucleus); \( V \) and \( W \) are the real and imaginary parts of the central potential; \( W_d \) is the surface part of the imaginary potential (the factor \( 4a' \) is put in so that the surface form factor \( 4a'df/dr \) has a maximum value of unity); \( f(r, r', a) \)
and \( f(r, r', a') \) are the form factors which contain the radial dependence of the potentials; and the last term is a real Thomas type spin-orbit potential. The Coulomb potential is given by
\[ (Z_i Z_T e^2 / 2R_c) \left[ 3 - (r^2 / R_c^2) \right] \quad \text{for} \quad r < R_c, \]
and by
\[ Z_i Z_T e^2 / r^2 \quad \text{for} \quad r > R_c, \]

where \( Z_i \) and \( Z_T \) are the atomic numbers of the incident particle and target nucleus respectively. The functions \( f(r, r', a) \) and \( f(r, r', a') \) are
the usual Saxon-Woods form factors,
\[ f(r, r', a) = \left[ 1 + \exp \left[ (r - R) / a \right] \right]^{-1}, \]
and
\[ f(r, r', a') = \left[ 1 + \exp \left[ (r - R') / a' \right] \right]^{-1}. \]

Here, \( R = r_o A^{1/3} \) and \( R' = r'_o A^{1/3} \) are the radii of the real and imaginary potential wells, and \( a \) and \( a' \) are the surface diffuseness parameters of the real and imaginary wells. Both \( V \) and \( W \) are normally positive numbers. Positive \( V \) gives a refractive index less than unity so that the incoming particle speeds up inside the nucleus while positive \( W \) provides
for the absorption of particles from the beam. The surface part of the imaginary potential ($W_D$) and the spin-orbit potential are also normally positive numbers. These terms are localized near the nuclear surface by multiplying them by a form factor which is the derivative of the Saxon-Woods form. The quantity $(1/r)(h/m_\pi c)^2$ is a dimensional factor which makes it possible to express $V_{so}$ in MeV. The pion mass is used instead of, say, the proton mass on theoretical grounds and serves the additional purpose of making the magnitude of $V_{so}$ reasonable. The potential which gave the best fits to all of the $^{12}_C(d,d)$ data was obtained from the general potential by setting $W_D = V_{so} = 0$. The $1.365, 1.548,$ and $2.378$ MeV $^{13}_C(p,p)$ data could be fit best by using $W = 0$, $W_D \neq 0$, and $V_{so} \neq 0$ in the general potential. At $1.917$ and $2.195$ MeV the best $^{13}_C(p,p)$ fits were obtained with a purely real potential, that is, $V \neq 0$, $V_{so} \neq 0$, and $W = W_D = 0$.

The potentials used to generate the entrance channel wave functions for the DWBA calculations were of the same form as those which gave the best fits to the deuteron elastic scattering data. The potentials used to generate the exit channel wave functions were different from those which gave the best fits to the proton elastic scattering data because the spin-orbit term had to be dropped. The DWBA programs available at Oak Ridge which include spin-orbit coupling do not calculate $p-\gamma$ angular correlation coefficients.
B. Results

1. Angular Distributions. The DWBA fits to the $C^{12}(d, p_{2, 3})$ data are shown in Figures 16, 17, and 18 along with the corresponding optical model fits to the $C^{12}(d, d)$ and $C^{13}(p, p)$ data. The elastic scattering fits labeled $a$ are the best fits obtained with the parameter-search program. The only difference between the potentials which gave the best fits to the $C^{13}(p, p)$ data and those which gave the fits labeled $a'$ is that $V_{so} = 0$ for the latter. Since there was no $C^{13}(p, p)$ data at $E_p = 1.734$ MeV, an appropriate potential was obtained from the best fit potentials at 1.548 and 1.917 MeV by interpolation. The DWBA fits labeled $a$ were produced by using the potentials which gave the deuteron and proton elastic scattering fits labeled $a$ and $a'$ respectively. The DWBA fits labeled $b$ were obtained by modifying the best elastic scattering potentials. These curves represent the best DWBA fits that were found. The elastic scattering fits corresponding to the modified potentials (labeled $b$) were calculated by SALLY. The plane wave stripping fits labeled $c$ (in the case of $p_3$ at 3.2 MeV) were obtained at Duke (Gerke, 1961). Unlike the elastic scattering fits, the stripping fits were arbitrarily normalized to the data.

The potential parameters which were used in the calculation of the DWBA and modified elastic scattering fits are listed in tables 1 and 2. The cut-off radii which were used for each DWBA fit are also listed in these tables. A cut-off radius of 4.49 femis was used for all of the plane wave fits. As mentioned before, $V_{so} \neq 0$ for the best proton elastic scattering fits. The strengths of the real spin-orbit potentials corre-
Figure 16. DWBA and Plane Wave Fits to the 2.8 MeV $^1{C}^{12}(d, p_{2,3})^1{C}^{13}$ Angular Distribution Data and Optical Model Fits to the Corresponding $^1{C}^{12}(d, d)^1{C}^{12}$ and $^1{C}^{13}(p, p)^1{C}^{13}$ Angular Distribution Data.
\begin{align*}
&\text{\textbf{C}^{12}(d, p) \text{C}^{13}} \\
&E_{d} = 2.8 \text{ MeV}, Q = -0.96 \text{ MeV}, J = 1 \\
&\text{a} \text{ Cut-Off DWBA, Best Elastic Parameters} \\
&\text{b} \text{ Cut-Off DWBA, Best Fit} \\
&\text{c} \text{ Butler Curve} \\
&10^2 \text{ to } 10^8 \\
&\text{d} \frac{d\sigma}{d\Omega} \text{ (mb/sr)} \\
&\text{\textbf{C}^{12}(d, d) \text{C}^{12}} \\
&E_{d} = 2.8 \text{ MeV} \\
&\text{a} \text{ Best Fit} \\
&\text{b} \text{ Modified Fit} \\
&\text{\textbf{C}^{13}(p, p) \text{C}^{13}} \\
&E_{p} = 1.548 \text{ MeV} \\
&\text{a} \text{ Best Fit} \\
&\text{a'} \text{ Best Fit (Minus Effects Of Spin-Orbit)} \\
&\text{\textbf{C}^{13}(p, p) \text{C}^{13}} \\
&E_{p} = 1.365 \text{ MeV} \\
&\text{a} \text{ Best Fit} \\
&\text{a'} \text{ Best Fit (Minus Effects Of Spin-Orbit)} \\
&\text{\textbf{C}^{12}(d, p) \text{C}^{13}} \\
&E_{d} = 2.8 \text{ MeV}, Q = -1.131 \text{ MeV}, J = 2 \\
&\text{a} \text{ Cut-Off DWBA, Best Elastic Parameters} \\
&\text{b} \text{ Cut-Off DWBA, Best Fit} \\
&\text{c} \text{ Butler Curve}
\end{align*}
Figure 17. DWBA and Plane Wave Fits to the 3.2 MeV $^{12}\text{C}(d, p_2, \lambda)^{13}\text{C}$ Angular Distribution Data and Optical Model Fits to the Corresponding $^{12}\text{C}(d, d)^{12}\text{C}$ and $^{13}\text{C}(p, p)^{13}\text{C}$ Angular Distribution Data.
Figure 18. DWBA and Plane Wave Fits to the 3.7 MeV $C^{12}(d, p_{2,3})C^{13}$ Angular Distribution Data and Optical Model Fits to the Corresponding $C^{12}(d, d)C^{12}$ and $C^{13}(p, p)C^{13}$ Angular Distribution Data.
Table 1
Optical Model Potentials and Cut-Off Radii Used in the DWBA Analysis of $^{12}\text{C}(d,p)^{13}\text{C}$

<table>
<thead>
<tr>
<th></th>
<th>Best Elastic Parameters</th>
<th></th>
<th>Best Fit Parameters</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$E_d$</td>
<td>2.8</td>
<td>3.2</td>
<td>3.7</td>
</tr>
<tr>
<td>$V_d$</td>
<td>29.83</td>
<td>59.65</td>
<td>55.68</td>
<td>29.97</td>
</tr>
<tr>
<td>$W_d$</td>
<td>34.98</td>
<td>9.00</td>
<td>5.72</td>
<td>24.85</td>
</tr>
<tr>
<td>$r_{o_d}^d$</td>
<td>2.57</td>
<td>1.67</td>
<td>1.80</td>
<td>2.57</td>
</tr>
<tr>
<td>$r_{o_d}'$</td>
<td>0.40</td>
<td>1.49</td>
<td>1.97</td>
<td>0.63</td>
</tr>
<tr>
<td>$a_d$</td>
<td>0.31</td>
<td>0.57</td>
<td>0.53</td>
<td>0.20</td>
</tr>
<tr>
<td>$a_d'$</td>
<td>1.25</td>
<td>0.41</td>
<td>0.14</td>
<td>1.31</td>
</tr>
<tr>
<td>$V_p$</td>
<td>50.49</td>
<td>43.32</td>
<td>92.05</td>
<td>50.49</td>
</tr>
<tr>
<td>$W_{D_p}$</td>
<td>7.41</td>
<td>0.00</td>
<td>0.90</td>
<td>7.41</td>
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<tr>
<td>$r_{o_p}^p$</td>
<td>1.16</td>
<td>1.56</td>
<td>0.91</td>
<td>1.16</td>
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<tr>
<td>$r_{o_p}'$</td>
<td>1.09</td>
<td>1.56</td>
<td>3.42</td>
<td>1.09</td>
</tr>
<tr>
<td>$a_p$</td>
<td>1.13</td>
<td>0.29</td>
<td>0.58</td>
<td>1.13</td>
</tr>
<tr>
<td>$a_p'$</td>
<td>0.57</td>
<td>0.29</td>
<td>0.83</td>
<td>0.57</td>
</tr>
<tr>
<td>Cut-Off</td>
<td>3.89</td>
<td>5.00</td>
<td>3.89</td>
<td>3.89</td>
</tr>
</tbody>
</table>

$W_{D_d} = W_p = 0 \quad r_{c_d} = 1.40 \quad r_{c_p} = 1.25$

All energies are in MeV, and all lengths are in fermis.
Table 2

Optical Model Potentials and Cut-Off Radii Used in the DWBA Analysis of

\( \text{C}^{12}(d,p)\text{C}^{13} \)

<table>
<thead>
<tr>
<th></th>
<th>Best Elastic Parameters</th>
<th>Best Fit Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2.8 3.2 3.7</td>
<td>2.8 3.2 3.7</td>
</tr>
<tr>
<td>( E_d )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( V_d )</td>
<td>29.83 59.65 55.68</td>
<td>29.83 --- 55.68</td>
</tr>
<tr>
<td>( W_d )</td>
<td>34.98 9.00 5.72</td>
<td>34.98 --- 5.72</td>
</tr>
<tr>
<td>( r_{od} )</td>
<td>2.57 1.67 1.80</td>
<td>2.67 --- 1.80</td>
</tr>
<tr>
<td>( r'_{od} )</td>
<td>0.40 1.49 1.97</td>
<td>0.40 --- 1.97</td>
</tr>
<tr>
<td>( a_d )</td>
<td>0.31 0.57 0.53</td>
<td>0.31 --- 0.53</td>
</tr>
<tr>
<td>( a'_d )</td>
<td>1.25 0.41 0.14</td>
<td>1.25 --- 0.14</td>
</tr>
<tr>
<td>( V_P )</td>
<td>35.32 48.00 66.51</td>
<td>35.32 --- 62.22</td>
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<tr>
<td>( W_{Dp} )</td>
<td>5.57 3.50 0.00</td>
<td>5.57 --- 0.50</td>
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<tr>
<td>( r_{op} )</td>
<td>1.65 1.32 1.13</td>
<td>1.65 --- 1.16</td>
</tr>
<tr>
<td>( r'_{op} )</td>
<td>1.30 1.24 1.13</td>
<td>1.30 --- 1.25</td>
</tr>
<tr>
<td>( a_p )</td>
<td>0.94 0.81 0.64</td>
<td>0.94 --- 0.59</td>
</tr>
<tr>
<td>( a'_p )</td>
<td>0.46 0.39 0.64</td>
<td>0.46 --- 0.47</td>
</tr>
<tr>
<td>Cut-Off</td>
<td>3.89 5.00 3.89</td>
<td>3.89 --- 3.89</td>
</tr>
</tbody>
</table>

\[ W_{D_d} = W_{P} = 0 \quad r_{c_d} = 1.40 \quad r_{c_P} = 1.25 \]

All energies in MeV.

All lengths in fermis.
sponding to these fits were $V_{so} = 3.54$, $12.09$, $8.02$, $1.99$, and $8.00$ MeV
at $E_p = 1.365$, $1.548$, $1.917$, $2.195$, and $2.728$ MeV respectively. Figure
19 shows the radial dependence of the optical potentials which correspond
to the parameters given in tables 1 and 2.

2. **Spectroscopic Factors.** The relationship between the experimental
and predicted stripping cross sections given at the beginning of this
chapter can be used to obtain absolute spectroscopic factors when all
of the necessary nuclear level structure information is known. The
isobaric-spin coupling factor which appears in this expression is equal
to unity for the $C^{12}_{2d, 3p, 3}$ reactions because $T_f = T_{zi} = 0$ and $T_f = T_{zf} = 1/2$
(Lane, 1953). A further simplification results from the fact that only
one value of the angular momentum transfer is important in each of the
two reactions. This means that the sum over $l$, $s$, and $j$ reduces to one
term. The spectroscopic factor for the $C^{12}_{2d, 3p, 3}$ reactions can therefore be written

$$S = (1.48) \left[ \frac{(d\sigma/d\Omega)^{\text{expt}}}{\sigma(\theta)} \right] \left[ \frac{(2J_i + 1)}{(2J_f + 1)} \right]$$

The multiplicative factor 1.48 must be introduced when the Hulthen form
rather than the exponential form of the deuteron wave function is used in
SALLY (Bass et al., 1962). The Hulthen deuteron wave function was
used so that the DWBA spectroscopic factors obtained here could be
compared with the plane wave spectroscopic factors reported by
Macfarlane and French (1960). The ratio $(d\sigma/d\Omega)^{\text{expt}}/\sigma(\theta)$ was evaluated
at or near the peaks of the stripping angular distributions. The specific
angles which were used are listed in table 3 along with the calculated
spectroscopic factors and the plane wave factors of Macfarlane and
French. In view of the uncertainties in the values of the absolute stripping
Figure 19. Optical Model Potentials Used in the DWBA Analysis.
### Table 3

**Spectroscopic Factors**

#### Distorted Wave Born Approximation

<table>
<thead>
<tr>
<th>$E_d$</th>
<th>Level Excitation Energy</th>
<th>$l_n$</th>
<th>$J_f^\pi$</th>
<th>$\theta^*$</th>
<th>Cut-Off Radius</th>
<th>Spectroscopic Factors</th>
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<tr>
<td>2.8</td>
<td>3.68</td>
<td>1</td>
<td>$3/2^-$</td>
<td>20</td>
<td>3.89</td>
<td>0.20</td>
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<td></td>
<td></td>
<td>0.19</td>
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<td>3.2</td>
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<td></td>
<td></td>
<td>20</td>
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<td>0.21</td>
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<tr>
<td>3.7</td>
<td></td>
<td></td>
<td></td>
<td>35</td>
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<td>0.22</td>
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<td></td>
<td>0.17</td>
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<tr>
<td>2.8</td>
<td>3.85</td>
<td>2</td>
<td>$5/2^+$</td>
<td>55</td>
<td>3.89</td>
<td>2.03</td>
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<td></td>
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<td>3.2</td>
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<td></td>
<td></td>
<td>50</td>
<td>5.00</td>
<td>1.03</td>
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<td>3.7</td>
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<td></td>
<td></td>
<td>45</td>
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<td></td>
<td></td>
<td>1.27</td>
</tr>
</tbody>
</table>

#### Plane Wave Born Approximation

| $E_d$ | Level Excitation Energy | $l_n$ | $J_f^\pi$ | $\theta^*$ | Spectroscopic Factors ($S = \frac{\sigma}{\Omega}$)**
<table>
<thead>
<tr>
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<th></th>
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<tr>
<td>8.0</td>
<td>3.68</td>
<td>1</td>
<td>$3/2^-$</td>
<td>4.2</td>
<td>0.10</td>
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<tr>
<td>9.0</td>
<td></td>
<td></td>
<td></td>
<td>4.5</td>
<td>0.22</td>
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<tr>
<td>14.8</td>
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<td></td>
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<td>4.0</td>
<td>0.10</td>
</tr>
<tr>
<td>8.0</td>
<td>3.85</td>
<td>2</td>
<td>$5/2^+$</td>
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<td>1.58</td>
</tr>
<tr>
<td>9.0</td>
<td></td>
<td></td>
<td></td>
<td>4.5</td>
<td>1.18</td>
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<tr>
<td>14.8</td>
<td></td>
<td></td>
<td></td>
<td>5.4</td>
<td>1.14</td>
</tr>
</tbody>
</table>

*Center-of-mass reaction angle at which $(d\sigma/d\Omega)_{expt}/(d\sigma/d\Omega)_{SALLY}$ was evaluated (in deg).

**Obtained with the Best Elastic Parameters.

***Obtained with the Best Fit Parameters.

****Obtained from table in Macfarlane and French (1960).

All energies in MeV, and all lengths in fermis.
cross sections, it is estimated that the values of \( S \) obtained are accurate to about \( \pm 25\% \).

3. **Angular Correlation Parameters.** The plane wave deuteron stripping theory predicts that \((d, p\gamma)\) angular correlations measured in the reaction plane will be symmetric about the recoil axis \((k_r\text{ in Figure 20})\), that correlations measured in the azimuthal plane will be isotropic about the recoil axis, and that anisotropies observed in the reaction plane will only depend on the angular momenta involved (Satchler and Spiers, 1952). However, the DWBA theory allows shifts of the reaction plane symmetry axis \((\varphi_3\text{ in Figure 20})\) away from the recoil axis, azimuthal anisotropy about the recoil axis, and energy dependent attenuation of the reaction plane anisotropy (Huby et al., 1958). For \( \ell = 1 \) captures the latter effect is characterized by the attenuation or distortion parameter \( \lambda \). Predicted values of \( \lambda \) lie between zero (complete attenuation) and unity (zero attenuation). In the plane wave limit \( \lambda = 1 \). For \( \ell = 2 \) captures the DWBA theory predicts the possibility of two reaction plane symmetry axes. The positions of these axes with respect to the beam direction are usually denoted by \( \alpha_{22} \) and \( \alpha_{44} \) (Satchler and Tobocman, 1960). In the plane wave limit \( \alpha_{22} \) and \( \alpha_{44} \) coincide with the recoil angle.

The curves labeled \( \_a \) and \( \_b \) in Figures 21 and 22 were calculated by SALLY at the same time as the \( C^{12}(d, p_2) \) stripping fits labeled \( \_a \) and \( \_b \) in Figure 17. The plane wave predictions are labeled \( \_c \) in Figures 21 and 22. The cut-off radius is 5.00 fermis for the DWBA curves and 4.49 fermis for the plane wave curves. Figure 23 shows the effect of varying the DWBA cut-off radius. The potentials used in this calculation are the ones which gave fit \( \_a \) in Figure 17. Since the symmetry axis
Figure 20. Angular Correlation Geometry.
Figure 21. DWBA and Plane Wave Fits to the 3.2 MeV $^{12}$C$(d, p_{2}\gamma)^{13}$

Angular Correlation Symmetry Axis Position Data.
$^{12}\text{C}(d,p_2\gamma)^{13}\text{C}$

$E_d = 3.2 \text{ MeV}, Q = -0.961 \text{ MeV}, \ell = 1$

a Cut-Off DWBA, Best Elastic Parameters
b Cut-Off DWBA, Best Fit
c Butler Curve

SYMMETRY AXIS POSITION $\phi_0$ (C.M.)

PROTON ANGLE $\theta_{dp}$ (C.M.)
Figure 22. DWBA and Plane Wave Fits to the 3.2 MeV $^{12}\text{C}(d, p\gamma)^{13}\text{C}$

Angular Correlation Distortion Parameter Data.
$C^{12}_{d,p^2\gamma}C^{13}$

$E_d = 3.2\text{ MeV}, Q = -0.961\text{ MeV}, l = 1$

a) Cut-Off DWBA, Best Elastic Parameters
b) Cut-Off DWBA, Best Fit
c) Butler Curve

DISTORTION PARAMETER $\lambda$

PROTON ANGLE $\theta_{dp}(C.M.)$
Figure 23. The Effect of Various Cut-Off Radii on the DWBA Fits to the 3.2 MeV $^1_2C(\text{d}, p^\gamma)^{13}_2$ Angular Distribution, Symmetry Axis Position, and Distortion Parameter Data.
position is physically unchanged by the addition or subtraction of \(180^\circ\),
it is permissible to shift the data points up or down \(180^\circ\). The two open
points in Figure 23 were obtained in this manner. The DWBA curves
shown in Figure 24 were calculated at the same time as the \(C^{12}(d, p_3)\)
fits labeled \(a\) in Figures 16, 17, and 18. The plane wave curves were
calculated with a cut-off radius of 4.49 fermis. The angular correlation
data shown in Figures 21 through 24 were measured by Fletcher et al.
(1962).
Figure 24. The Effect of Various Cut-Off Radii on the DWBA Fits to the
2.8, 3.2, and 3.7 MeV $^{12}$C($d,p_{3}\gamma$)$^{13}$C Angular Correlation
Symmetry Axis Position Data.
$^{12}\text{C}(d,p^{3})^{13}\text{C}$

$Q = -1.131\text{ MeV}, \lambda = 2$

Butler Curve

Cut-Off DWBA,

Best Elastic Parameters

Cut-Off (fm) = b 0.00, c 3.83, d 2.88

$E_d = 2.8\text{ MeV}$

$E_d = 3.2\text{ MeV}$

$E_d = 3.7\text{ MeV}$

PROTON ANGLE $\theta_{dp}$ (C.M.)
CHAPTER V

DISCUSSION OF RESULTS AND CONCLUSIONS

A. Angular Distributions

The DWBA fits to the stripping angular distribution data (Figures 16, 17, and 18) are clearly better than the corresponding Butler fits in all cases except \((d, p_3')\) at 2.8 MeV. The improvement in the fits to the \((d, p_2')\) angular distributions is especially noticeable. A number of workers have found that the agreement between calculated and experimental angular distributions tends to diminish with increasing values of \(l_n\) (Tobocman and Gibbs, 1962; Smith and Ivash, 1963; Macefield et al., 1963). This tendency is not evident in the present data. Frequently, \(l_n = 1\) peaks are observed for which the small angle data dips below the \(l_n = 1\) Butler curves. In some cases the corresponding DWBA fits do not have this defect (Scott, 1961), but the DWBA fits to the 3.7 MeV \((d, p_2')\) data is no better than the Butler fit in this respect. The lack of agreement between the DWBA curves and the 2.8 MeV \((d, p_3')\) data is probably due to the resonance-like structure which appears in the \((d, p_3')\) yield near this energy (see Figure 14 on page 49). Since similar structure occurs in the \((d, p_2')\) yield (see Figure 13 on page 47), the good fit obtained at 2.8 MeV is probably fortuitous. Several studies have been
reported (Miller, Wegner, and Hall, 1961; Hinds et al., 1962; Macefield et al., 1963) in which a dramatic improvement in the DWBA fits was obtained by modifying the optical potentials which gave the best fits to the elastic scattering data. The DWBA fits to the $G^{12}(d,p_{2,3})$ data were not markedly improved by using modified potentials. The fact that the best fits were obtained by cutting off the stripping integrals near the nuclear surface is somewhat disturbing because one of the main features of the DWBA theory is the elimination of the cut-off radius parameter $r_0$ which appears in the Butler theory. However, the use of non-zero cut-off radii to improve fits obtained with SALLY is not unusual (Bassel, 1963). If the zero-range interactions used in SALLY for calculational convenience are replaced by finite-range interactions, contributions from the nuclear interior are suppressed to some extent without using radial cutoffs (Halbert et al., 1961; Austern et al., unpublished). The absolute cross sections calculated by SALLY were in much better agreement with the data than those calculated by using the plane wave formalism. The Butler cross sections were on the order of 50 to 100 times larger than the DWBA values. The fact that the Butler curves generally reproduce the shapes of measured angular distributions near the stripping peak in a more or less satisfactory manner but completely fail to reproduce the absolute cross sections is well known but not well understood (French, 1961). The absolute cross section discrepancy is accounted for by introducing an empirical parameter $\theta_0^2$. All of the shortcomings of the Butler theory are lumped together in this parameter (Macfarlane and French, 1960). In this scheme the reduced width is expressed as the product of the spectroscopic factor and the
empirical parameter, that is, \( \theta^2 = S \theta_0^2 \). The values \( \theta_0^2 \) which are appropriate for the \( C^{12}(d, p_2, p_3) \) reactions studied here are on the order of 0.06 (Macfarlane and French, 1960). The absolute reduced widths tabulated by Macfarlane and French (1960) were divided by \( \theta_0^2 \) to obtain the plane wave spectroscopic factors shown in table 3. The absolute spectroscopic factors (or reduced widths in single particle units) obtained from the DWBA analysis fall between the limits of 1/4 and 1 (within experimental error). Many of the absolute spectroscopic factors obtained by other workers who used the DWBA formalism also lie in this range (Macfarlane, 1963). The anomalously large spectroscopic factor obtained from the 2.8 MeV \( (d, p_3) \) angular distribution can probably be explained by the fact that the DWBA fit was rather poor in this case.

The deuteron elastic scattering angular distributions show more structure than the proton elastic scattering angular distributions (see Figures 16, 17, and 18). This is not surprising because the \((d, d)\) measurements were made at energies comparable to or greater than the height of the Coulomb barrier of \( C^{12} \) (about 2.9 MeV) while the \((p, p)\) measurements were made at energies lower than the \( C^{13} \) Coulomb barrier (about 3.0 MeV). The optical model fits to the \((d, d)\) data were much more sensitive to the potentials used than the \((p, p)\) fits as would be expected. The relatively good fit obtained for the 2.8 MeV \((d, d)\) data is probably accidental because there is a pronounced resonance in the \((d, d)\) yield near this energy (see Figure 10 on page 41). The values of \( \chi^2 \) corresponding to the best \((d, d)\) fits at 2.8, 3.2, and 3.7 MeV are 771, 1403, and 1701 respectively. The values of \( \chi^2 \) corresponding to the best \((p, p)\)
fits at 1.365, 1.548, 1.917, 2.195, and 2.378 MeV are 16, 9, 90, 21, and 151 respectively. The small values of $\chi^2_p$ are another indication that the $(p, p)$ scattering is dominated by the Coulomb field. As would be expected, the potentials which gave the best fits to the $(p, p)$ data contain either very small surface absorption terms or no absorption terms at all. These potentials are shown in Figure 21. The best fits to the $(d, d)$ data were obtained with potentials which contain volume absorption terms. The large absorption term needed to fit the 2.6 MeV data is due to the resonance near this energy. The discrepancy between the fits and the data at large angles may be due to the fact that the optical model does not take compound elastic scattering into account. Attempts to fit angular distributions of particles elastically scattered from light nuclei at low energies with the optical model are generally unsuccessful (Bjorklund, Campbell, and Fernbach, 1961; Nodvik, Duke, and Melkanoff, 1962; Smith and Ivash, 1963) so the results obtained here are not unusual.

B. Angular Correlation Parameters

The $p_2 - \gamma$ correlation parameters $\varphi^r_0$ and $\lambda$ calculated by SALLY are more sensitive to the cut-off radius used than to the potentials used (see Figures 21, 22, and 23). This also seems to be true for the $p_3 - \gamma$ correlation parameters $\alpha^{22}$ and $\alpha^{44}$ (see Figure 24), but the lack of data in this case makes it impossible to say anything definite. Figure 24 was included here to show the dependence of the parameters calculated by SALLY on the different cut-off radii and potentials used.
The symmetry axis position predicted by the Butler theory and the data points were shown for comparison purposes only. The calculated $p_2 - \gamma$ correlation parameters definitely reproduce the trend of the data better than the Butler predictions, but the fits obtained are not particularly impressive.

C. Conclusions

The DWBA calculations made with SALLY reproduced both the shapes and the absolute magnitudes of the measured $^{12}\text{C}(d, p_{2,3})$ angular distributions much better than the Butler calculations. Reasonable absolute reduced widths were obtained without introducing the empirical parameter $\theta_0^2$ which appears in the plane wave calculations of Macfarlane and French (1960). However, the optical model potentials which were used in the DWBA calculations are not well behaved. They do not change smoothly over the energy range studied and the potential parameters do not always lie within the limits which are considered to be physically reasonable (Hodgson, 1963a; Hodgson, 1963b). The unsatisfactory behavior of the potentials is not unexpected because resonant compound nucleus processes are known to contribute to the $^{12}\text{C} + d$ reaction yield (Bonner et al., 1956; McEllistrem et al., 1956; Evans, Kuehner, and Almqvist, 1963). The $^{12}\text{C} + d$ yields measured in this study also show the effects of compound nucleus contributions. No attempt was made to correct the stripping data for these effects because a usable form of reaction theory which incorporates both compound nucleus and direct reaction mechanisms does not exist yet.

The fits to the $C^{12}(d,p_2\gamma)$ angular correlation parameters $\varphi_0'$ and $\lambda$ obtained by using SALLY reproduced the general trend of the data much better than the Butler predictions, but no quantitative fits were found. Rook and Hodgson (1961) and Garg et al., (1961b) have reported detailed DWBA analyses of the $\text{Be}^9(d,p\gamma)\text{Be}^{10}$ and $\text{Be}^{11}(d,\alpha\gamma)C^{12}$ reactions respectively. Quantitative fits to the correlation parameters were not obtained in either of these studies. If accurate values of $\lambda$ could be found by making detailed DWBA calculations, the quantity which contains all of the nuclear spectroscopic information (nuclear spins, angular momentum transfer, and multipole mixtures) could be found, in principle, no matter how large the distortions were (Satchler and Tobocman, 1960). Since it was not possible to calculate $\lambda$ accurately for the $C^{12}(d,p_2\gamma)$ reaction, the DWBA method is of no help in extracting spectroscopic information from the correlations. The DWBA fits are in fairly good agreement with the Butler curves near the stripping peak. If angular correlations are measured on the stripping peak, the errors involved in extracting spectroscopic information by using the Butler formalism are probably no larger than the errors involved in calculating $\lambda$ by using the DWBA formalism.
APPENDIX

DISTORTED WAVE CALCULATIONS

The DWBA formalism upon which the program SALLY is based is outlined below. The discussion was taken from Bassel, Drisko, and Satchler (1962). The distorted wave theory of a direct nuclear reaction \( A(a, b)B \) is based on a transition amplitude of the form

\[
T = \int \frac{d\vec{r}_a}{a_A} \frac{d\vec{r}_B}{b_B} \varphi_b^*(\vec{k}_b, \vec{r}_B) \langle b, B | V | a, A \rangle \varphi_a(\vec{r}_a)
\]  

(1)

where \( \vec{k}_a \) is the relative momentum of the incident particle and \( \vec{k}_b \) is the relative momentum of the outgoing particle. The \( \varphi \) are the distorted waves (plane plus spherical scattered waves, outgoing or incoming denoted by the superscript (\( ^\uparrow \)) respectively). The distorted waves are assumed to be functions only of the separation of the centers of mass of the colliding pair. The effects of spin-orbit coupling are neglected. In practice the distorted waves are generated from the optical model potentials which describe the observed elastic scattering. The distorted waves satisfy Schrödinger equations of the form

\[
[\gamma^2 + k^2 - (2\mu/\hbar^2) \ U(r) - (2k\eta/r)] \ \varphi(\vec{k}, \vec{r}) = 0
\]

(2)

where \( U(r) \) is the optical model potential, \( \mu \) the reduced mass of the pair, and \( \eta \) the Coulomb parameter, \( \eta = (Z \ Z'/e^2)/h\nu \). The remaining factor in (1) is the matrix element of the interaction taken between the
internal states of the colliding pairs,

$$
(b, B | V | a, A) = \int \psi_b^* \phi_B^* V \psi_a \phi_A \ d\tau.
$$

(3)

This factor is the effective interaction which induces the transition between the initial and final elastic scattering states. It contains all the information on nuclear structure, angular momentum selection rules, and the type of direct reaction under consideration. The effective interaction is in general a function of both $\mathbf{r}_{aA}$ and $\mathbf{r}_{bB}$. The zero-range approximation can be introduced at this point by putting the two vectors parallel, that is,

$$
\mathbf{r}_{bB} = \left( \frac{m_A}{m_B} \right) \mathbf{r}_{aA}
$$

where $m_A$ and $m_B$ are the masses of $A$ and $B$. With this approximation the amplitude (1) reduces to a single 3-dimensional integral. In a stripping reaction, $a = b + x$ and $B = A + x$ where $x$ is the particle captured. The interaction is generally assumed to be $V_{bx}$. The factor (3) then reduces to a "spectroscopic amplitude" (or relative reduced width) times the quantity

$$
\phi_x(\mathbf{r}_{xA}) D(\mathbf{r}_{bx})
$$

(4)

where $\phi_x$ is the bound-state wave function for the captured particle $x$ within $B$, and $D$ is the product of the interaction and the internal wave function for the relative motion of $b$ and $x$ within $a$. The usual zero-range approximation used for stripping reactions consists of setting $D$ equal to a delta-function which leads to $\mathbf{r}_{bB} = \left( \frac{m_A}{m_B} \right) \mathbf{r}_{aA}$.

Bassel et al. (1962) show that the differential cross section can be written

$$
d\sigma/d\Omega = \frac{\int \frac{m_B M_A m_a}{m_b M_B |T|^2}}{(2\pi \hbar)^2 k_a (2J_A + 1)(2s_a + 1)}
$$

(5)
where \( J_A, J_B, s_a, \) and \( s_b \) are the spins of the particles and \( M_A, M_B, m_a, \) and \( m_b \) are the \( z \)-components of these spins. If the transition amplitude is expressed in terms of the quantities \( A_{t,sj} \) and \( \beta^m_{sj} \), the differential cross section becomes

\[
\frac{d\sigma}{d\Omega} = \frac{m_B^4 (2J_B + 1) \sum_\beta \left| A_{t,sj} \right|^2 \left| \beta^m_{sj} \right|^2}{\left[ \alpha^4 (2J_A + 1)(2s_a + 1) k_b^3 k_a \right]}.
\]

(6)

The coefficient \( A_{t,sj} \) contains the strength of the interaction \( V \) and a spectroscopic amplitude which depends on the internal nuclear structure. The \( \beta^m_{sj} \) are the matrix elements for the transfer of definite angular momenta \( t, s, \) and \( j \). SALLY computes the quantity \( \sigma_{t,sj} ^{(0)} \) which has the form

\[
\sigma_{t,sj} ^{(0)} = \sum_m \frac{m_A^3 (m_a + m_A)(m_b + m_B) k_b^3 k_a}{(m_B + 9.268) \sum_\beta \left| \beta^m_{sj} \right|^2}.
\]

(7)

If energies are in MeV, lengths in fermis, and masses in proton mass units, \( \sigma_{t,sj} ^{(0)} \) is in mb/str. The differential cross section now becomes

\[
\frac{d\sigma}{d\Omega} = \frac{(2J_B + 1)/(2J_A + 1)}{\sum_\beta \left| A_{t,sj} \right|^2 / (2s_a + 1) 5.093 \times 10^3} \sigma_{t,sj} ^{(0)}.
\]

(8)

This normalization is so chosen that for deuteron stripping (where \( s_a = 1 \) and \( s = 1/2 \)) the quantity multiplying \( \sigma_{t,sj} ^{(0)} \) inside of the sum is just the spectroscopic factor \( S(t,sj) \). Thus, for deuteron stripping the differential cross section becomes

\[
\frac{d\sigma}{d\Omega} = \frac{(2J_B + 1)/(2J_A + 1)}{\sum_\beta S(t,sj) \sigma_{t,sj} ^{(0)}}.
\]

(9)

This is the expression that was used in Chapter IV except that the isobaric-spin coupling factor does not appear here. For simplicity,
isobaric spin does not appear explicitly in SALLY. Macfarlane and French (1960) show that isobaric spin can be accounted for by introducing the Clebsch-Gordon coefficient \( T_A \frac{1}{2} T_B \frac{1}{2} | T_{zA} T_{zB} \).

The \((d, p)\) differential cross sections calculated by SALLY are subject to the following assumptions and approximations:

(a) Weak coupling is assumed, that is, it is assumed that elastic scattering is the most important process which occurs, and that inelastic or reaction events can be treated as small perturbations. The transition amplitude is calculated in first order perturbation theory with initial and final states described by optical model wave functions. This seems to be a good assumption except at quite low energies.

(b) Spin independent distorting potentials are assumed. When the \(^{13}C(p, p)^{13}C\) data were fit with the parameter-search program, it was found that the best fits were obtained with non-zero spin-orbit terms. However, since the strengths of these spin-orbit potentials were considerably smaller than the strengths of the central potentials, it is not expected that their inclusion would drastically alter the shapes of the calculated angular distributions.

(c) The zero-range approximation is made. For the reaction \(A(d, p)B\) this means that the product of the \(n-p\) interaction with the deuteron internal wave function is replaced by a delta-function. This approximation also implies that \(Y_{np}\) is local and central, that only the s-state of the deuteron is taken into account, and that all exchange effects are ignored. The general applicability of this approximation is now under study (Austern et al., to be published).

(d) The neutron bound state wave function is obtained by matching
a harmonic oscillator wave function for the internal region to the spherical Hankel function that is appropriate to the binding energy of the captured neutron \( Q + 2.226 \text{ MeV} \).

(c) All nuclei and distorting potentials are assumed to be spherical. This should be valid for the present experiment.
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