AN ANGULAR-CORRELATION STUDY OF THE

$^{24}\text{Mg} \left(^3\text{He, a }\gamma\right)^{23}\text{Mg}$ REACTION

by

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Date: March 4, 1968

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A dissertation submitted in partial fulfillment of
the requirements for the degree of Doctor of
Philosophy in the Department of Physics
in the Graduate School of Arts and
Sciences of Duke University

1968
ABSTRACT
(Physics)

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Nine excited states of $^{23}\text{Mg}$ have been investigated with the $^{24}\text{Mg}(^{3}\text{He}, \alpha \gamma)^{23}\text{Mg}$ reaction at $^{3}\text{He}$ bombarding energies of 6.37 and 8.05 MeV using the angular-correlation method of axial symmetry. The observed $\alpha$ particles were detected in an annular semiconductor counter positioned at 180° relative to the beam direction. From the analysis of the experimental angular correlations, and in conjunction with known $J^\pi$ values for $^{23}\text{Mg}$, spin and parity assignments of $5/2^+$, $7/2^{+}\ (3/2)$, $1/2^+$, $9/2^{+}\ (5/2)$, $1/2^-\ (3/2^-),\ 3/2^-,\ (5/2^+)$, and $1/2^+$ were established for the levels at 0.451, 2.048, 2.356, 2.712, 2.768, 3.792, 3.968, and 4.353 MeV, respectively. Gamma-ray branching ratios were determined for each of these levels as were the $\gamma$-ray multipole mixing ratios for most of the transitions. Coincidence measurements performed with a 20.6 cc Ge(Li) detector led to the determination of the decay modes of the 2.712- and 2.768-MeV levels. Yield curves of $\alpha$ particles detected in the
annular counter and leading to the $^{23}\text{Mg}$ ground state and the levels at 0.451, 2.048, and 2.356 MeV were measured for $^3\text{He}$ bombarding energies between 6.0 and 8.2 MeV.

The predictions of a strong-coupling collective model are compared with the experimental results. The ground state and the levels at 0.451, 2.048, and 2.712 MeV can be regarded as members of a $K^\pi = 3/2^+$ rotational band based on Nilsson orbit 7 with the ground state being the band head. The observed mixing and branching ratios for the $\gamma$-ray transitions among these levels agree with the model calculations, assuming the levels to be unperturbed by band mixing. A prolate deformation of $\eta \approx 4$ is indicated. The levels at 2.356, 2.904, and 3.968 MeV can be identified as members of a $K^\pi = 1/2^+$ rotational band based on Nilsson orbit 9 with the 2.356-MeV level being the bandhead. Band mixing between this band and the $K^\pi = 3/2^+$ band was attempted in order to calculate the mixing and branching ratios for the 2.356- and 2.904-MeV levels. The negative parity levels at 2.768 and 3.792 MeV are possible members of a $K^\pi = 1/2^-$ rotational band based on Nilsson orbit 4 with the 2.768 MeV level, representing a core-excited state, being the bandhead. The observed level at 3.856 MeV appears to be the third member of this band with $J^\pi = 5/2^-$. The level at 4.353 MeV can be interpreted as the bandhead of a $K^\pi = 1/2^+$ rotational band formed by promotion of a core nucleon from Nilsson orbit 6 into orbit 7.

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ACKNOWLEDGMENTS

I wish to express my gratitude to Dr. N. R. Roberson who suggested this project and has given his continued interest and support during all phases of this work. I am indebted to Dr. D. R. Tilley for many helpful discussions related to both the measurements and the analysis of the data. I wish to thank Dr. R. V. Poore for his continued help with the computer programming, many helpful discussions concerning experimental problems, his assistance in taking data, and particularly the Ge(Li) detector data which he laboriously took.

The assistance of Drs. V. H. Webb and M. B. Lewis in taking the data is deeply appreciated. I would like to thank Mr. S. E. Edwards for his help with the electronics and Mr. R. L. Rummel and the entire Nuclear Structure Group for the assistance with the accelerator. I am obliged to Mrs. Joseph Bailey who did an excellent job in preparing the illustrations. Thanks are also due to Mr. A. W. Lovette for many helpful suggestions concerning the design of the evaporator.

I would like to thank Dr. E. K. Warburton for the use of his angular correlation code and Dr. A. Waltner for providing the $^{24}\text{Na}$ and
ThC" sources. Special thanks are also due to Dr. R. E. Grove at Randolph-Macon College and Dr. R. H. Rohrer and Mr. R. W. Carter at Emory University who offered much help and encouragement during my academic career. I wish also to thank my parents who have offered continuous encouragement.

I am grateful to Drs. H. W. Newson and E. G. Bilpuch and the Nuclear Structure Laboratory for providing me with the research assistantship. This work was supported in part by the Atomic Energy Commission.

L. C. H.
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$^{24}\text{Mg}(^{3}\text{He}, \alpha \gamma)^{23}\text{Mg}$ REACTION
Chapter I

INTRODUCTION

A. General

One of the major contributions to the understanding of nuclear behavior has been the nuclear collective model, for which the formulation was first carried out in detail in 1952 by A. Bohr (1) and was later extended by Bohr and Mottelson (2) and by Nilsson. (3) This model was originally formulated to explain phenomena such as the large static quadrupole moments and enhanced E2 transition probabilities encountered in the study of heavy nuclei. Considerable success was achieved within the atomic mass regions $150 \leq A \leq 190$ and $A \geq 222$.

Nilsson (3) treated quantitatively the behavior of a single particle within the non-spherical potential provided by a deformed core. According to this formulation, the spectra of nuclei having the same odd nucleon number, either N or Z, should be similar. Although this model was not expected to be valid below $A = 100$, Litherland et al. (4, 5) established in 1956 the applicability of this model to light nuclei in their interpretation of the level
schemes of the mirror pair $^{25}_{13}\text{Al}_{12}$ ($Z = 13$) and $^{25}_{12}\text{Mg}_{13}$ ($N = 13$).

The nuclear systems with $N$ or $Z = 11$ consist of $^{19}_{8}\text{O}_{11}$, $^{21}_{10}\text{Ne}_{11}$, $^{21}_{11}\text{Na}_{10'}$, $^{23}_{12}\text{Na}_{12'}$, $^{23}_{12}\text{Mg}_{11}$, and $^{25}_{11}\text{Na}_{14'}$. The Nilsson model has been used with success regarding the interpretation of the static properties of $^{21}\text{Ne}$, $^{21}\text{Na}$, and $^{23}\text{Na}$ but only recently has there been sufficient experimental information upon which to base a detailed comparison of the dynamic-level properties of the Nilsson model. Howard, Allen, and Bromley (6) and more recently Poletti and Start (7) have shown that the Nilsson model successfully predicts both the phase and magnitude of the multipole mixing ratios involving $\gamma$-ray transitions among the three lowest-lying levels of $^{21}\text{Ne}$, $^{21}\text{Na}$, and $^{23}\text{Na}$. The latter authors have also made Nilsson model predictions for the analogous transitions in $^{23}\text{Mg}$, assuming that the spin sequence for the ground and first two excited states is identical to that in the mirror nucleus $^{23}\text{Na}$. They remark that the predictions should be quite accurate and point out that it would be quite surprising if the phases of the mixing ratios were found to be incorrect.

The energy levels of $^{23}\text{Mg}$ (8) have been determined mainly by means of the $^{24}\text{Mg}(^3\text{He}, \alpha)^{23}\text{Mg}$ reaction. This nucleus is radioactive (9) and decays primarily to the $^{23}\text{Na}$ ground state. This decay is super-allowed ($\log ft = 3.7$) which determines the ground state spin and parity of $^{23}\text{Mg}$ as $J^\pi = 3/2^+$. Angular distributions of $\alpha$-particle groups populating the ground and first-excited state lead to $J^\pi = (3/2, 5/2)^+$ for both states. (10)
In view of the lack of experimental information concerning the $^{23}\text{Mg}$ nucleus and the success in interpreting the static and dynamic properties of the low-lying states of the other "central" $N$ or $Z = 11$ nuclei by means of the Nilsson model, it was clear that an angular-correlation study of this nucleus was needed. Of nine nuclear reactions which could populate levels in $^{23}\text{Mg}$, only the $^{24}\text{Mg}(^3\text{He}, \alpha)^{23}\text{Mg}$ reaction ($Q_\circ = 4.048$ MeV) was applicable using the Duke University 4-MeV Van de Graaff accelerator. The remaining reactions had large ($\leq -4$ MeV) negative $Q$-values (with the exception of the $^{21}\text{Ne}(^3\text{He}, n)^{23}\text{Mg}$ reaction) and would have required a tandem accelerator or cyclotron to initiate them. The $^{24}\text{Mg}(^3\text{He}, \alpha)^{23}\text{Mg}$ reaction, however, was well suited for a study of $^{23}\text{Mg}$ for several reasons. In 1965 Roberson, Tilley, and Weller (11) developed a $^3\text{He}$ ion source for the Duke University Van de Graaff accelerator which could provide up to 500 nA of doubly-ionized $^3\text{He}$ ions at energies of up to 8.2 MeV. Since the reaction $Q$-value of the $^{23}\text{Mg}$ 4.353-MeV tenth-excited state was only slightly negative ($Q_{10} = -0.305$ MeV), it was felt that most of the low-lying levels of $^{23}\text{Mg}$ could be populated with the available $^3\text{He}$ beam. The major advantage in using this reaction is that it is well suited to the angular-correlation method of axial symmetry.

Historically, the utilization of the symmetry of the reacting system in a nuclear reaction to simplify the theoretical analysis associated with an angular-correlation experiment was first pointed out by Biedenharn, Arfken, and Rose (12) in 1951. The method was first applied by Warburton
and Rose (13) in a $\gamma - \gamma$ angular-correlation experiment. The method of axial symmetry as applied to particle-$\gamma$ angular correlations has been attributed to J. Newton. (14) This whole method has been put in a more useful form by Litherland and Ferguson (15) and has been widely used.

Essentially, the particle-$\gamma$ angular-correlation method is one of imposing constraints on the angular distribution of the de-excitation $\gamma$ rays from an excited state formed in a nuclear reaction by detecting the reaction particles in an axially-symmetric detector positioned along the incident beam axis. These constraints depend on both the spin of the $\gamma$-ray emitting state and the multipole mixing ratio of the $\gamma$ rays. The analysis of the $\gamma$-ray angular distributions can often lead to a unique set of these parameters. An important advantage of the method is that the analysis of the particle-$\gamma$ angular correlations is independent of the reaction mechanism for the formation of the $\gamma$-ray emitting state. These remarks will be expanded in Chapter III.

A particle angular-distribution experiment was not considered for the present study as DWBA theory is not very reliable at the beam energies available in this laboratory. Such an experiment could, in principle, yield $J_n$ values and spectroscopic parameters which would be quite useful information.
B. The Present Experiment and Other Recent Work on $^{23}\text{Mg}$

An investigation of the $^{23}\text{Mg}$ nucleus using the particle-$\gamma$ angular-correlation method of axial symmetry (referred to as Method II in Ref. 15) was begun in an effort to make rigorous spin assignments wherever possible or to limit rigorously the possible level spins, and to compare experimentally determined spins, decay modes, and mixing ratios with the predictions of the Nilsson model. In addition to elucidating the nuclear structure of $^{23}\text{Mg}$, such a study could help to remove some ambiguities regarding experimental spin assignments in the mirror nucleus $^{23}\text{Na}$ and thereby lead to a greater understanding of the nuclear systems with $N = 11$.

Approximately six months after this investigation was begun, work of a preliminary nature concerning the $^{23}\text{Mg}$ nucleus was reported by a number of groups. Joyce, Zurmuhle, and Fou (16) using the $^{24}\text{Mg}(^{3}\text{He},\alpha)^{23}\text{Mg}$ reaction, have made $J^\pi$ assignments to the angular distributions of $\alpha$ particles leading to several low-lying levels of $^{23}\text{Mg}$. Ganguly et al. (17) and Kozub and Kashy (18, 19) have measured deuteron angular distributions from the $^{24}\text{Mg}(p,d)^{23}\text{Mg}$ reaction ($Q_o = -14.328$ MeV) and have assigned $J^\pi$ values to a number of levels in $^{23}\text{Mg}$. Dubois and Barwaker (20) have performed an $\alpha-\gamma$ angular-correlation and $\alpha$-particle angular-distribution experiment using the $^{24}\text{Mg}(^{3}\text{He},\alpha\gamma)^{23}\text{Mg}$ reaction and report spin-parity assignments for the first three excited states of $^{23}\text{Mg}$ and limit the spins of three other levels to two possible values. A preliminary report on the
study of this same reaction at Duke University has been given. (21) Spin assignments and mixing ratios were given for the first two excited states and the isotropy of all of the $\gamma$ rays from the decay of the third-excited state was indicated.

Work of a more definite nature has been recently published by Hay and Kean (22) who found eleven new levels in $^{23}$Mg between 4.40 and 6.24 MeV excitation using the $^{24}\text{Mg}(^3\text{He}, \alpha)^{23}\text{Mg}$ reaction. DaSilva et al. (23) have reported the results of their study of this same reaction. They make spin assignments for the first two excited states of $^{23}\text{Mg}$ and limit the spins of two higher states to two possible values. Quite recently, Dubois and Earwaker (24) have published the results of their study of $^{23}\text{Mg}$.

It was realized that the work undertaken in this investigation would overlap the work of the various groups mentioned above and possibly of others due to the keen interest of nuclei in this region of the 2s-1d shell. However, it was felt that this independent study would be useful in that it could help to confirm or refute the work of other groups working simultaneously on $^{23}\text{Mg}$ and hopefully would provide additional information not determined by other groups.

In mid-1967 a large-volume Ge(Li) $\gamma$-ray detector was acquired by North Carolina State University. A coincidence measurement was performed utilizing the high intrinsic resolution of this detector to study the decay modes of $^{23}\text{Mg}$. This information greatly complemented the angular-correlation measurements.
Chapter II

EXPERIMENTAL APPARATUS AND PROCEDURE

A. Angular-Correlation Measurements

1. General

The present investigation was carried out using the Duke University 4-MeV Van de Graaff accelerator equipped with a $^3\text{He}$ ion source. This source made it possible to obtain a doubly-ionized $^3\text{He}$ beam of good intensity with energies up to 8 MeV. The $^3\text{He}$ beam, containing both doubly-ionized and singly-ionized components, was accelerated and then magnetically analysed by a switching magnet. This magnet has beam ports which make it possible to control the accelerator with the $^3\text{He}^+$ beam while using the less intense $^3\text{He}^{++}$ beam for bombardment. The terminal potential was monitored using a precision ac digital voltmeter and was maintained to within $\pm 10$ keV during all phases of this experiment.

After magnetic analysis, the $^3\text{He}^{++}$ beam passed through a set of remotely controlled slits which were located approximately 2 m from the switching magnet and were used to limit the beam intensity. Shielding
was provided around the exit of the slits to reduce extraneous radiation. The slits were followed by a magnetic-quadrupole lens and two beam deflectors which were utilized to guide the beam into the target chamber located approximately 5 m from the switching magnet. By optimizing the accelerator ion-source parameters and proper positioning of the lens and deflectors it was possible to focus the beam entirely through a 2.38-mm diameter collimator located approximately 30 cm from the center of the target chamber.

2. The Target Chamber

The angular-correlation chamber and associated collimation system are illustrated in Figure 1. The chamber itself was a 15.6-cm diameter brass cylinder with a 1.59-mm wall and a 9.53-mm bakelite top and bottom sealed to the cylinder by O-rings. The chamber was supported at each end by the beam pipe and could be rotated about the beam axis and also about a point defined by a ball joint positioned approximately 60 cm in front of the chamber. The top of the chamber contained a lucite window which allowed inspection of the target which was mounted on a 1.9-cm diameter, 1.59-mm thick stainless-steel ring. This ring was attached to a rod passing through the bottom of the chamber and could be rotated or pulled out of the path of the beam. The beam was collected in a Faraday cup located approximately 1.5 m from the target chamber and was measured using an Elcor Model A309B current integrator. (25) An oil diffusion
Figure 1. Top View of Angular-Correlation Chamber.
pump located between the Faraday cup and target chamber provided a vacuum of approximately 50 μtorr in the chamber.

The beam was initially collimated by a 2.38-mm diameter Ta collimator attached to a 14.7-cm long stainless-steel tube as shown in Figure 1. Additional collimation was provided by a 1.59-mm diameter Ta collimator attached to the other end of the tube. A 3.17-mm diameter stainless-steel insert, which was mounted in the solid-state detector holder, served as the anti-scattering collimator. Lead cylinders were placed in the tube to reduce the γ radiation caused by the beam striking the first collimator.

The charged-particle detector was mounted in a lucite holder with an attached snout which was designed to prevent radiation scattered by the chamber walls from striking the sensitive area of the detector. A small lip on the 3.17-mm diameter insert served to shield the rough inner edge of the detector from the back-scattered radiation.

3. Target Preparation

The $^{24}$Mg targets were prepared by evaporating MgO enriched to 99.96% $^{24}$Mg (26) onto plain 3 x 1-inch glass slides or onto glass slides which supported a thin carbon backing. The oxide was partially reduced by adding Ta powder to the MgO prior to evaporation from a Ta boat. The targets evaporated onto the carbon backings were readily floated off the slides in water and mounted on the stainless-steel rings. The self-supporting targets, however, could only be removed from the slides after
evaporating the MgO-Ta mixture at a rapid rate. This technique presumably caused a water-soluble oxide layer to be deposited onto the glass slides before the magnesium metal began to evaporate. It was then possible to float the shiny, metallic films from the slides in a water bath and mount them on the target rings. It has recently come to the attention of the author that BaI₂ and CsI are good release agents in removing evaporated isocopes from substrates. The slides can be uniformly coated with these water-soluble compounds by vacuum evaporation before evaporating the isotope. The targets can then, presumably, be readily floated from the slides in a water bath.

Both self-supporting and carbon-backed targets were used during the experiment. The latter targets, however, were easier to produce and were less likely to break under long bombardment and high beam currents. The thickness of the ²⁴Mg targets was of the order of 75 μgm/cm² and that of the carbon backings approximately 15 μgm/cm². The only criteria used in selecting a target thickness were that the resolution be adequate to resolve the α groups of interest and the thickness be sufficient to yield a reasonable coincidence counting rate.

4. Reaction Product Detection

a. Alpha-Particle Detector. The charged-particle spectrum from the target was detected by a silicon surface barrier detector mounted at 180° to the ³He beam and 4 cm from the target. This detector, supplied by ORTEC (27), was fabricated from 950 ohm-cm silicon, was of 100 mm²
area, and contained a 4-mm diameter hole through which the $^3$He beam passed. The detector subtended a solid angle of approximately 0.06 steradians at the center of the target. The inner and outer edges of the sensitive area subtended angles at the target center of 173° and 177°, respectively. The counter could stop approximately 14-MeV α particles and approximately 3.5-MeV protons at a depletion depth of 100 μ. The use of a low-resistivity counter was necessary to insure that the pulses from the detector would be of sufficiently short rise time to trigger the time pickoff unit which furnished the timing signals for the coincidence measurements. The use of a thin counter also insured that protons from the $(^3$He,p) reaction would make a negligible contribution to the charged-particle spectrum.

b. Gamma-Ray Detector. The γ-ray detectors were cylindrical Na(Tl) crystals furnished by Harshaw. (28) The initial phase of this experiment was performed using a 5.1 x 5.1-cm crystal. Later a 7.6 x 7.6-cm and a 10.2 x 12.7-cm crystal were employed. The larger crystal was used to improve the statistics of the weak ground-state transition of the $^{23}$Mg second-excited state, but the poor resolution of this crystal rendered the results questionable. The 7.6 x 7.6-cm crystal was found to possess the best resolution and efficiency for detecting the higher-energy γ rays encountered in this study. The faces of the 5.1 x 5.1-cm and 7.6 x 7.6-cm crystals were positioned 8.9 cm and 11.4 cm, respectively, from the target. Each crystal had a resolution of approximately 10%
FWHM for a 0.51-Mev $\gamma$ ray. The 5.1 x 5.1-cm crystal was mounted on an RCA 6810-A photomultiplier tube (28) while the 7.6 x 7.6-cm crystal was used with an Amperex 58-AVP photomultiplier tube. (30) Both tubes were enclosed in $\mu$-metal shields to minimize effects of external magnetic fields on the gain of the tubes. Optical coupling between the crystals and tubes was provided by Dow Corning 200 fluid (viscosity = $10^6$ centistokes). (31)

The $\gamma$-ray detector was mounted on a trolley which rotated about a vertical axis through the center of the target chamber. The detector angular settings were indicated on a 360° calibrated table mounted directly beneath the target chamber. The angular readings were made with an error not greater than 0.5°.

5. Electronics

Figure 2 is a block diagram of the coincidence circuit used to perform a two-parameter analysis of the $\alpha$-$\gamma$ coincidences from the $^{24}$Mg($^3$He, $\alpha \gamma$)$^{23}$Mg reaction. This circuit, in conjunction with an on-line computer, made it possible to obtain, simultaneously, angular-correlation data on up to ten excited states of $^{23}$Mg.

The Amperex 58-AVP photomultiplier tube and Model PA58 base was used for the majority of the angular-correlation measurements. This tube is a 14-stage, very fast, high-gain photomultiplier with a Cs-Sb, semi-transparent, curved cathode of approximately 12.7-cm diameter. The anode pulse has a rise time of 2 ns which makes the tube ideally suited for use with fast coincidence circuitry. It was observed with initial use of
Figure 2. Block Diagram of Electronics Used for the Angular-Correlation Measurements.
BLOCK DIAGRAM OF ELECTRONICS
the tube base that there were rather large pulse-height changes due to high counting rates in the output from the base. This problem was corrected by decreasing the total resistance of the resistor bank in order to increase the bleeder current. This modification acted to stabilize the voltage across each dynode which led to greater gain stability in the tube. Severe voltage fluctuations were also observed during initial use of the tube. This problem was corrected by fixing the conducting layer on the wall of the tube at the same negative potential as applied to the cathode. The circuit diagram showing these modifications is given in Reference 32.

Linear signals, used for energy analysis, were taken from the 14-th dynode of the photomultiplier and amplified by an ORTEC Model 113 preamplifier. These signals were then delayed approximately 500 ns, amplified by a Hamner Model N-301 linear amplifier (33), and used as one input to a TMC 1024-channel dual analog-to-digital converter (ADC). (34)

Fast $\gamma$-ray pulses were taken directly from the anode of the photomultiplier and driven directly to the accelerator console area through 50-ohm cable chosen for impedance matching. It was necessary to limit these pulses to protect the fast coincidence circuitry from excessively high voltages. The limiter circuit is given in Reference 35. The limited pulses were amplified by a Chronetics Model 106 pulse amplifier (36) having a fixed gain of ten and a rise time of 2 ns. These amplified pulses were then fed into a Chronetics Model 101 dual discriminator with a fixed 1.5 $\mu$s output pulse width, a width long enough to prevent double pulsing.
Linear signals from the annular detector were amplified by an ORTEC Model 109 charge-sensitive preamplifier. These signals were delayed approximately 400 ns and amplified by an ORTEC Model 410 pulse-shaping amplifier. This amplifier provides simultaneous unipolar (singly differentiated) and bipolar (doubly differentiated) outputs with RC or delay-line shaping. The bipolar signal was used as the input for both an ORTEC Model 408 and a RIDL Model 30-21 biased amplifier. (37) These amplifiers allowed only those portions of the spectrum which were of interest to be analysed. The unipolar signal was fed into an oscilloscope and was used as a visual monitor of pulses from the annular detector. The output of the ORTEC biased amplifier served as the input to the second half of the TMC dual ADC.

The time of arrival of particles detected by the annular detector was derived from an ORTEC Model 260 time pickoff used in series between the detector and the charge-sensitive preamplifier. A toroidal transformer in this unit inductively taps the leading edge of the detector signal without appreciably affecting the linear signal which is fed to the preamplifier. These timing signals, from the secondary of the transformer, then actuate a fast amplifier and tunnel diode discriminator. When used with the low-resistivity detector, the time pickoff proved to be an efficient source of timing signals. The fast signals were driven through 50-ohm cable to a Chronetics Model 114 fast discriminator.

The logic pulses from the fast-α and fast-γ discriminators
were clipped to a width of 20 ns and fed into a Chronetics Model 107 twofold coincidence circuit which furnished the fast coincidence pulses. Due to the relatively slow rise time of pulses from the NaI crystal (typically 200 ns) and electron transit time in the photomultiplier tube, it was necessary to delay the fast-\(\alpha\) signal 40 ns relative to the fast-\(\gamma\) signal in order for the two pulses to overlap in time. Accidental coincidences were determined by routing the clipped-\(\alpha\) and -\(\gamma\) logic signals into the second half of the Chronetics fast coincidence circuit but with the fast-\(\alpha\) signal delayed approximately 200 ns relative to the fast-\(\gamma\) signal. The overall time resolution of the fast coincidence circuitry was approximately 40 ns.

The output pulses from the true-plus-accidental (total) and accidental coincidence circuits were fed into a mixer circuit which generated a gate pulse for both the particle ADC and the \(\gamma\) ADC. This circuit also provided a signal by which accidental coincidences stored in the computer could be distinguished from total coincidences.

A Computer Control Company Model DDP-224 computer (38) was programmed as a 256 x 64-channel, two-parameter analyser with 64 channels assigned to the \(\gamma\)-ray spectrum and 256 channels assigned to the \(\alpha\)-particle spectrum. For each \(\alpha\) particle group stored along the 256-channel axis, the associated \(\gamma\)-ray coincidence spectrum (total and accidental) was stored along the 64-channel axis. The total and accidental coincidences were recorded in binary form on magnetic tape and later read back into the computer for data analysis. Only 128 channels of the 256-channel axis
could be read back at one time as the computer had only an 8K memory.

The effect of dead time in the \( \gamma \)-ray discriminators was determined by counting the number of \( \gamma \) rays detected during each angular measurement. This was accomplished by scaling the counting rate down by a factor of \( 10^5 \) using a Chronetics Model 109 100 mc prescaler in series with a modified RIDL scaler. The number of \( \gamma \) rays so detected was then stored in the computer along with the coincidence data. Using the measured 2.5 \( \mu \)s dead time for the \( \gamma \)-ray circuitry, a dead time correction was calculated. This correction was essentially insignificant as care was taken to maintain a constant beam current for each angular measurement.

The normalization of the \( \alpha-\gamma \) correlation data was based on a 512-channel \( \alpha \)-particle monitor spectrum recorded in a way which could account for changes in the bias level of the time pickoff unit. Pulses from the annular counter which were subsequently amplified by the RIDL biased amplifier were fed into a Victoreen 2048-channel ADC (39) which was gated by the time pickoff signal generated by a fast discriminator. In this manner changes in the bias level of the time pickoff, which affected the \( \alpha-\gamma \) coincidence rate, could be accounted for by normalizing the angular measurements for a particular \( \alpha \)-particle group to the number of counts recorded for that same group in the monitor spectrum. The monitor spectra were recorded in binary form on magnetic tape for later analysis. These spectra and also the two-dimensional data were displayed on an oscilloscope as the data were accumulated and provided a visual monitor of the experiment.
6. Procedure

a. Alignment. The target chamber was initially aligned by adjusting its orientation so that the $^3$He beam impinged upon the center of a quartz crystal positioned near the target chamber exit. The magnetic lens and deflectors were not used during this state of the alignment. The annular counter and target-ring support were then aligned using a carefully made brass rod designed to pass through an insert located at the target chamber exit, the target ring, the annular counter, and finally the 1.59 mm diameter collimator positioned at the target chamber entrance. This alignment was verified by bombardment of a carbon target with the $^3$He beam. The 1.59-mm diameter beam spot was found to be centered on the target.

The center of the calibrated table upon which the $\gamma$-ray detector rotated was positioned under the center of the target chamber using a plumb line attached to the target rod. Plumb lines suspended from both sides of the beam pipe were also used to position the 180° position of the calibrated table directly under the center of the beam pipe. The $\gamma$-ray detector was centered relative to the target by aligning the central axis of the cylindrical crystal and attached photomultiplier with the center of the target.

The overall alignment of the system was verified by measuring the $^{12}\text{C}(d,p_1\gamma)^{13}\text{C}$ angular correlation which is known to be isotropic as the $^{13}\text{C}$ 3.08-MeV first-excited state has $J^\pi = 1/2^+$. (40) This correlation was found to be isotropic to within 2%.
b. Excitation. Ten excited states in $^{23}\text{Mg}$ have been identified up to 4.353 MeV, primarily by means of the $^{24}\text{Mg}(^{3}\text{He}, \alpha)^{23}\text{Mg}$ reaction. (8) Recently many more levels have been identified at higher energies. (22, 24) The work in this study, however, is limited to the first ten excited states. Of these levels, the states at 0.451, 2.048, 2.356, and 4.353 MeV are well separated from nearby levels and consequently posed no special resolution problems. Two close-lying levels exist at 2.712 and 2.768 MeV, the latter level separated by 136 keV from the sixth-excited state at 2.904 MeV. Another set of close-lying levels exists at 3.792 and 3.856 MeV with the latter level separated by 112 keV from the ninth-excited state at 3.968 MeV. While it was impossible to fully resolve these two sets of three close-lying states an attempt was made to find bombarding energies at which some of these states would be preferentially excited. This was accomplished by evaporating a thin film of $^{24}\text{Mg}$ onto a thin carbon backing and observing the $\alpha$-particle yield in the annular counter as a function of $^{3}\text{He}$ bombarding energy. The levels populated were then identified with the aid of the reaction kinematics. Bombarding energies of 6.37 and 8.00 MeV were chosen as the optimum energies at which to perform the angular-correlation measurements. The $\alpha$-particle spectra recorded at these two energies are shown in Figure 3. At 6.37 MeV the first-, third-, sixth-, and tenth-excited states were populated with approximately the same cross section. The second-excited state was only weakly excited. The 2.712, 2.768-MeV doublet was weakly excited relative to the nearby sixth-excited state and
Figure 3. Alpha-Particle Spectra from the $^{24}\text{Mg}(^{3}\text{He}, \alpha)^{23}\text{Mg}$ Reaction Recorded in the Annular Detector at $^{3}\text{He}$ Bombarding Energies of 6.37 and 8.00 MeV. The $\alpha$-particle groups are labeled by the state in $^{23}\text{Mg}$ to which they lead. The arrows indicate the expected position of the corresponding $\alpha$-particle groups as determined from the reaction kinematics.
the ninth-excited state at 3.968 MeV appeared to be populated more than
the 3.792-, 3.856-MeV doublet. At a $^3$He energy of 8.00 MeV the second-
extcited state was reasonably well excited so that a correlation measure-
ment was feasible. The 2.712-, 2.768-MeV doublet was very strongly
populated relative to the 2.904-MeV level. The thin-target yield also
indicated that the 3.792-MeV seventh-excited state was fed more strongly
than the levels at 3.856 and 3.968 MeV. It thus appeared that for these
two bombarding energies information could be obtained on the decay modes
of nine excited states of $^{23}$Mg. It was doubtful that any information could
be obtained concerning the weakly excited 3.856-MeV level.

Yield curves of $\alpha$ particles detected at 180° to the beam axis
and leading to the $^{23}$Mg ground state and first three excited states are
shown in Figure 4. The rather large fluctuations in the yield suggest that
there is considerable compound nucleus formation in the beam-energy
region (6.0 - 8.2 MeV) studied here. This same behavior was observed
by Dubois and Earwaker (24) for the $^3$He energy range 8-11 MeV with the
$\alpha$ particles detected at 10° relative to the beam axis. The small relative
cross section for exciting the 2.048-MeV level suggests that the spin of
this level differs from that for the ground state and first- and third-
excited states. The spins of these states, to be discussed later, are given
in Figure 4 for comparison.

c. Delays. Coincidences between the fast-$\alpha$ and fast-$\gamma$ logic
signals were established by delaying the $\alpha$ signal 40 ns relative to the $\gamma$
Figure 4. \(^{24}\text{Mg}(^{3}\text{He}, \alpha)^{28}\text{Mg}\) Excitation Spectra Recorded in the Annular Detector for the \(^{3}\text{He}\) Energy Range 6.0-8.2 MeV.
$^{24}\text{Mg}(^{3}\text{He},\alpha)^{23}\text{Mg}$ EXCITATION AT $\theta_{c}=180^\circ$

$\alpha_0$

$0.451~\frac{3}{2}^+$

$^{23}\text{Mg}$

$\alpha_1$

$0.451~\frac{5}{2}^+$

$\alpha_2$

$2.048~\frac{3}{2}^+(4)$

$\alpha_3$

$2.356~\frac{1}{2}^+$

NUMBER OF COUNTS

$E_{3\text{He}}$ (MeV)

6.0 6.5 7.0 7.5 8.0 8.5
signal. This delay was determined using a Chronetics Model 105 time-to-pulse-height (TPH) converter. This device is designed to convert the time overlap of two standardized pulses to a pulse whose height is proportional to the time overlap between the two pulses. A pulse from the fast-a discriminator, clipped to a pulse width of 100 ns, was split and fed into the two inputs of the converter. The output was then fed into a TMC 400-channel analyser for pulse height analysis. The peak channel so determined corresponded to that expected for two perfectly overlapping pulses. The fast-a and fast-γ logic signals, both clipped to 100 ns pulse width, were then routed into the TPH converter. Delay was introduced between the two pulses until the coincidence peak fell in the channel of the analyser expected for perfect overlap. As previously mentioned, the accidental coincidences were established by delaying the fast-a signal 200 ns relative to the fast-signal.

d. Timing Measurements. Initial use of the time pickoff unit indicated that the device was extremely sensitive to noise pulses which existed in the laboratory. There was also a tendency of the discriminator level of the tunnel diode to drift downwards. Both of these effects tended to cause oscillation of the tunnel diode discriminator. A circuit was devised which prevented data from being stored by the computer while the discriminator was oscillating. The noise pulses were considerably reduced by surrounding the pickoff unit, the charge-sensitive preamplifier, and the target chamber with brass screen.
An upwards drift of the discriminator level was also observed occasionally during the runs. This effect was of little concern since it caused only a decreased efficiency in the time pickoff which would be reflected in the monitor spectra to which the data were normalized. The question arose, however, as to the effect of these drifts, in the time pickoff triggering level, on the resolving time of the coincidence circuitry. To resolve this question a two-parameter analysis was performed of the TPH-converter signals as a function of α-particle energy. Linear, amplified pulses from the annular counter and logic pulses from the converter were fed into separate sections of the TMC dual ADC, both of which were gated by the fast coincidence pulses. Several two-dimensional spectra were recorded in this manner for a wide range of time pickoff bias levels. These measurements revealed that the time resolution of the coincidence circuit was not a function of the pickoff bias level.

A similar measurement was performed for the converter timing signals as a function of γ-ray energy. It was observed that a somewhat longer resolving time was required to record α-γ coincidences for γ rays with energies less than approximately 1 MeV than that required at higher energies. This effect is a result of the long rise time of pulses from the NaI crystal and is one of the limitations of leading-edge timing. However, this measurement revealed that a resolving time of \( 2\gamma = 40 \text{ ns} \) was sufficient to record coincidences for γ rays with energies as low as approximately 400 keV.
e. Angular Measurements. The angular-correlation measurements consisted in determining the intensity of α-γ coincidences as a function of the direction of emission of the γ rays relative to the beam axis with the α-particle detector fixed at 180° relative to the beam. The measurements were performed at five angles of the γ-ray counter between 26° and 90°. These angles were chosen in a random order to reduce the effect of possible drifts in the system. The measurements were also performed on the opposite side of the chamber to compensate for any asymmetries associated with rotation of the target, fluctuations in the beam, or in positioning of the γ-ray counter. At least two or more of the angular measurements were repeated on each side of the target chamber. The target was oriented at 30° to the beam axis and faced the NaI detector. The target was rotated when the NaI detector was moved to the opposite side of the chamber. This avoided any absorption of low-energy γ rays by the stainless-steel target ring. Each angular measurement lasted for five hours. Beam currents of 100 nA were employed. The measurements performed at a 3He energy of 6.37 MeV were repeated once and those at 8.05 MeV were repeated twice.

f. Extraction of the Angular Correlations. The total and accidental coincidence spectra obtained for each angular measurement were recorded on magnetic tape and later read back into the computer for analysis. Figure 5 shows the x- and y-axis projections of a typical two-dimensional spectrum obtained at θ = 90° and at a 3He energy of 8.05 MeV.
Figure 5. X- and Y-Axis Projections of 2-Dimensional Coincidence Spectrum from the $^{24}$Mg($^3$He, a $\gamma$)$^{23}$Mg Reaction. The x-axis projections, labeled A, B, and C, are the $\gamma$ rays coincident with selected portions of the $\alpha$-particle spectrum shown on the left and denoted by A, B, and C. The $\alpha$-particle groups are labeled by the energy in MeV of the level in $^{23}$Mg to which they lead. The $\gamma$ rays are labeled by transition energies given in MeV. The $^{23}$Mg level schemes, appropriate to the selected regions A, B, and C of the particle spectrum, are also shown. These spectra are the result of a single measurement made at $\theta = 80^\circ$ and at a $^3$He energy of 8.05 MeV. Accidental coincidences have been subtracted.
The α-particle spectrum, displayed on the y axis, was obtained by performing a block sum of the entire 64 x 128-channel spectrum and projecting the result onto the y axis. The coincident γ-ray spectrum associated with a particular α-particle group was obtained by summing the region of the y axis corresponding to that particle group and projecting the sum onto the x axis. The accidental coincidence spectrum was subtracted by reading the accidental coincidence spectrum for the same run into the computer and performing the block sum with the same y-axis coordinates. The resulting x-axis projection was then subtracted from the initial projection resulting in the true coincidence spectra shown in Figure 5. Summed γ-ray spectra, which were used to determine branching ratios, were obtained by accumulating the γ-ray spectra recorded at all angles for a particular α-particle group. The corresponding accidental coincidence spectra were subtracted.

The individual γ-ray spectra corresponding to each particle group were plotted, and in cases where gain shifts existed on the γ-ray axis the energy calibration for each spectrum was used to sum a particular energy interval for each γ ray. In some cases the γ-ray spectra were each normalized to the same gain before summing. These sums were then normalized using the monitor spectra discussed previously. Gamma-ray spectral shapes were used where necessary to extract the angular correlations. The procedure used to analyse each particular correlation will be discussed in more detail in Chapter III.
B. Ge(Li) Detector Coincidence Measurements

1. General

High resolution $\gamma$-ray coincidence measurements were performed with the Ge(Li) detector to help determine the decay modes of the low-lying levels of $^{23}$Mg and particularly the 2.712-, 2.763-MeV doublet. These measurements also led to a confirmation of the energies of many of the $^{23}$Mg levels.

2. The Ge(Li) Detector

The Ge detector was an ORTEC lithium-drifted germanium diode in the form of a right circular cylinder with both ends open. The total active volume was 20.6 cc. The measured total resolution for 1.33-MeV $\gamma$ rays using an ORTEC Model 118A preamplifier and Model 410 pulse-shaping amplifier (2 $\mu$s time constant, single RC shaped) was 4.1 keV FWHM. The detector was mounted on an ORTEC Model 81 right angle cryostat which contained liquid nitrogen. The detector-cryostat assembly was furnished with a Model 118A preamplifier mounted directly to the cryostat to minimize the preamplifier input capacitance.

3. Target Chamber

The target chamber was a nickel-plated copper cylinder 7.63-cm in diameter with 3.17-mm thick walls. Aluminum plates 9.53-mm thick were sealed to the top and bottom of the chamber by o-rings. Targets
were mounted in this chamber as in the correlation chamber. The annular counter was mounted at 180° to the beam axis and was supported by a holder similar to that used in the correlation chamber. The Ge(Li) detector could be positioned flush with the wall of the chamber. Beam collimation was provided by two 1.59-mm diameter Ta collimators and a 3.17-mm diameter anti-scattering collimator inserted in the annular counter support. The beam was collected in a Faraday cup located approximately 1.5 m from the chamber.

4. Electronics

The electronic circuit used to observe coincidences between a particles detected in the annular counter and γ rays detected with the Ge detector was similar to that used for the angular-correlation measurements. The main difference between the two circuits was the manner in which the fast signals were obtained from the Ge detector. A block diagram of the electronics associated with this detector is illustrated in Figure 6.

Following a suggestion by ORTEC, the fast-γ signal was obtained from an ORTEC time pickoff inserted into the circuit as shown in the figure. By placing the pickoff between the preamplifier and pulse-shaping amplifier a low triggering threshold may be achieved with little energy resolution degradation and with the time resolution affected only slightly by the preamplifier rise time. The plug-in inductor (L3) in the ORTEC Model 260 time pickoff unit was changed from 0.56 μh to 5.6 μh
Figure 6. Block Diagram of Electronics Associated with the Ge(Li) Detector.
to prevent multiple trigger from the wide input signal. The fast-$\gamma$ pulses were fed through two Chronetics Model 101 discriminators and then into a Chronetics Model 107 coincidence unit.

Pulses from the ORTEC Model 118A preamplifier, used for energy analysis, were amplified by the linear amplifier stage of an ORTEC Model 220 linear pulse analysis system and then by the bipolar stage. The bipolar pulse permits a much higher counting rate with less baseline distortion than would be possible with a unipolar signal. The first and second differentiation and integration time constants employed were each 1 $\mu$s. Signals from the bipolar amplifier were delayed 1.4 $\mu$s by the delay amplifier and fed into the linear gate which was gated with the fast coincidence signal. Gating the linear gate with this low counting rate signal reduced the number of pulses being fed to the $\gamma$-ray ADC and led to better energy resolution. In addition, the linear gate contained a dc baseline restoration network without which good energy resolution could not have been achieved. The linear gate pulses were fed into a biased amplifier followed by a pulse stretcher. The pulse stretcher stretches the peak voltage of the input signal, thereby reducing the bandwidth requirements of the ADC which in turn leads to improved linearity. Finally, the stretched pulses served as the input for the 2048-channel Victoreen ADC.

For these measurements the TPH converter spectrum was stored along with the $\alpha$-$\gamma$ coincidence spectrum so that the computer served as a three-parameter analyser. This procedure was necessary in order to
record the low-energy $\gamma$ rays. A two-dimensional spectrum of the TPH converter signals versus $\gamma$-ray energy revealed that a much longer resolving time was required to detect $\alpha-\gamma$ coincidences for $\gamma$ rays with energies below approximately 1 MeV than needed to detect higher energy ones. As with the NaI crystal, this effect is due to the long rise time of low-energy pulses from the Ge detector. In order to record $\gamma$ rays with energies in the range 0.4 to 4.5 MeV a resolving time of approximately 100 ns would have been required. This would cause an inordinate number of accidental coincidences in the spectrum, masking the true coincidence events. However, by storing the TPH converter spectrum along with the coincidence spectrum smaller time intervals, e.g. 15 ns, could be chosen for a particular $\gamma$ ray of interest. In this way a greatly reduced accidental/true ratio was obtained. Using three-parameter analysis the coincident $\gamma$ rays could be labeled with the $\alpha$-particle coordinates to which they corresponded and also with their time coordinates. This method of analysis clearly provides much more information than with the two-parameter analysis used for the correlation measurements.

The fast-$\alpha$ and fast-$\gamma$ signals were both clipped to a pulse width of 100 ns and fed into the TPH converter after delaying the $\alpha$ signal 31 ns relative to the $\gamma$ signal. The timing signals from the converter were amplified and fed into one side of the TMC ADC. The linear-$\alpha$ signals were routed into the other half of the ADC and the linear-$\gamma$ signals were fed into the Victoreen ADC. Each ADC was gated by the fast coincidence gate pulse.
The data storage program provided 256 channels for the TPH axis, 1024 channels for the α-energy axis, and 2048 channels for the γ-energy axis. The TPH signals were delayed relative to the linear-γ signals so that the peak in the TPH spectrum fell approximately in the middle of the axis. This allowed the full time interval on the TPH axis to be seen.

The target chamber and annular detector were aligned so that the beam was centered on a carbon-backed $^{24}$Mg target. The Ge detector was placed flush with the wall of the chamber and at $90^\circ$ to the beam axis. The coincidence measurement was performed at a $^3$He energy of 8.00 MeV. Each measurement lasted 5 hours after which the data were recorded on magnetic tape. A total of 150 hours of data were taken. There were no significant gain changes during this time on either the α- or γ-energy axis. Energy calibration spectra were recorded at frequent intervals using a RdTh (ThC") source. This source emitted γ-rays with energies of $2.61425 \pm 0.00050$ MeV (photopeak) (41), $1.59237$ MeV (double escape peak), and $0.51094 \pm 0.00007$ MeV (annihilation radiation). (42) These γ-rays were used for the calibration. The γ-ray energy axis had the range of approximately 300 keV to 4.5 MeV with an energy dispersion of 2.070 keV/channel. The resolution for a $1.751$-MeV γ-ray was 19 keV FWHM.

5. Reduction of the Data

The first step in analysing the three-dimensional data was to read several runs into the computer to determine the coordinates of the
α-particle groups. The 2048-channel γ-ray axis was then divided, arbitrarily, into three regions: channels 37-400, 400-800, and 800-2048. The TPH spectra for several runs for γ rays in the channel range 800-2048 and for a particular set of particle coordinates were read into the computer. The peak observed then defined the time coordinates for this particular γ-ray energy interval. Each of the five hour runs was then searched for the γ rays with these particle and time coordinates. The resulting γ-ray spectra were summed. This procedure was followed for the γ-ray channels 400-800 and 37-400. The three summed spectra resulted in a final composite spectrum. A three-point running average of this final spectrum was made to reduce the effects of point scatter in the data. The peaks of known energy from the ThC\textsuperscript{u} spectrum were used to obtain an energy calibration curve containing both a linear term and a quadratic term which accounted for any non-linearity in the ADC. The center of the coincident γ-ray peaks was estimated and the energies of these peaks found from the calibration curve.
Chapter III

DATA ANALYSIS AND EXPERIMENTAL RESULTS

A. Method of Analysis

1. Gamma-Ray Angular Distribution Equations

The experimental method used for the $\alpha-\gamma$ correlation measurements and the method of theoretical analysis were primarily developed by Litherland and Ferguson. (15) The basis of their procedure is to exploit the simplifications brought about by making the reacting system in a nuclear reaction axially symmetric. A $\gamma$-ray emitting state formed in this way is aligned and can usually be described by a small number of population parameters for the magnetic substates of the residual nucleus. This type of alignment can be achieved in a nuclear reaction of the type $X(h_1, h_2)Y^*$, $Y^* \rightarrow Y + \gamma$ if $h_2$ is detected in a counter axially symmetric about the beam axis. This reaction is illustrated in Figure 7 where it is schematically presented in the form

$$a \, \ell_1 \, s_1 \rightarrow b \rightarrow \tau_1 \, \ell_2 \, s_2.$$  

Here $a$ and $b$ symbolize, respectively, the spin of the target nucleus $X$.

(43)
Figure 7. Schematic Reaction-Angular Momenta Diagram.
and the spin of some intermediate state assumed to be formed in the reaction. The state \( b \) is usually not a level of sharp spin and parity but is normally several overlapping levels. The spin of the \( \gamma \)-ray emitting state \( Y^* \) is symbolized by \( J_1 \) and that of the final state by \( J_2 \). The orbital angular momentum and intrinsic spin of the incident and emergent particles, \( h_1 \) and \( h_2 \), respectively, are represented by the symbols \( \lambda_1, s_1, \lambda_2, \) and \( s_2 \). The interfering multipolarities of the de-excitation \( \gamma \) rays are denoted by \( L \) and \( L' \). The incident particle is assumed to be unpolarized. The axis of quantization is taken to be the beam axis. The appropriate angular momentum relations for this reaction are

\[
\begin{align*}
\overrightarrow{R}_1 &= \overrightarrow{a} + \overrightarrow{s}_1 , \\
\overrightarrow{b} &= \overrightarrow{R}_1 + \overrightarrow{\lambda}_1 = \overrightarrow{R}_2 + \overrightarrow{\lambda}_2 , \\
\overrightarrow{R}_2 &= \overrightarrow{J}_1 + \overrightarrow{s}_2
\end{align*}
\]

and

\[
\overrightarrow{R}_1 = \overrightarrow{a} + \overrightarrow{s}_1 + \overrightarrow{\lambda}_1 - \overrightarrow{s}_2 - \overrightarrow{\lambda}_2 .
\]

If the state \( Y^* \) is formed by the absorption of an unpolarized particle in the direction of the quantization axis (the beam axis) and followed by the emission of a second particle which is detected along this same axis, then the magnetic substates of the final state which can be populated do not exceed the sum of the spins of \( X, h_1, \) and \( h_2 \). This theorem follows from the fact that the orbital angular momentum contained in plane waves in the direc-
tion of the beam axis has zero projection along this axis. This is clear since the orbital angular momentum of a particle, \( \mathbf{l} = r \times \mathbf{p} \), is always perpendicular to the linear momentum \( \mathbf{p} \) and hence has zero projection on the \( \mathbf{p} \) axis. This same point can be obtained if the incident and emergent beams are treated as plane waves and expanded in spherical harmonics \( Y^l_m(\cos \theta) \). Then with \( \theta = 0^\circ \) or \( 180^\circ \) the only contributing terms to this expansion are those for which \( m = 0 \), i.e., zero projection. If the magnetic quantum numbers of the \( \gamma \)-ray emitting state of spin \( J_1 \) are denoted by \( a \), then from Equation 2 with \( l_1 \) and \( l_2 \) equal to zero along the beam axis the maximum value of \( a \) is

\[
\alpha_{\text{max}} = a + s_1 + s_2 .
\]

This equation is the basis for the previous comment that an aligned state may be described by a small number, depending on the reaction producing the alignment, of population parameters for the magnetic substates of the residual nucleus.

Litherland and Ferguson (15) give the expression for the angular distribution of a primary \( \gamma \) ray originating from a nucleus aligned in the manner described. Poletti and Warburton (43) have presented the identical relation in a form which is more amenable to analysis and it is this formulation which will be presented here.

The angular distribution of a \( \gamma \) ray which decays from a state of spin \( J_1 \) and magnetic quantum numbers \( a \) to a state of spin \( J_2 \) is given by
\[ W(\theta) = \sum_k a_k P_k(\cos \theta) = \sum_k \rho_k(J_1) F_k(J_1 J_2) Q_k P_k(\cos \theta) \] (4)

where \( \theta \) is the angle between the beam axis and the direction of emission of the \( \gamma \) rays. The \( P_k(\cos \theta) \) is a Legendre polynomial and the summation index \( k \) assumes even values from 0 to \( 2J_1 \). The \( \rho_k(J_1) \) are statistical tensors which describe the alignment of the initial state and the \( F_k(J_1 J_2) \) depend only upon the \( \gamma \)-ray cascade. The \( Q_k \) are attenuation coefficients for the \( \gamma \)-ray detector. The calculation of these quantities for the present study, outlined in Reference 32, yielded the values 1.0, 0.9315, 0.7858, 0.5929, and 0.3882 for \( k \) ranging from 0 to 8, respectively.

The \( \rho_k(J_1) \) are given by a weighted sum over the population parameters, \( P(a) \), of the \( 2J_1 + 1 \) magnetic substates associated with \( J_1 \):

\[ \rho_k(J_1) = \sum_a \rho_k(J_1, a) P(a) \] (5)

where \( a \) extends from \(-J_1\) to \(J_1\). The \( P(a) \) specify the fraction of aligned nuclei in the magnetic substates of \( J_1 \) and have the important restriction that they must be positive. The normalization is such that \( \sum_a P(a) = 1 \) so that the \( P(a) \) lie in the range \( 0 \leq P(a) \leq 1/2 \) if the \( a_{\text{max}} \) of Equation 3 are non-zero. The \( \rho_k(J_1, a) \) in Equation 5 are given by the ratio

\[ \langle J_1 a, J_1 - a | k 0 \rangle / \langle J_1 a, J_1 - a | 0 0 \rangle. \]

The state \( J_1 \) is assumed to be unpolarized so that \( P(a) = P(-a) \) and the \( \gamma \)-ray angular distributions from the \( \pm a \) magnetic substates are identical.

The \( \gamma \)-ray angular distribution from the state \( J_1 \) is assumed
to contain multipolarities \( L \) and \( L' \) where \( L \) and \( L' \) take on integer values from \( |J_1 - J_2| \) to \( J_1 + J_2 \). If only the two lowest allowed multipole admixtures are assumed to be present in the \( \gamma \)-ray transition, as is assumed in this study, then the \( F_k(J_1 J_2) \) of Equation 4 are given by

\[
F_k(J_1 J_2) = \frac{F_k(LL J_2 J_1) + 2x F_k(LL' J_2 J_1) + x^2 F_k(L'L' J_2 J_1)}{1 + x^2}
\]  

(6)

where \( L \) is the lowest allowed value and \( L' = L + 1 \). The multipole mixing ratio, \( x \), is given by the ratio of reduced matrix elements:

\[
x = \frac{\langle J_2 || L' || J_1 \rangle}{\langle J_2 || L || J_1 \rangle}.
\]  

(7)

The \( F_k(LL' J_2 J_1) \) of Equation 6 are tabulated functions given in Reference 43. These coefficients further limit \( k \) to \( k \leq \min(2L, 2L', 2J_1) \).

The phase convention for \( x \) chosen here is Convention II of Poletti and Start (7) which results from choosing a positive sign for the term linear in \( x \) in Equation 6. This convention, opposite to that of Reference 15 and 43, was chosen as a result of the careful study of phases of mixing ratios carried out by Poletti and Start concerning \( \gamma \)-ray transitions from the first two excited states of \(^{21}\text{Ne}, ^{21}\text{Na}, \) and \(^{23}\text{Na} \). The phases obtained for \(^{25}\text{Mg} \) should then be more readily compared with the phases found for the other \( N \) or \( Z = 11 \) nuclei. Rose and Brink (44) have recently treated the whole problem of the phases of mixing ratios by making a phase consistent derivation of angular distributions for \( \gamma \) rays emitted in the decay of an aligned
nuclear state.

Using Convention II the phases of the mixing ratios reported in this work are given assuming that the multipole mixture is a "natural one", i.e., a mixed E(L + 1) + ML transition, regardless of the known or suspected nature of the radiation. The phases of M2 + E1 mixtures presented here therefore are of the wrong sign and account must be taken of this when comparing these phases with theoretical predictions.

The angular distribution of the second $\gamma$ ray in a cascade of the type $J_1 \rightarrow J_2 \rightarrow J_3$ with the first $\gamma$ ray unobserved is given as

$$W(\theta) = \sum_k \rho_k(J_1) U_k(J_1J_2) F_k(J_2J_3) Q_k P_k \cos \theta$$  \hspace{1cm} (8)

where for the two lowest allowed multipolarities

$$U_k(J_1J_2) = \frac{U_k(LJ_1J_2) + x^2 U_k(L'J_1J_2)}{1 + x^2}$$  \hspace{1cm} (9)

The $U_k(LJ_1J_2)$ are tabulated functions (43) and the other quantities have been previously defined.

2. Data Fitting Procedure

In the $^{24}\text{Mg}(^3\text{He}, \alpha)^{23}\text{Mg}$ reaction with the $\alpha$ particles detected at $180^\circ$ with respect to the beam direction it is clear from Equation 3 that only the $\alpha =\pm 1/2$ magnetic substates of $^{23}\text{Mg}$ can be populated and each component is assumed to be equally populated. The angular correlations of the de-excitation $\gamma$ rays are, for this situation, independent of the details of the formation of the excited states of $^{23}\text{Mg}$. Consequently, the
angular distribution coefficients, i.e., the $a_k$ coefficients of Equation 4, depend, essentially, only upon the spins of the levels of $^{23}\text{Mg}$ and the $\gamma$-ray mixing ratios.

The measured $\gamma$-ray angular distribution for a transition of the type $J_1 \rightarrow J_2$ was fitted with the theoretical distribution given in Equation 4 using a linear least-squares fitting code run on the DDP-224 computer. In this procedure the spins $J_1$ and $J_2$ take on all allowed values and for a given set of spins the fit is made for discrete values of $x$. The best fit corresponds to that value (or values) of $x$ which minimizes $\chi^2$, where

$$\chi^2 = \frac{1}{n} \sum_{i} \left[ \frac{Y(\theta_i) - W(\theta_i)}{\Delta Y(\theta_i)} \right]^2 .$$

(10)

In this equation $\Delta Y(\theta_i)$ is the statistical uncertainty assigned to the $\gamma$-ray yield $Y(\theta_i)$ at angle $\theta_i$ and $n$ is the number of degrees of freedom. This latter quantity is given by the number of data points less the number of variable parameters which for the present case is the normalization between $Y(\theta)$ and $W(\theta)$. The mixing ratio is not considered to be a variable parameter in the fit since $x$ is fixed for each calculation of $\chi^2$. A plot of $\chi^2$ versus $x$ will then show dips corresponding to possible solutions for the assumed spins $J_1$ and $J_2$. The relative probability that a particular set of $J_1$, $J_2$, and $x$ is the correct solution can be found by referring to $\chi^2$-probability tables. (45) In this study for a fit to be regarded as an acceptable one, a value of $\chi^2$ corresponding to a confidence level of
0.1% or greater had to be achieved. For five angles, \( n = 4 \) and \( \chi^2 (0.1\%) = 4.6. \)

Since the limits on \( x \) are \( \pm \infty \), the variable used in the fitting procedure was \( \arctan x \) which was varied in \( 5^\circ \) steps from \(-85^\circ\) to \(85^\circ\). Each minimum in the \( \chi^2 \) curves was then studied by varying \( \arctan x \) in \( 1^\circ \) steps. The computer program could only accept one set of data at a time but could treat transitions of the type \( J_1 \rightarrow J_2 \rightarrow J_3. \) In this case the correlation of one of the \( \gamma \) rays of the cascade is fitted using Equation 8 with the mixing ratio of the unobserved \( \gamma \) ray fixed. In order to present the experimental data in a compact form the program initially fitted each angular correlation with a Legendre polynomial expansion of the form

\[
W(\theta) = a_0 + a_2 P_2 (\cos \theta) + a_4 P_4 (\cos \theta) .
\]  

(11)

The \( a_0 \) coefficients generated by this least-squares fit were used in conjunction with tables of efficiencies and photopeak fractions for \( \gamma \) rays detected in Na(Tl) crystals (46) to obtain branching ratios for the decay modes of \( ^{23}\text{Mg} \).

The statement that only the \( \alpha = \pm 1/2 \) substates of \( ^{23}\text{Mg} \) are populated using the \( ^{24}\text{Mg} (^3\text{He}, \alpha)^{23}\text{Mg} \) with collinear geometry is not quite true. The \( ^3\text{He} \) beam from the accelerator has a finite emittance which implies that not all of the \( ^3\text{He} \) ions impinging upon the target will be going in exactly the same direction, i.e., some of the particles will have a projection of orbital angular momentum along the beam axis. In addition, the
annular counter is not an ideal counter located at 180° with respect to the beam direction but is an annulus whose inner and outer edges subtend angles of 177° and 173°, respectively, at the target center. Litherland and Ferguson (15) have considered these effects which, for the present case, allow magnetic substates with $\alpha > 1/2$ to be populated. Their results indicate that the population of the $\alpha = \pm 3/2$ substates should be proportional to $L^2 \epsilon^2$ where $L$ is the orbital angular momentum of the $^3$He beam and $\epsilon$ is the half angle in radians subtended by the annular counter at the target center. This result reflects the fact that the response of the system is proportional to the approximate area of the counter, i.e., $\pi \epsilon^2$. With $\epsilon = 0.015$ radians and $L = 2$ there should be approximately a 5% population of the $\alpha = \pm 3/2$ substates. To investigate this effect the $\chi^2$ test was performed first with $P(3/2) = 0$. The minima in the resulting $\chi^2$ curves were then studied by fitting the data with $P(3/2) = 0.05 P(1/2)$. This procedure was followed in fitting each of the angular correlations and the so called Finite Size Effect (FSE) was found to be negligible.

B. Experimental Results

1. General

For convenience in discussing the experimental results the levels of $^{23}$Mg will be labeled by numbers beginning with 0 for the ground state. The mention of the parameter $x_{10}$, for example, will then refer to the mixing ratio of the $\gamma$ ray for the transition from the first-excited state (1) to
the ground state (0). The symbol J will refer to the total angular momentum (spin) of the initial nuclear state in a transition. For cases where more than one transition is involved the symbol J will be labeled with the number of the particular level being discussed. The phases of mixing ratios presented here will be strictly experimental phases and will conform to Convention II. (7) The $^{23}$Mg level scheme was taken from the work of Hinds and Middleton (8) who studied the $^{24}$Mg($^{3}$He, α)$^{23}$Mg reaction using a broad-range magnetic spectrometer to detect the α particles. Spins of 11/2 or greater were not considered in the following analysis as they are not expected on the basis of the systematics of neighboring nuclei and because they would lead to level lifetimes incompatible with the effective resolving time of the coincidence circuit. This last remark will be discussed in Section 3.

2. The $^{23}$Mg Ground State

Most of the published information concerning the $^{23}$Mg ground state has been summarized by Endt and Van der Leun. (47) The $^{23}$Mg nucleus is radioactive with a half-life of 12 s. The β⁻-decay proceeds mainly to the $^{23}$Na ground state with approximately 91% intensity. This decay is super-allowed (log ft = 3.7) which determines the ground-state spin and parity of $^{23}$Mg as $^3I^\pi = 3/2^+$. Gamma-annihilation coincidence measurements indicate a 9% branch to the $^{23}$Na 0.44-MeV first-excited state with log ft = 4.5. The angular distribution from the $^{24}$Mg($^{3}$He, α)$^{23}$Mg reaction has been measured at $^{3}$He energies of 5.5 MeV (10), 10 and
12 MeV (24), and at 15 MeV. (16) The analysis of these angular distributions leads to an \( l_n = 2 \) assignment for the picked-up neutron and a \( J^\pi = (3/2, \ 5/2)^+ \) assignment to the ground state. The \( d_o \) angular distribution from the \(^{24}\text{Mg}(p, d)\ ^{23}\text{Mg}\) reaction has been measured at \( E_p = 33.6 \) MeV and is also characterized by \( l_n = 2 \) in agreement with the \(^3\text{He}, d\) results. (18) In the following analysis the ground state spin of \(^{23}\text{Mg}\) will be assigned \( J^\pi = 3/2^+ \). The mirror nucleus \(^{23}\text{Na}\) also has \( J^\pi = 3/2^+ \) for the ground state as does \(^{21}\text{Ne}\) and \(^{21}\text{Na}\). (6)

3. The 0.451-MeV Level

The decay of the 0.451-MeV first-excited state was studied at \(^3\text{He}\) bombarding energies of 6.37 and 8.05 MeV. A typical coincidence decay spectrum recorded in the NaI crystal is shown in Figure 8. From the spectrum recorded with the Ge(Li) detector, the energy of this transition was found to be \( 0.451 \pm 0.004 \) MeV in agreement with the value \( 0.451 \pm 0.010 \) MeV given in Ref. 8. Two separate angular correlations taken at a \(^3\text{He}\) energy of 6.37 MeV and one taken at 8.05 MeV were each fitted by the method of least squares with a Legendre polynomial expansion including even terms up to \( P_4 (\cos \theta) \). The average value of the coefficients of \( P_2 (\cos \theta) \) and \( P_4 (\cos \theta) \) from the three measurements was \( a_2/a_0 = -0.46 \pm 0.02 \) and \( a_4/a_0 = -0.02 \pm 0.02 \). The errors given here are statistical. No finite geometry corrections were made in obtaining these coefficients.
Figure 8. The 0.451-MeV Level Gamma-Ray Decay Spectrum.

This figure illustrates the spectrum of γ rays observed at 26° to the beam in coincidence with α particles leading to the 23Mg 0.451-MeV level in the 24Mg(3He, αγ)23Mg reaction at a 3He energy of 6.37 MeV. The solid circles represent true coincidences and the open circles represent accidental coincidences. A true-to-accidental ratio of approximately four was obtained for this transition.
0.451-MeV LEVEL $\gamma$-RAY SPECTRUM

COUNTS / CHANNEL

CHANNEL NUMBER

0.451

100

0

$^{23}\text{Mg}$

• TRUE

○ ACCIDENTAL
The large anisotropy observed eliminates \( J = 1/2 \) for this level and the absence of a marked \( P_4(\cos \theta) \) term is incompatible with \( J = 7/2 \). Spin assignments of \( 9/2 \) or greater were not considered as probable for this level since the lifetime of the level would then be long compared with the resolving time of the coincidence circuit. Assuming for the moment that the 0.451-MeV level has \( J = 3/2 \), then the fastest possible transition would have E3 multipolarity. The Weisskopf estimate for such a transition, \( (48) \) using a radius of 1.2 \( f \), leads to a mean lifetime of \( \tau_m \approx 10^{-2} \) s. Including a factor of 100 for possible enhancement leads to an upper bound of \( \tau_m \approx 10^{-4} \) s for the transition speed. The relative number of decaying nuclei remaining after a time \( t \) is \( e^{-t/\tau_m} \). With \( t \) equal to the effective resolving time of \( 10^{-8} \) s, the relative number of nuclei decaying during this time is quite small. In addition, the nuclear orientation of a level with \( \tau_m \approx 10^{-4} \) s would not be expected to be preserved. Spins of \( 9/2 \) or greater are also excluded by an \( l_n = 2 \) assignment to this level, to be discussed later, and also on the basis of the systematics of neighboring nuclei. From these arguments the experimental data can only be satisfied by \( J = 3/2 \) or 5/2.

The \( \chi^2 \) test was applied to each of the three correlations assuming \( J \) to be 1/2, 3/2, 5/2, and 7/2. Minima were found in each of the resulting curves although these solutions did not dip below the 0.1\% confidence level. The fact that better fits were not achieved was believed to be due to instrumental difficulties associated with detecting this low-energy
\( \gamma \) ray. The \( \chi^2 \) test only allows for statistical error whereas the statistics obtained for each of the correlations was quite good. Rather than average the mixing ratios obtained from the three separate fits, the three \( \chi^2 \) curves were added together after weighing each with the appropriate number of degrees of freedom. Two of the angular correlations were taken at five angles to the beam \((n = 4)\) and the third one at four angles \((n = 3)\). The combined \( \chi^2 \) was then taken to be

\[
\chi^2 = (4/11) \chi_1^2 + (4/11) \chi_2^2 + (3/11) \chi_3^2
\]

(12)

where \( \chi_1^2 \), \( \chi_2^2 \), and \( \chi_3^2 \) are the individual \( \chi^2 \) fits from the three correlations.

From the combined fit, shown in Figure 9, the most probable solutions are \( J = 5/2 \) with \( x_{10} = \tan (-4 \pm 3)^0 = -0.070 \pm 0.052 \) and \( J = 3/2 \) with \( x_{10} = \tan (-62 \pm 5)^0 = -1.88 \pm 0.09 \) or \( \tan (-42 \pm 5)^0 = -0.90 \pm 0.09 \). The solutions for \( J = 1/2 \) and \( 7/2 \) are seen in the figure to be excluded with a high probability. The errors on \( x_{10} \) were calculated from the difference between the value of \( x_{10} \) found from the combined fit and the \( x_{10} \) values found from the individual fits using the method of weighted averages as discussed in the propagation of errors. (49) Without additional information, e.g., lifetime measurements, it is not possible to discriminate between the two solutions \( J = 3/2 \) and \( 5/2 \) for the 0.451-MeV level.

The \( \alpha \) angular distribution from the \(^{24}\text{Mg}(^3\text{He}, \alpha)^{23}\text{Mg} \) reaction has been measured at \(^3\text{He} \) energies of 5.5 MeV (10), 10 and 12 MeV (24),
Figure 9.  $E_\gamma = 0.45$-MeV Angular Correlation and Combined $\chi^2$ versus Arctan $x_{10}$ for the 0.451-MeV Level.

The combined $\chi^2$ was formed by summing the $\chi^2$ fits to each of the three angular correlations measured for the 0.451-MeV level. Each $\chi^2$ was weighted with the appropriate number of degrees of freedom as described in the text. Spins of $J = 1/2$, $3/2$, $5/2$, and $7/2$ were assumed for the 0.451-MeV level with $J = 3/2$ and $5/2$ yielding the most probable solutions. At the top of the figure is shown the angular correlation measured at a $^3$He energy of 6.37 MeV. The solid line through these data is the fit for $J = 5/2$ and arctan $x_{10} = -4^\circ$. For each of the three fits $P(3/2) = 0$ with the FSE being insignificant. Note that the data are plotted as a function of $\cos^2 \theta$.  


and at 15 MeV. (16) In each case the distribution was characterized by \( l_n = 2 \) which leads to a \( J^\pi = (3/2, 5/2)^+ \) assignment to the 0.451-MeV level. Dubois and Earwaker (24) report a noticeable difference in the shape of the \( a_0 \) and \( a_1 \) angular distributions and remark that the effect may be due to a \( J \)-dependence of the pickup reaction. (50) Kozub (51) has measured the \( d_0 \) and \( d_1 \) angular distributions from the \( ^{24}\text{Mg}(p, d)^{23}\text{Mg} \) reaction at \( E_p = 33.6 \) MeV and found that both distributions were characterized by \( I_n = 2. \) A \( J \)-dependence effect was observed which appeared as a difference in the overall slope of the differential cross-section versus angle curves for the two distributions. In contrast to \(^3\text{He}, a\) reactions, \( J \)-dependence effects in \((p, d)\) reactions have been shown to be quite distinct and pronounced as demonstrated by the work of Glassenburger (52) and Legg and Rost. (53) The work of Kozub thus lends strong support to a \( J^\pi = 5/2^+ \) assignment to the \(^{23}\text{Mg} 0.451\)-MeV level.

Further support for this assignment comes from experiments on the analogous state in the mirror nucleus \(^{22}\text{Na} \). Poletti and Start (7) have made a rigorous \( 5/2^+ \) assignment to the first excited state in \(^{23}\text{Na} \) which differs by less than 50 keV from the energy of the \(^{23}\text{Mg} 0.451\)-MeV level. They found that \( x_{10} = 0.08 \pm 0.02 \). It is interesting to note that the \(^{21}\text{Ne} \) first excited state at 0.35 MeV has \( J^\pi = 5/2^+ \) with \( x_{10} = -0.08 \pm 0.03 \) (54) and the analogous level in \(^{21}\text{Na} \) at 0.34 MeV has \( J^\pi = 5/2^+ \) with \( x_{10} = 0.05 \pm 0.05 \). (7) Additional support for the \( 5/2^+ \) assignment comes from the theoretical prediction (55) that when collective effects
enhance the transition strengths for E2 radiations it is expected that E2/M1 mixing ratios for corresponding mirror transitions will most probably differ in sign. For a 5/2\(^+\) assignment the E2/M1 mixing ratio \(x_{10}\) in \(^{23}\)Na and \(^{23}\)Mg do indeed differ in sign whereas for a 3/2\(^+\) assignment the phase of \(x_{10}\) is negative for both nuclei.

4. The 2.904-MeV Level

The analysis of the angular correlations from the 2.904-MeV level will be presented at this time as it will permit a unique spin assignment of \(J = 5/2\) for the 0.451-MeV level.

This level was studied at a \(^3\)He energy of 6.37 MeV as the yield for this state, as shown in Figure 3, was greater than that for the nearby fourth and fifth excited states at 2.712 and 2.768 MeV, respectively. The contribution to the 2.904-MeV level \(\gamma\)-ray spectra from the low-energy portion of these two states was considered to be negligible. The coincident \(\gamma\)-ray spectrum, composed of nine spectra summed over five different angles to the beam, is shown in Figure 10. The 2.904-MeV level was found to decay (60 \(\pm\) 4)\% to the ground state and (40 \(\pm\) 4)\% to the first excited state. No \(\gamma\) rays resulting from the decay of the 2.712- and 2.768-MeV levels can be identified in this spectrum. A 2.75-MeV \(\gamma\)-ray spectral shape from a \(^{24}\)Na source was used as a spectral shape for the 2.90-MeV \(\gamma\) ray in order to extract the 2.45-MeV \(\gamma\) ray. These branching ratios were obtained from the \(a_0\) coefficients of the Legendre polynomial fit to the 2.90- and 2.45-MeV angular correlations. The errors reflect
Figure 10. The 2.904-MeV Level Gamma-Ray Decay Spectrum. This figure illustrates the spectrum of $\gamma$ rays in coincidence with the $\alpha$ group populating the $^{23}$Mg 2.904-MeV level in the $^{24}$Mg($^3$He, $\alpha$ $^{23}$Mg reaction at a $^3$He energy of 6.37 MeV. This spectrum is the sum of nine spectra taken at five different angles to the beam. The $\gamma$-ray peaks correspond to the transitions indicated in the decay scheme. A 2.90-MeV spectral shape, shown sketched in, was used to extract the 2.45-MeV $\gamma$ ray. Accidental coincidences have been subtracted.
2.904-MeV LEVEL $\gamma$-RAY SPECTRUM

COUNTS / CHANNEL

CHANNEL NUMBER

0 0.45 2.904 60±4 40±4 0.45 0 $^{23}$Mg

2.45 2.90
both statistical uncertainty and the uncertainty in extracting the 2.45-MeV γ-ray.

From the spectrum of γ rays detected at a ³⁷He energy of 8 MeV in the Ge(Li) detector, and in coincidence with α particles leading to the 2.712-, 2.768-, and 2.904-MeV levels, γ rays with energies of 2.902 ± 0.008 MeV and 2.454 ± 0.008 MeV could be identified as the primary transitions from the decay of the 2.904-MeV level. The 2.904-MeV value is in agreement with the value 2.904 ± 0.010 MeV given in Ref. 8 for the energy of the sixth excited state. This level was weakly populated at the ³⁷He energy of 8 MeV as shown in the Ge(Li) detector spectrum of Figure 20.

The 2.90-MeV angular correlation was anisotropic and was characterized by \( a_2/a_0 = 0.11 \pm 0.03 \) and \( a_4/a_0 = 0.15 \pm 0.04 \). When the \( \chi^2 \) test was applied to this correlation, with assumed spins of \( J = 1/2, 3/2, 5/2, 7/2, \) and \( 9/2 \) for the 2.904-MeV level, only the solutions \( J = 3/2 \) with \( x_{60} = \tan (-12 \pm 2)^{\circ} \) (\( = 0.213 \pm 0.035 \)) and \( J = 5/2 \) with \( x_{60} = \tan (14 \pm 2)^{\circ} \) (\( = 0.249 \pm 0.035 \)) had a confidence level of 0.1% or less. The errors were taken to be the width of the \( \chi^2 \) curve for a given \( J \) between the minimum and the 0.1% confidence level. This criterion was used to assign an error to \( x \) in all cases where a solution had a confidence level of less than 0.1%. The results of the \( \chi^2 \) analysis and the experimental correlation are shown in Figure 11.

The 2.45-MeV correlation was anisotropic and was characterized by \( a_2/a_0 = -0.05 \pm 0.05 \) and \( a_4/a_0 = -0.24 \pm 0.07 \). When the \( \chi^2 \)
Figure 11. \( \gamma = 2.90\text{-MeV Angular Correlation and } \chi^2 \) versus Arctan \( X_{80} \) for the 2.904-MeV Level. The data were fitted assuming \( J \), the spin of the 2.904-MeV level, to be 1/2, 3/2, 5/2, 7/2, and 9/2, resulting in the five curves illustrated in the figure. Only the solutions for \( J = 3/2 \) and 5/2 are seen to have a confidence level of 0.1% or less. In the top portion of the figure is shown the 2.90-MeV correlation taken at a \(^3\)He energy of 6.37 MeV. The solid lines through the data are the fits for \( J = 3/2 \) and 5/2 with specific multipole mixtures. The probabilities of \( \chi^2 \) exceeding the 0.1%, 1%, and 10% confidence levels are 0.1%, 1%, and 10%, respectively, for a correct solution.
test was applied to this angular distribution, the spin of the 0.451-MeV level was assumed to be \( J_1 = 3/2 \) or 5/2 as determined from fitting the \( a_1 - \gamma \) angular correlation discussed earlier. The spin of the 2.904-MeV level was then assumed to be \( J = 3/2 \) or 5/2 as was just shown. The results of this analysis along with the experimental data are shown in Figure 12.

For the spin sequence \( J \rightarrow J_1 = 5/2 \rightarrow 3/2, \ x_{61} = \tan \ (10 \pm 3)^0 \ (\approx \ 0.176 \pm 0.052) \) and for \( 3/2 \rightarrow 3/2, \ x_{61} = \tan \ (-15 \pm 5)^0 \ (\approx -0.268 \pm 0.087) \) or \( \leq \tan \ (-85)^0 \ (\leq -11.43) \). For the sequence \( J \rightarrow J_1 = 3/2 \rightarrow 5/2, \ x_{61} = \tan \ (-74 \pm 8)^0 \ (\approx -3.49 \pm 0.14) \) or \( \tan \ (-4 \pm 7)^0 \ (\approx -0.070 \pm 0.123) \), and for \( 5/2 \rightarrow 5/2, \ x_{61} = \tan \ (-24 \pm 9)^0 \ (\approx -0.445 \pm 0.158) \) or \( \tan \ (82 \pm 10)^0 \ (\approx 7.12 \pm 0.13) \).

In order to discriminate between the solutions \( J_1 = 3/2 \) and 5/2 for the 0.451-MeV level, the 0.45-MeV cascade transition was extracted and was found to be characterized by \( a_2/a_o = -0.27 \pm 0.02 \) and \( a_2/a_o = -0.01 \pm 0.03 \). The procedure followed in fitting this correlation was to fix \( x_{61} \) at one of the values which gave an acceptable solution for the 2.45-MeV correlation and allow \( \arctan \ x_{10} \) to take on discrete values between \(-85^0 \) and \( 85^0 \). For a transition of this type the data are fitted with the theoretical correlation given in Equation 8. For a consistent solution, the values of \( x_{10} \) so obtained must agree with the values of \( x_{10} \) found from analysing the \( a_1 - \gamma \) angular correlation. The results of this analysis are shown in Figure 13. In the left hand portion of this figure the spin of the 0.451-MeV level has been assigned \( J_1 = 3/2 \) and the spin of the 2.904-MeV
Figure 12. $E_\gamma = 2.45$-MeV Angular Correlation and $\chi^2$ versus Arctan $X_{61}$ for the 2.904-MeV Level. In the left hand portion of the figure the spin of the 0.451-MeV level was assumed to be $J_1 = 3/2$ and $J$, the spin of the 2.904-MeV level, was assumed to be 3/2 or 5/2 resulting in the two curves illustrated in the figure. Directly above these curves are the experimental data taken at a $^3$He energy of 6.37 MeV. The solid lines through these data are the theoretical fits for $J = 3/2$ and 5/2 with specific mixing ratios. In the right hand portion of the figure is shown the results of a similar analysis but with $J_1 = 5/2$. Directly above these results are shown the experimental data and the fits for $J = 5/2$ and 3/2 with specific mixing ratios given in the figure.
Figure 13. $E \gamma = 0.45$-MeV Angular Correlation and $\chi^2$ versus Arctan $X_{10}$ for the 2.904-MeV Level. In the left hand portion of the figure is shown the results of fitting the angular distribution of the 0.45-MeV cascade transition from the decay of the 2.904-MeV level with the spin of the 0.451-MeV level equal to $3/2$. The mixing ratio $x_{61}$ was fixed at a value found from fitting the 2.45-MeV correlation for $J$, the spin of the 2.904-MeV level, equal to $3/2$ or $5/2$. There is seen to be no value of $x_{61}$ which yields a value of $x_{10}$ consistent with that found from fitting the $\alpha_1-\gamma$ correlation (shown shaded). The solution for $J = 5/2$ and $J_1 = 3/2$ with $x_{61} = \tan (10^\circ)$ yields a value of $x_{10}$ with a confidence level of approximately 0.1% within the allowed range of $x_{10}$. In the right hand portion of the figure it is evident that a consistent solution is obtained if the spin of the 0.451-MeV level is $5/2$, the confidence level being 50%.
level was assumed to be \( J = 3/2 \) and \( 5/2 \). For these spin choices there is no value of \( x_{61} \) which yields a value of \( x_{10} \) for which the corresponding \( \chi^2 \) has a confidence level of 0.1% or less within the range of \( x_{10} \) found from the \( a_1 - \gamma \) angular correlation. Recall that for \( J_1 = 3/2 \) the \( a_1 - \gamma \) angular correlation, when fitted, yielded \( x_{10} = \tan (-62 \pm 5)^\circ \) or \( \tan (-42 \pm 5)^\circ \). It is the arctangent of these values which is the "allowed range of \( x_{10} \)" shown shaded in the left hand portion of Figure 13. In the top left hand portion of this figure is shown the 0.45-MeV angular distribution. The solid line through these data are the fits for \( J = 5/2 \), \( \arctan x_{61} = 10^\circ \) and \( \arctan x_{10} = -37^\circ \) and -67°, the two fits being superimposed. The two values of \( \arctan x_{10} \) used in these fits correspond to the smallest value of \( \chi^2 \) for these solutions within the allowed range of \( x_{10} \). In the right hand portion of Figure 13 is shown the results of the \( \chi^2 \) analysis assuming that \( J_1 = 5/2 \). With \( J \), the spin of the 2.904-MeV level, equal to 3/2 or 5/2 and using the values of \( x_{61} \) found for these spin choices from the 2.45-MeV correlation, it is seen that there are two solutions which are consistent with the allowed range of \( x_{10} \). Recall that from applying the \( \chi^2 \) test to the \( a_1 - \gamma \) correlation a solution was found for \( J_1 = 5/2 \) with \( x_{10} = \tan (-4 \pm 3)^\circ \). The arctangent of this value is shown shaded in the right hand portion of Figure 13. The two solutions consistent with this result are \( J = 3/2 \) with \( x_{61} = \tan (-4 \pm 7)^\circ \) and \( J = 5/2 \) with \( x_{61} = \tan (-24 \pm 9)^\circ \). In the top right hand portion of the figure is shown the experimental data. The solid lines through these data are the fits for \( J \to J_1 = \)
$5/2 \rightarrow 5/2$ with $\arctan x_{61} = -24^\circ$ and $\arctan x_{10} = -3^\circ$, and $J \rightarrow J_1 = 3/2 \rightarrow 5/2$ with $\arctan x_{61} = -4^\circ$ and $\arctan x_{10} = -1^\circ$, the two fits being superimposed. The values of $\arctan x_{10}$ used in these fits correspond to the smallest value of $\chi^2$ for these solutions within the allowed range of $x_{10}$.

These results demonstrate that $J = 5/2$ is the most probable spin for the $0.451$-MeV level. This spin assignment is further supported by the firm $5/2^+$ assignment for the corresponding level in the mirror nucleus $^{23}$Na, the $J$-dependence effect observed by Kozub, and by the approximate rule for the sign of $E2/M1$ mixtures for corresponding mirror transitions.

It should be mentioned that the $0.45$-MeV radiation resulting from the decay of the $2.904$-MeV level contains an unknown component of $0.51$-MeV annihilation radiation. This radiation results primarily from true coincidences following pair production in material in the vicinity of the NaI crystal. No account was taken of this component when extracting the $0.45$-MeV angular correlation. However, since it was possible to obtain a fit to the $a_6 - \gamma_{0.45}$ correlation which was consistent with that found from fitting the $a_1 - \gamma_{0.45}$ correlation, which did not contain any appreciable annihilation radiation, it was believed that the $a_6 - \gamma_{0.45}$ correlation was not seriously distorted by this source of radiation.

Dubois and Earwaker (24) have measured the $a_6$ angular distribution from the $^{24}$Mg($^3$He, a)$^{23}$Mg reaction at a $^3$He energy of 10 MeV and found it to be characterized by $I_n = 2$ which leads to a $J^\pi = (3/2, 5/2)^+$
assignment to the 2.904-MeV level. The mirror-pair analog of this level in $^{23}$Na is assumed to be the 2.98-MeV level. Many attempts have been made to determine the spin of this level and the results favor $3/2$ only slightly over a $5/2$ assignment. (7) Dubois (56) has measured the $d_6$ angular distribution from the $^{22}$Ne($^3$He, d)$^{23}$Na at a $^3$He energy of 10 MeV. The distribution was characterized by $q_p = 2$ which implies $J^m = (3/2, 5/2)^+$ for the $^{23}$Na 2.98-MeV level. The $3/2^+$ assignment is favored as a result of the approximate rule for E2/M1 mixtures discussed previously. For a $3/2^+$ assignment to the $^{23}$Na 2.98-MeV level and to the $^{25}$Mg 2.904-MeV level, the mixing ratios $x_{60}$ differ in sign (7) whereas for a $5/2^+$ assignment to both levels the signs are identical.

5. The 2.048-MeV Level

The 2.048-MeV level was studied at a $^3$He energy of 8.05 MeV as this level was weakly populated at other energies between 6 and 8 MeV as shown in Figure 4. The summed $\gamma$-ray decay spectrum, composed of seven spectra taken at five different angles to the beam, is shown in Figure 14. The dominant mode of decay is through the first excited state, $(85 \pm 3)\%$, with a ground state decay of $(15 \pm 3)\%$. These branching ratios were determined from the summed $\gamma$-ray coincidence spectrum and from the $a_0$ coefficients of the Legendre polynomial fit to the 2.05-MeV and 1.60-MeV angular correlations. In analysing the 2.05-MeV decay spectrum, the 2.03-MeV $\gamma$ ray from the $^{28}$Si(d, p$2\gamma$)$^{29}$Si reaction was used as a spectral shape to extract the 1.60-MeV $\gamma$ ray. The $^{29}$Si 2.03-MeV
Figure 14. The 2.048-MeV Level Gamma-Ray Decay Spectrum. This figure illustrates the spectrum of $\gamma$ rays in coincidence with the $\alpha$ group populating the $^{23}$Mg 2.048-MeV level in the $^{24}$Mg($^3$He, $\alpha$$\gamma$)$^{23}$Mg reaction at a $^3$He energy of 8.05 MeV. This spectrum is the sum of seven spectra taken at five different angles to the beam. The $\gamma$-ray peaks correspond to the transitions indicated in the decay scheme. A 2.05-MeV spectral shape, shown sketched in, was used to extract the 1.60-MeV $\gamma$ ray. Accidental coincidences have been subtracted. Note that the scale used to depict the 0.45-MeV $\gamma$ ray has been reduced by a factor of two.
2.048-MeV LEVEL $\gamma$-RAY SPECTRUM

COUNTS / CHANNEL

CHANNEL NUMBER

0.45

1.60

2.05

$2^3$Mg

$0.45 \pm 0.01$

$1.60 \pm 0.01$

$2.05 \pm 0.01$

$85 \pm 3$

$15 \pm 3$

$2.048$
level decays approximately 100% to the ground state. The error in the branching ratios reflects both statistical uncertainty and the uncertainty in extracting the 1.60-MeV $\gamma$ ray.

From the spectrum of $\gamma$ rays detected in the Ge(Li) detector at a $^3$He energy of 8 MeV, and in coincidence with a particles populating the 2.048-MeV level, $\gamma$ rays with energies of $0.451 \pm 0.004$ MeV and $1.599 \pm 0.004$ MeV were identified with transitions from the decay of the $^{23}$Mg second-excited state. The ground state branch was too weak to be clearly discerned above the background. The two $\gamma$ rays observed lead to an energy of $2.050 \pm 0.006$ MeV for the $^{23}$Mg second-excited state in agreement with the result $2.048 \pm 0.010$ MeV given in Ref. 8.

The angular correlation of the 1.60-MeV $\gamma$ ray was extracted using the 2.03-MeV spectral shape and was found to be characterized by $a_{2}/a_{0} = -0.66 \pm 0.03$ and $a_{4}/a_{0} = 0.01 \pm 0.04$ where these coefficients are the average values from three separate measurements. In the $\chi^2$ analysis of this correlation the spin of the 0.451-MeV level was fixed at $J_{1} = 5/2$ and the spin, $J$, of the 2.048-MeV level was allowed to assume values from 1/2 to 9/2. The resulting $\chi^2$ curves from fitting one of the three angular correlations taken at a $^3$He energy of 8.05 MeV are illustrated in the left hand portion of Figure 15. Solutions with confidence levels of 0.1% or less were found for $J = 7/2$ with $x_{21} = \tan (-10.3 \pm 2.2)^{0}$ ($= -0.182 \pm 0.038$) or $J = 3/2$ with $x_{21} = \tan (33.3 \pm 3.0)^{0}$ ($= 0.657 \pm 0.052$) or $\tan (68.3 \pm 3.0)^{0}$ ($= 2.51 \pm 0.05$). These mixing ratios are the weighted
Figure 15. $\text{E}_{\gamma} = 1.60$, $0.45$-MeV Angular Correlations and $\chi^2$ versus $\text{Arctan } X_{21}$, $\text{Arctan } X_{10}$ for the $2.048$-MeV Level. In the left hand portion of the figure is shown the results of fitting one of the three measured $1.60$-MeV correlations with $J_1 = 5/2$ and $J = 1/2$ through $3/2$. Spins of $J = 7/2$ and $3/2$ are seen to yield acceptable solutions. One of the sets of data taken at a $^3\text{He}$ energy of $8.05$ MeV is shown in the top portion of the figure. The solid line through these data is the theoretical fit for $J = 7/2$ and $x_{21} = \tan \left(-10^\circ\right)$. In the right hand portion of the figure is shown the results of fitting one of the $0.45$-MeV cascade transitions with $x_{21}$ fixed at the average values given in the text. The shaded region is the range of $x_{10}$ determined from the analysis of the $a_1 - \gamma$ $0.45$ correlation with $J_1 = 5/2$, i.e., $x_{10} = \tan \left(-4 \pm 3^\circ\right)$. It is estimated that the $J = 7/2$ solution is approximately 80 times more consistent with this result than is the solution for $J = 3/2$. One of the $0.45$-MeV correlations is shown at the top of the figure with the fits for $J = 7/2$ (solid line) and $3/2$ (dashed line) with specific multipole mixtures.
averages of three measurements and were calculated using the procedure
given in the discussion of the propagation of errors in Ref. 49.

In order to obtain discrimination against either the $J = 7/2$ or
$3/2$ solutions the 0.45-MeV cascade correlation was extracted and was
characterized by $a_2/a_0 = 0.35 \pm 0.02$ and $a_4/a_0 = -0.04 \pm 0.03$. In analysing this angular distribution the mixing ratio $x_{21}$ was fixed at the values
which gave acceptable solutions for the 1.60-MeV correlation and the mixing ratio $x_{10}$ was allowed to vary. For a consistent solution the values of
$x_{10}$ so obtained must again agree with the value of $x_{10}$ found from fitting
the $a_1 - \gamma$ angular correlation. These results are shown in the right
hand portion of Figure 15. Only the solution for $J = 7/2$ with $x_{21} = \tan
(-10.3)\degree$ yields a $\chi^2$ with a confidence level of 0.1% or less within the
allowed range of $x_{10}$, shown shaded. For $J = 3/2$ and $x_{21} = \tan (33.3)\degree$ the
minimum value of $\chi^2$ within the allowed range of $x_{10}$ is 6 which cor-
responds to a confidence level of approximately 0.05%. The minimum value
of $\chi^2$ for $J = 7/2$ within the allowed range of $x_{10}$ has a confidence level
of 4%. Therefore $J = 7/2$ is approximately 80 times more probable than
is $3/2$ for the spin of the 2.048-MeV level. The fits to one of the three
measured 0.45-MeV correlations is shown in the top right hand portion of
Figure 15. For these fits the values of $x_{21}$ used are the weighted average
of three measurements and the $x_{10}$ values correspond to the smallest value
of $\chi^2$ for the solutions within the allowed range of $x_{10}$.

The 2.05-MeV transition, though weak, was extracted with con-
considerable difficulty. The angular distribution was found to be characterized by $a_2/a_0 = 0.17 \pm 0.10$ and $a_4/a_0 = -0.33 \pm 0.15$. These coefficients are only consistent for a spin of $5/2$ or greater for the 2.048-MeV level and support the results above that $J = 7/2$ is more probable than $J = 3/2$. The $\chi^2$ curves for $J = 7/2$ and $3/2$ for one of the sets of experimental data taken at a $^3$He energy of 8.05 MeV are shown in Figure 16. For $J = 7/2$, $x_{20} = \tan(-8 \pm 14)^0 (= -0.141 \pm 0.249)$ and for $J = 3/2$, $x_{20} = \tan(-6 \pm 13)^0 (= -0.105 \pm 0.231)$. Unfortunately, both solutions have a confidence level of approximately 10% which negates the above argument concerning the $a_2$ and $a_4$ coefficients. The large statistical uncertainty in the data is seen to introduce a large error in $x_{20}$. The $x_{20}$ value for $J = 7/2$ conflicts with an earlier measurement in this laboratory (21) which yielded an impossibly large ($x_{20} = \tan(-25 \pm 3)^0$) mixing ratio for this transition, if the parity of the level is positive, although this fit led to an unambiguous $J = 7/2$ assignment to the 2.048-MeV level. The present result, $x_{20} = \tan(-8 \pm 14)^0$, is favored. DaSilva et al. (23) have fitted this same correlation and report $x_{20} = 0.06 \pm 0.05$ for $J = 7/2$. The weighted average (49) of the two results is $x_{20} = 0.05 \pm 0.05$. This small value of $x_{20}$ suggests that the multipolarity of the ground state decay is $M3 + E2$ which would lead to a $\pi = +$ assignment to the 2.048-MeV level. Dubois and Earwaker (24) obtained a good $l_n = 4$ fit to the $a_2$ angular distribution which leads to $J^\pi = (7/2, 9/2)^+$ for this level. However, they remark that the fit may be fortuitous since the level is not strongly populated in the ($^3$He, a) reaction.
Figure 16. $E\gamma = 2.05$-MeV Angular Correlation and $\chi^2$ versus Arctan $x_{20}$ for the 2.043-MeV Level. This figure shows the $\chi^2$ fits to the 2.05-MeV angular distribution with $J = 7/2$ and $3/2$. In the top portion of the figure is shown the experimental correlation taken at a $^3$He energy of 8.05 MeV. The solid line through these data is the fit for $J = 7/2$ with $x_{20} = \tan \left( -8^\circ \right)$ and the dashed line is the fit for $J = 3/2$ with $x_{20} = \tan \left( -6^\circ \right)$. The $7/2$ fit appears to be slightly better than that for $J = 3/2$. 
The assumed analog of this level in $^{23}$Na is the 2.08-MeV level. Poletti and Start (7) have studied the angular correlations from this level and find that $J = 7/2$ is the most probable spin for this level although $J = 3/2$ cannot be rigorously eliminated. If the $^{23}$Na 2.08-MeV level and the $^{23}$Mg 2.048-MeV level have positive parity, as suspected, then the mixing ratios $x_{21}$ are $E2 + M1$ mixtures. For both $J = 3/2$ and $7/2$ assignments to these two levels the mixing ratios $x_{21}$ differ in sign for the corresponding transitions. The approximate rule for $E2/M1$ mixing ratios, discussed previously, does not in this case allow discrimination between spin assignments of $3/2$ or $7/2$ for the two mirror levels.

Considering all of the above information the spin of the 2.048-MeV level is most probably $7/2$ although $3/2$ cannot be rigorously excluded. The observed branching seems more consistent with $J = 7/2$ since with this assignment it is more likely that the 2.048-MeV level would decay strongly to the 0.451-MeV level with $\Delta J = 1$ than decay to the ground state with $\Delta J = 2$. The parity of the level is most likely positive since for a negative parity assignment the mixing ratio $x_{21}$ for $J = 7/2$ would imply an improbably large $M2$ component in the 1.60-MeV radiation. Therefore $J^\pi = 7/2^{(+)}(3/2)$ for the $^{23}$Mg 2.048-MeV level.

6. The 2.356-MeV Level

The 2.356-MeV level was studied at $^3$He energies of 6.37 and 8.05 MeV. The coincident $\gamma$-ray spectrum, taken at 6.37 MeV and summed over five different angles, is shown in Figure 17. In analysing this
Figure 17. The 2.356-MeV Level Gamma-Ray Decay Spectrum. This figure illustrates the spectrum of $\gamma$-rays in coincidence with the $\alpha$ group populating the $^{23}$Mg 2.356-MeV level in the $^{24}$Mg($^3$He, $\alpha\gamma$)$^{23}$Mg reaction at a $^3$He energy of 6.37 MeV. This spectrum is the sum of five spectra taken at five different angles to the beam. The $\gamma$-ray peaks correspond to the transitions indicated in the decay scheme. A 2.36-MeV spectral shape, shown sketched in, was used to extract the 1.91-MeV $\gamma$ ray. Accidental coincidences have been subtracted.
spectrum the 1.91-MeV $\gamma$ ray was extracted using the 2.43-MeV $\gamma$ ray from the $^{28}$Si($d, p_3 \gamma$)$^{29}$Si as a spectral shape for the 2.36-MeV $\gamma$ ray. The $^{28}$Si 2.43-MeV level decays approximately 100% to the ground state. The $^{23}$Mg 2.356-MeV level was found to decay to the ground and first excited state in the proportions $(29 \pm 4)\%$ and $(71 \pm 4)\%$, respectively. These branching ratios were obtained from the $a_0$ coefficients of the least-squares fit to the 2.36- and 1.91-MeV correlations. The errors in the branching ratios reflect both statistical uncertainty and the uncertainty in extracting the 1.91-MeV $\gamma$ ray.

The spectrum of $\gamma$ rays detected in the Ge(Li) detector at a $^3$He energy of 8 MeV, and in coincidence with the a group leading to the 2.356-MeV level, revealed $\gamma$ rays with energies of $0.453 \pm 0.004$ MeV, $1.909 \pm 0.004$ MeV, and $2.359 \pm 0.004$ MeV which could be identified with transitions from this level. The latter energy agrees with the value $2.356 \pm 0.015$ MeV given in Ref. 8 as the energy of the $^{23}$Mg third-excited state.

The 2.36-MeV angular correlation, taken at a $^3$He energy of 8.06 MeV, was characterized by $a_2/a_0 = -0.01 \pm 0.05$ and $a_4/a_0 = -0.03 \pm 0.08$. In applying the $\chi^2$ test to this correlation the spin, $J$, of the 2.356-MeV level was assumed to range from 1/2 to 9/2. The $\chi^2$ curves for these spin choices are illustrated in Figure 18. Acceptable solutions were found for $J = 1/2$ with $x_{30}$ undetermined, $J = 3/2$ with $x_{30} = \tan (-15 \pm 8)^\circ$($= -0.268 \pm 0.141$, or \(\leq\) $\tan (-79)^\circ$($\leq -5.14$, or $\geq\) $\tan (81)^\circ$ ($\geq 6.31$), and $J = 5/2$ with $x_{30} = \tan (11 \pm 6)^\circ$($= 0.194 \pm 0.105$).
Figure 18.  \( E_\gamma = 2.36\text{-MeV} \) Angular Correlation and \( \chi^2 \) versus \( \arctan X_{30} \) for the 2.356-MeV Level. In the lower portion of the figure is illustrated the results of the \( \chi^2 \) analysis of the 2.36-MeV angular distribution assuming the spin, \( J \), of the 2.356-MeV level to be 1/2, 3/2, 5/2, 7/2, and 9/2. Acceptable solutions were found for \( J = 1/2 \) with \( x_{30} \) undetermined and \( J = 3/2 \) and 5/2 for specific multipole mixtures. The experimental angular correlation, taken at a \(^3\)He energy of 8.00 MeV, is shown in the top portion of the figure. The solid line through these data is the fit for \( J = 1/2 \). The fits for \( J = 3/2 \) and 5/2 fit equally well. The percentages shown give the probability that a correct solution has a \( \chi^2 \) of the corresponding value or larger.
The 1.91-MeV correlation was extracted after subtracting the isotropic background of the 2.36-MeV γ ray. This correlation was characterized by \( a_2/a_0 = 0.00 \pm 0.03 \) and \( a_4/a_0 = -0.03 \pm 0.04 \). In the \( \chi^2 \) analysis of this correlation the spin of the 2.356-MeV level was assumed to be \( J = 1/2, 3/2, \) and \( 5/2 \). The results of this analysis, shown in the left hand portion of Figure 19, lead to acceptable solutions of \( J = 1/2 \) with \( x_{31} \) undetermined, \( J = 3/2 \) with \( x_{31} = \tan (-72 \pm 5)^0 (\pm 3.08 \pm 0.09) \) or \( \tan (-5 \pm 5)^0 (\pm 0.087 \pm 0.087) \) and \( J = 5/2 \) with \( x_{31} = \tan (-21 \pm 5)^0 (\pm 0.384 \pm 0.087) \).

The 0.45-MeV cascade correlation was extracted and was found to be characterized by \( a_2/a_0 = 0.02 \pm 0.02 \) and \( a_4/a_0 = -0.03 \pm 0.03 \). In applying the \( \chi^2 \) test to this correlation, \( x_{31} \) was fixed at one of the values given above and \( \arctan x_{10} \) was treated as a variable. The results of this analysis, shown in the right hand portion of Figure 19, show that only the solution for \( J = 1/2 \) yields a value of \( x_{10} \) for which the corresponding \( \chi^2 \) has a confidence level of less than 0.1% within the allowed range of \( x_{10} \). The solution for \( J = 3/2 \) with \( x_{31} = \tan (-72)^0 \) has a confidence level slightly above the 0.1% level within the allowed range. This analysis indicates that a spin of \( J = 1/2 \) for the 2.356-MeV level is approximately four times more probable than is a \( 3/2 \) assignment. For the latter spin the transition to the \( J = 5/2, 0.451-\)MeV level must be largely quadrupole (i.e., \( T_{31} (E2) = 9.5 T_{31} (M1) \)) if the parity of the level is positive. For negative parity and \( J = 3/2 \), the M2 transition strength would be improbably
Figure 19. \( E_\gamma = 1.91-, 0.45\)-MeV Angular Correlations and \( \chi^2 \) versus \( \arctan x_{31} \). \( \arctan x_{10} \) for the 2.356-MeV Level. The \( \chi^2 \) analysis of the 1.91-MeV correlation, with the results illustrated on the left, was made assuming the spin, \( J \), of the 2.356-MeV level to be 1/2, 3/2, and 5/2. The spin of the 0.451-MeV level was fixed at \( J_1 = 5/2 \). Each of these spin choices is seen to yield acceptable solutions, the 3/2 and 5/2 choices for specific mixtures \( x_{31} \). The top portion of the figure shows the experimental data taken at a \( ^3\text{He} \) energy of 8.05 MeV. The solid line through these data is the fit for \( J = 1/2 \). The results of analysing the 0.45-MeV cascade correlation (with \( x_{31} \) fixed at the values found from analysing the 1.91-MeV correlation) is illustrated on the right. Only \( J = 1/2 \) yields a consistent value of \( x_{10} \) with a confidence level less than 0.1%, the solution with \( J = 5/2 \) and \( x_{31} = \tan (-72^\circ) \) being slightly above the 0.1% confidence level. The experimental data is shown at the top of the figure along with the fit for \( J = 1/2 \).
large.

Strong support for the \( J = 1/2 \) assignment comes from the work of Joyce et al. (16) and Dubois and Earwaker (24) who found the \( a_3 \) angular distribution from the \( ^{24}\text{Mg}(^3\text{He}, a)^{23}\text{Mg} \) reaction to be characterized by an \( \mathbf{l}_n = 0 \) pickup pattern. In addition, Ganguly et al. (17) and Kozub (18) found the \( d_3 \) angular distribution to also be characterized by \( \mathbf{l}_n = 0 \). The 2.356-MeV level thus has an unequivocal \( J^\pi = 1/2^+ \) assignment as was favored from the analysis of the angular correlations from this level.

The assumed mirror level in \( ^{23}\text{Na} \) is at 2.39 MeV. Dubois (56), using the \( ^{22}\text{Ne}(^3\text{He}, d)^{23}\text{Na} \) reaction, has found the \( d_3 \) angular distribution to be well fitted with an \( \mathbf{l}_p = 0 \) pickup pattern which leads to a \( J^\pi = 1/2^+ \) assignment to this level. Richter and von Witsch (57), in a different experiment, also report \( J^\pi = 1/2^+ \) for this level. Poletti and Start (7) have studied the angular correlations from this level and concluded that \( J = 1/2 \) (3/2).

7. The 2.712- and 2.765-MeV Levels

The 2.712-MeV level was studied in conjunction with the 2.768-MeV level as it was impossible to separately resolve the \( a \) groups populating these levels using only a semiconductor detector. A magnetic spectrometer would have been required as was employed by Dubois and Earwaker (24). A \(^3\text{He} \) energy of 8.05 MeV was used to populate these levels as at this energy the close-lying 2.904-MeV level was weakly populated (see Fig. 3).

In order to determine the decay modes of these two levels a
spectrum of the $\gamma$ rays detected in the Ge(Li) counter and in coincidence with the a groups leading to the 2.712-, 2.768-, and 2.904-MeV levels was taken. This spectrum, recorded at a $^3$He energy of 8 MeV, is shown in Figure 20. A three-point running average has been made of the data illustrated in this figure. The energies of the $\gamma$-ray peaks shown were obtained using the $^{24}$Na calibration curve discussed in Chapter II. In obtaining this spectrum three TPH intervals were used as previously discussed. The slanted lines at channels 400 and 800 in the figure are the boundaries of the three $\gamma$-ray spectra which were obtained for the different TPH intervals. Accidental coincidences have not been subtracted from the spectrum although this will not affect the conclusions drawn. Gamma rays with energies of $2.767 \pm 0.004$ MeV and $1.751 \pm 0.004$ MeV (double escape peak) can be identified with a ground state branch from the 2.768-MeV level. These two energies agree with the result $2.768 \pm 0.010$ MeV given in Ref. 8 for the energy of the fifth-excited state. Gamma rays with energies of $2.261 \pm 0.006$, $0.451 \pm 0.004$, $0.661 \pm 0.004$, and $1.603 \pm 0.005$ MeV can be identified with transitions from the 2.712-MeV level through the first and second excited states as indicated in the decay scheme. There is no evidence for a ground state branch from the 2.712-MeV level. The two energies 2.261 and 0.451 MeV lead to an energy of $2.712 \pm 0.007$ MeV for the fourth excited state in agreement with the result $2.712 \pm 0.010$ MeV given in Ref. 8. Gamma rays with energies of $2.902 \pm 0.008$, $2.454 \pm 0.008$, and $0.451 \pm 0.004$ MeV can be identified with transitions from the
Figure 20. The 2.712-, 2.768-, and 2.904-MeV Level Gamma-Ray Decay Spectrum Recorded with the Ge(Li) Detector. This figure illustrates the spectrum of $\gamma$ rays taken at $\theta = 90^\circ$ to the beam with the Ge(Li) detector, and in coincidence with the $\alpha$ groups leading to the $^{23}\text{Mg}$ 2.712-, 2.768-, and 2.904-MeV levels in the $^{24}\text{Mg}(^3\text{He}, \alpha\gamma)^{23}\text{Mg}$ reaction at a $^3\text{He}$ energy of 8 MeV. The $\gamma$-ray peaks correspond to the transitions indicated in the decay scheme. The branching ratios given in the decay scheme were found from a summed $\gamma$-ray spectrum taken with a NaI crystal. The energies in the decay scheme are from Ref. 8 whereas the energies of the $\gamma$-ray peaks in the figure were deduced from a $^{24}\text{Na}$ calibration curve. A three-point running average has been made of the data illustrated here. Accidental coincidences have not been subtracted. See the accompanying text and Section B of Chapter II for further details.
decay of the 2.904-MeV level. In conclusion, this spectrum is consistent with the assumption that the 2.768-MeV level decays 100% to the ground state and that the 2.712-MeV level decays only through the first and second excited states. The statistics are low and weak branches may have been missed.

Using these decay modes the branching ratios were determined from the spectrum of these γ-ray detected in the NaI counter. A typical NaI spectrum is shown in Figure 21. From a summed γ-ray spectrum it was found that the 2.712-MeV level decays (66 ± 5)% to the 0.451-MeV level and (34 ± 5)% to the 2.048-MeV level. In obtaining these branching ratios a 2.75-MeV γ-ray from a 24Na source was used as a spectral shape for the 2.77-MeV γ-ray to extract the 2.26-MeV γ-ray. This shape is shown dashed in Figure 21. An approximate flat-line background was subtracted in extracting the 0.66-MeV γ-ray as shown in the figure. The error in the branching ratios reflects both statistical uncertainty and the uncertainty in extracting the 0.66- and 2.26-MeV γ-rays.

The 2.77-MeV correlation was characterized by $a_2/a_0 = 0.00 \pm 0.01$ and $a_4/a_0 = 0.00 \pm 0.02$. The results of the $\chi^2$ analysis are shown in Figure 22. The observed isotropy is consistent with solutions of $J = 1/2$ with $x_{50}$ undetermined, $J = 3/2$ with $x_{50} = \tan (-14 \pm 2)^0 (= 0.249 \pm 0.035)$ and $J = 5/2$ with $x_{50} = \tan (11 \pm 2)^0 (= 0.194 \pm 0.035)$. The absence of branching to the 0.451-MeV level favors a $J = 1/2$ assignment. If the parity of the 2.768-MeV level is positive then the ratio of transition
Figure 21. The 2.712-, 2.768-MeV Level Gamma-Ray Spectrum Recorded with the Na(Tl) Detector. This figure illustrates the spectrum of γ-rays taken at θ = 50° to the beam with the Na(Tl) detector, and in coincidence with the α particles leading to the $^{23}\text{Mg}$ 2.712- and 2.768-MeV levels in the $^{24}\text{Mg}(^3\text{He},\gamma)^{23}\text{Mg}$ reaction with a $^3\text{He}$ energy of 8.05 MeV. The γ-ray peaks correspond to the transitions indicated in the decay scheme deduced from the Ge(Li) detector spectrum. A 2.77-MeV spectral shape, shown sketched in, was used to extract the 2.26-MeV γ-ray. The manner in which the 0.66-MeV γ-ray was extracted is also indicated. The branching ratios were deduced from a NaI γ-ray spectrum summed over four different angles to the beam. Accidental coincidences have been subtracted from the spectrum shown here.
2.712-2.768-MeV LEVEL \( \gamma \)-RAY SPECTRUM

\[
\begin{align*}
\text{COUNTPS/CHANNEL} & \quad \text{CHANNEL NUMBER} \\
0.45 & \quad 0 \quad 20 \quad 40 \quad 60 \quad 80 \\
2.768 & \quad 2.712 & \quad 2.048 & \quad 1.60 & \quad 2.26 & \quad 2.77 \\
23\text{Mg} & \quad 34^{+5}_{-5} & \quad 100 & \quad 66^{+5}_{-5} & \quad 0 & \quad 0
\end{align*}
\]
Figure 22. $E_\gamma = 2.77$-MeV Angular Correlation and $\chi^2$ versus \arctan x$_{50}$ for the 2.768-keV Level. In analysing the 2.77-MeV correlation the spin, J, of the 2.768-MeV level was assumed to range from 1/2 through 3/2, resulting in the five curves shown in the figure. Acceptable solutions are found for $J = 1/2$ with $x_{50}$ undetermined, and for $J = 3/2$ and 5/2 with specific mixtures given in the text. In the top portion of the figure is shown the experimental correlation taken at a $^3$He energy of 8.05 MeV. The solid line through these data is the fit for $J = 1/2$, the 3/2 and 5/2 fits also resulting in an isotropic distribution. The probability of $\chi^2$ exceeding the 90% confidence level is 90% for a correct solution.
strengths for \( J = 3/2 \) and \( 5/2 \) is \( T_{50}(E2)/T_{50}(M1) = 0.06 \) and 0.04, respectively, which are not unusual values for E2 + M1 mixtures. For a negative parity assignment to the 2.768-MeV level the \( J = 3/2 \) and \( 5/2 \) assignments imply minimum M2 components of 4.6% and 2.5%, respectively, whereas the radiation would be expected to be pure E1. Based on this information alone the spin of the 2.768-MeV level is probably \( J = 1/2 \) although \( J = 3/2 \) and \( 5/2 \) cannot be rigorously excluded. Discrimination between these three spins is provided by the work of Dubois and Earwaker (24) who have fitted the \( a_5 \) angular distribution at a \( ^3\text{He} \) energy of 10 MeV with \( \lambda_n = 1 \) which implies \( j^\pi = (1/2, \ 3/2)^- \) for the 2.768-MeV level. With this restriction \( j^\pi = 1/2^- \) is favored as a result of the absence of branching to the 0.451-MeV level and that the radiation is expected to be pure E1.

The correlation of the 2.26-MeV \( \gamma \) ray was extracted using the 2.75-MeV \( \gamma \) ray from a \( ^{24}\text{Na} \) source as a spectral shape for the 2.77-MeV \( \gamma \) ray and also the fact that the background of the 2.77-MeV \( \gamma \) ray was isotropic. This correlation was characterized by \( a_2/a_0 = 0.06 \pm 0.14 \) and \( a_4/a_0 = -0.68 \pm 0.21 \). The marked anisotropy eliminates spins of \( 1/2 \) and \( 3/2 \) for the 2.712-MeV level. The \( \chi^2 \) test is shown in Figure 23. In this analysis the spin of the 2.712-MeV level was assumed to be \( J = 5/2, \ 7/2, \) and \( 9/2 \). The \( 7/2 \) solution was eliminated at the 0.1% confidence level.

Acceptable solutions were found for \( J = 9/2 \) with \(-11.43 \leq x_{41} \leq 0.02 \) and \( J = 5/2 \) with \( x_{41} = 5.67 \pm 0.04, \ -0.344 \pm 0.158 \), or \( \leq -9.51 \). The large errors on \( x_{41} \) are a result of the poor statistics obtained for this correla-
Figure 23. $E\gamma = 2.26$-MeV Angular Correlation and $\chi^2$ versus Arctan $X_{41}$ for the 2.712-MeV Level. In analysing the 2.26-MeV correlation the spin of the 2.712-MeV level was assumed to be $5/2$, $7/2$, and $9/2$, resulting in the three curves shown in the lower portion of the figure. Spins of $1/2$ and $3/2$ are eliminated by the marked anisotropy of the correlation. Acceptable solutions are found for $J = 9/2$ and $5/2$ with the $7/2$ solution eliminated at the 0.1% confidence level. Due to the large statistical uncertainty the restriction on the mixing ratios found for the $9/2$ and $5/2$ solutions is not stringent. The experimental correlation taken at a $^3$He energy of 8.05 MeV is shown at the top of the figure along with the fits for $J = 9/2$ (solid line) and $7/2$ (dashed line). The $9/2$ assignment is favored as for a $5/2$ assignment the 2.712-MeV level would be expected to exhibit a measurable ground state branch.
tion. From the observed branching of the 2.712-MeV level the \( J = 9/2 \) result is favored as for a \( 5/2 \) assignment a measurable decay to the ground state would be expected. With \( J = 9/2 \) a positive parity assignment is favored on the basis of the strong branch to the \( 5/2^+ \) 0.451-MeV level. In addition, the assumed mirror level in \(^{23}\text{Na}\) has been assigned \( \pi = + \) as discussed below.

In the mirror nucleus \(^{23}\text{Na}\) there exist two close-lying levels at 2.64 and 2.71 MeV. Poletti and Start \( (7) \) have studied the angular correlations from these levels assuming that the 2.64-MeV level decays 100\% to the ground state. They then find that the 2.71-MeV level decays \( (66 \pm 4)\% \) to the 0.44-MeV first excited state and \( (32 \pm 4)\% \) to the 2.06-MeV second excited state. They obtained an isotropic correlation for the 2.64-MeV transition and favored \( J^\pi = 1/2^+ \) for this level. The analysis of the 2.27-MeV correlation led to spin assignments to the 2.71-MeV level of \( 9/2 \) with \( x_{51} = -0.05 \pm 0.07 \) and \( 5/2 \) with \( x_{51} = 1.30 \pm 0.35 \), the \( 9/2 \) solution being favored. The small mixing ratio found for the \( 9/2 \) solution implies that the 2.27-MeV radiation has \( M3 + E2 \) multipolarity which in turn leads to a \( \pi = + \) assignment to the 2.71-MeV level. In the recent compilation of Endt and Van der Leun \( (47) \) the 2.71-MeV level has been assigned positive parity. Dubois \( (56) \) has found that the \( d_4 \) angular distribution from the \(^{22}\text{Ne}(^3\text{He},d)^{23}\text{Na}\) is best fitted by an \( J_p^\pi = 1 \) pickup pattern which implies \( J^\pi = (1/2, 3/2)^- \) for the 2.64-MeV level. Poletti and Start \( (7) \) reported that \( x_{40} = -0.26 \pm 0.05 \) for \( J = 3/2 \) which leads to a minimum
M2 component of 4.4% in the 2.64-MeV radiation whereas the transition would be expected to be pure E1. For this reason and also the apparent absence of branching to the 0.44-MeV level, a $J^\pi = 1/2^-$ assignment is favored for the $^{23}$Na 2.64-MeV level.

It thus appears that the $^{23}$Mg 2.768-MeV level and the $^{23}$Na 2.64-MeV are mirror levels, both with a most probable spin and parity of $J^\pi = 1/2^-$. The $^{23}$Mg 2.712-MeV level would then be the mirror of the $^{23}$Na 2.71-MeV level, both with a most probable spin and parity of $J^\pi = 9/2^+$. There is apparently an inversion of levels when going from $^{23}$Na to $^{23}$Mg. Both the existence of the $1/2^-$ and $9/2^+$ levels in $^{23}$Na and $^{23}$Mg and their inversion is expected on the basis of the systematics of neighboring nuclei. (58) Support for the existence of a $9/2^+$ level in $^{23}$Mg comes from the study of $^{21}$Ne for which the odd nucleon is a neutron as in $^{23}$Mg. The low-energy spectra of these two nuclei is expected to be similar.

With a ground state spin-parity of $3/2^+$, the $0.35^+$, $1.75^+$, and 2.87-MeV levels of $^{21}$Ne have been assigned $5/2^+$, $7/2$, and $9/2$, respectively. (54, 59)

Additional support for the spin assignment to the 2.712-MeV level could be obtained by analyzing the 0.66-MeV angular correlation. This correlation, however, could not be reliably extracted. The 0.45-MeV correlation was extracted and found to be anisotropic. However, this correlation is the sum of the 0.45-MeV cascade correlations from the decay of both the 2.712- and 2.048-MeV levels and is not readily amenable
to analysis. In principle this correlation would provide the necessary discrimination between the 9/2 or 5/2 spin choice for the 2.712-MeV level.

8. The 3.792-MeV Level

The 3.792-MeV seventh excited state was studied at a $^3$He energy of 8.05 MeV as at this energy the thin-target yield (Fig. 3) indicated that this level was populated more strongly than the 3.856- and 3.968-MeV levels. In the analysis the high-energy side of the composite $a_{7,8,9}$ peak was taken as corresponding primarily to the population of the 3.792-MeV level. The decay modes of this level were deduced from a $\gamma$-ray spectrum, shown in Figure 24, composed of six spectra taken at five different angles. The level was found to decay to the ground state and first and fifth excited states in the proportions $(6 \pm 3)\%$, $(36 \pm 5)\%$, and $(6 \pm 5)\%$, respectively. In Fig. 24 the $7 \rightarrow 5$ transition was identified by the presence of the 1.02-MeV $\gamma$ ray. The 100% decay $5 \rightarrow 0$ transition is obscured by the strong first escape peak of the 3.34-MeV $\gamma$ ray resulting from a $7 \rightarrow 1$ transition. The small peaks at approximately 1.60 and 1.90 MeV could not be identified with transitions from the 3.792-MeV level and may result from the decay of the 3.968-MeV level to be discussed. In the figure is sketched an estimation of the contribution of the 3.79-MeV $\gamma$ ray to the 3.34-MeV $\gamma$ ray as a spectral shape was not available for the 3.79-MeV $\gamma$ ray. The assumed shape of the 3.34-MeV $\gamma$ ray is also shown sketched in the figure as is the estimated shape of the background underlying the 1.02-MeV $\gamma$ ray.
Figure 24. The 3.792-MeV Level Gamma-Ray Decay Spectrum. This figure illustrates the spectrum of $\gamma$ rays in coincidence with, primarily, the $\alpha$ group leading to the $^{23}\text{Mg}$ 3.792-MeV level in the $^{24}\text{Mg}(^3\text{He}, \gamma)^{23}\text{Mg}$ reaction at a $^3\text{He}$ energy of 8.05 MeV. This spectrum is the sum of six spectra taken at five different angles. The $\gamma$-ray peaks correspond to the transitions indicated in the decay scheme with the exception of the two peaks at 1.60 and 1.92 MeV. These latter two peaks may correspond to a small population of the 3.968-MeV level. The manner in which the 3.79-, 3.94-, and 1.02-MeV $\gamma$ rays were extracted is shown by the dashed lines in the figure which are estimates of the peak shapes. Accidental coincidences have been subtracted.
From the spectrum of \( \gamma \) rays detected in the Ge(Li) detector at a \(^3\)He energy of 8 MeV, and in coincidence with the \( \alpha \) groups leading to the 3.792-, 3.856-, and 3.968-MeV levels, \( \gamma \) rays with energies of \( 0.451 \pm 0.004 \), \( 2.322 \pm 0.004 \), and \( 3.338 \pm 0.004 \) MeV were identified with the decay of the 3.792-MeV level through the 0.451-MeV level, the 2.322-MeV peak being the double escape peak of the 3.338-MeV peak. The energies 0.451 MeV and 3.338 MeV lead to 3.789 \( \pm 0.006 \) MeV as the energy of the \(^{23}\)Mg seventh excited state in agreement with the value 3.782 \( \pm 0.010 \) MeV given in Ref. \( \text{8} \). It was not possible to identify other transitions from the 3.792-MeV level due to the very weak branching of the 3.79- and 1.02-MeV transitions.

The 3.79-MeV correlation was extracted using those \( \gamma \)-ray spectra for which this peak was resolved from the 3.34-MeV \( \gamma \) ray. The correlation was characterized by \( a_2/a_0 = -0.47 \pm 0.08 \) and \( a_4/a_0 = 0.06 \pm 0.12 \). The large anisotropy eliminates \( J = 1/2 \) for the spin of the 3.792-MeV level. The \( \chi^2 \) analysis of this correlation is shown in Figure 29 with \( J \) ranging from 1/2 to 9/2. Solutions with confidence limits of less than 0.1% were found for \( J = 3/2 \) with \( -82^\circ \leq \arctan \chi_{70} \leq 22^\circ \) and \( J = 5/2 \) with \( \chi_{70} = \tan (-72 \pm 5)^\circ \) or \( \tan (-3 \pm 9)^\circ \). These mixing ratios should not be taken too seriously due to both the large statistical uncertainty and the approximate nature in which the correlation was obtained. However, it is felt that the correlation is indeed anisotropic which excludes \( J = 1/2 \) for the 3.792-MeV level.
Figure 25. $E \gamma = 3.79$-MeV Angular Correlation and $\chi^2$ versus Arctan $x_{70}$ for the 3.792-MeV Level. In applying the $\chi^2$ test to the 3.79-MeV correlation the spin of the 3.792-MeV level was assumed to range from $J = 1/2$ to $9/2$ resulting in the five curves illustrated in the figure. Solutions with confidence levels of less than 0.1% are found for $J = 3/2$ and $5/2$. The mixing ratios found for these two solutions may have substantial errors due to both statistical uncertainty and the approximate manner in which the correlation was extracted. The experimental correlation is shown at the top of the figure. The solid line through these data is the fit for $J = 3/2$ with $x_{70} = \tan (-39)^0$. The salient feature of this correlation is its anisotropy which excludes $J = 1/2$ for the 3.792-MeV level.
The 3.34-MeV correlation was characterized by $a_2/a_0 = -0.09 \pm 0.03$ and $a_4/a_0 = 0.00 \pm 0.05$, the anisotropy excluding $J = 1/2$ for the 3.792-MeV level. In the $\chi^2$ analysis the spin of the 0.451-MeV level was taken to be 5/2 and the spin of the 3.792-MeV level was assumed to take on values from 3/2 to 9/2. The results of this analysis are shown in Figure 26. Good fits to the data were found for $J = 3/2$ with $x_{71} = \tan (-78 \pm 8)^\circ (= -4.71 \pm 0.14)$, or $x_{71} = \tan (0 \pm 8)^\circ (= 0.00 \pm 0.14)$, $J = 5/2$ with $x_{71} = \tan (-28 \pm 6)^\circ (= -0.488 \pm 0.105)$, and $J = 7/2$ with $x_{71} = \tan (7 \pm 4)^\circ (= 0.128 \pm 0.070)$.

The 1.02-MeV correlation was extracted and found to be characterized by $a_2/a_0 = -0.33 \pm 0.18$ and $a_4/a_0 = -0.04 \pm 0.31$, the anisotropy eliminating $J = 1/2$ for the 3.792-MeV level. In the $\chi^2$ analysis the spin of the 2.768-MeV level was taken to be $J_5 = 1/2$ and the spin $J$ of the 3.792-MeV level was assumed to be 3/2, 5/2, and 7/2. The results of this analysis, shown in Figure 27, yield solutions for $J = 3/2$ with $x_{75} = \tan (5 \pm 12)^\circ (= 0.087 \pm 0.213)$ or $x_{75} = \tan (-65 \pm 12)^\circ (= -2.15 \pm 0.21)$. The errors on $x_{75}$ given here were taken to be the half-width of the $\chi^2$ curve for $J = 3/2$ between the minimum and the 30% confidence level. Even though the statistical uncertainty associated with this correlation is large, the data clearly favor a $J = 3/2$ assignment to the 3.792-MeV level.

Dubois and Earwaker (24) have measured the $\alpha_A$ angular distribution at a 3He energy of 12 MeV and found this distribution to be characterized by an $I_n = 1$ pickup pattern which implies $J^\pi = (1/2, 3/2)^-$ for the
Figure 26. $E_\gamma = 3.34$-MeV Angular Correlation and $\chi^2$ versus Arctan $X_{71}$ for the 3.792-MeV Level. For the $\chi^2$ analysis of the 3.34-MeV correlation the spin of the 3.792-MeV level was assumed to range from $J = 3/2$ to 9/2 with $J = 1/2$ excluded by the anisotropy of the angular distribution. The resulting $\chi^2$ curves, shown illustrated in the figure, reveal good fits to the data for $J = 3/2$, 5/2, and 7/2. The experimental angular correlation taken at a $^3$He energy of 8.05 MeV is shown at the top of the figure. The solid line through these data is the fit for $J = 3/2$ with $x_{71} = 0.0$. 
Figure 27. \( E\gamma = 1.02\)-MeV Angular Correlation and \( \chi^2 \) versus Arctan \( x_{75} \) for the 3.792-MeV Level. In the analysis of the 1.02-MeV correlation the spin of the 2.793-MeV level was assumed to be 1/2 and the spin of the 3.792-MeV level was taken to be \( J = 3/2, 5/2 \), and 7/2. A spin of \( J = 1/2 \) is excluded by the anisotropy of the correlation. The resulting \( \chi^2 \) curves shown in the figure indicate a clear preference for a \( J = 3/2 \) assignment to the 3.792-MeV level although the error in \( x_{75} \) is substantial. The experimental data are shown at the top of the figure and the two fits show the extent to which \( J = 3/2 \) is favored over \( J = 5/2 \).
\[
\cos^2 \theta
\]

- \( J = \frac{3}{2} \)
  - ARCTAN \( x_{75} = 5^\circ \)

- \( J = \frac{5}{2} \)
  - ARCTAN \( x_{75} = -35^\circ \)

\( \chi^2 \)

- \( J = \frac{3}{2} \)
  - \( x_{75} \)
  - \( \frac{5}{2} \)
  - \( \frac{3}{2} \)

- \( J = \frac{5}{2} \)
  - \( x_{75} \)
  - \( \frac{5}{2} \)
  - \( \frac{3}{2} \)
  - \( ^{23}\text{Mg} \)
3.792-MeV level. They comment that this level was strongly populated and that there is good reason to believe the DWBA fit. Accepting this \( J^m = 3/2^- \) value, the data just presented clearly lead to \( J^m = 3/2^- \) for the 3.792-MeV level.

The 0.46-MeV cascade correlation was extracted and was found to be characterized by \( a_2/a_0 = -0.32 \pm 0.01 \) and \( a_4/a_0 = 0.01 \pm 0.02 \). In the \( \chi^2 \) analysis of this correlation the 3.792-MeV level was assigned \( J = 3/2^- \), \( x_{71} \) was fixed at either -4.71 or 0.00, and arctan \( x_{10} \) was allowed to vary. The results of this analysis, shown in Figure 38, show the \( x_{71} = 0.00 \) solution to be more consistent with the results of the \( c_1 - \gamma_{0.46} \) correlation than is the solution with \( x_{71} = -4.71 \). Even though the point-scatter in the data is rather drastic, the preference of the smaller value of \( x_{71} \) over the larger one is clear. The solution for \( x_{71} = -4.71 \) can also be rejected on the grounds that it leads to an impossibly large M2 component in the 3.34-MeV radiation.

In light of the probable negative parity for the 3.792-MeV level, the ground state transition could be a mixture of M2 + E1 radiation and the large mixing ratios obtained for \( J = 3/2^- \) would be improbably large. However, as pointed out, these mixing ratios could have sizeable errors associated with them. The salient feature of the 3.79-MeV correlation is its anisotropy which excludes \( J = 1/2^- \) for this level. The 3.34-MeV transition could contain a mixture of M2 + E1 radiation as the 0.462-MeV level has positive parity. This radiation would be expected to be pure E1.
Figure 23. $E \gamma = 0.45$-MeV Angular Correlation and $\chi^2$ versus Arctan $X_{10}$ for the 3.792-MeV Level. For the analysis of the 0.45-MeV cascade correlation the $\chi^2$ test was applied first with $J = 3/2$ and $\alpha_{71} = -4.71$ and then with $\alpha_{71} = 0.00$. In both cases arctan $X_{10}$ was allowed to vary. For a consistent solution the values of $X_{10}$ so obtained must agree with the value of $X_{10}$ found from the $a_{1} - \gamma 0.45$ correlation. This latter result is shown shaded in the figure. Although the point scatter in the data, shown at the top of the figure, is quite bad the smaller value of $\alpha_{71}$ leads to the most consistent solution. The extent to which the smaller value is favored is indicated by the fits to the data given by the solid ($\alpha_{71} = 0.00$) and dashed ($\alpha_{71} = -4.71$) lines drawn through these data.
and this is borne out by the preference of the \( x_{71} = 0.00 \) value over the larger value as was just demonstrated. Finally, if the spin-parity of the 2.768-MeV level is indeed 1/2\(^-\), then the 1.02-MeV radiation could be a mixture of E2 + M1 components. Of the two values of \( x_{75} \) found for this transition, i.e., \( x_{75} = -2.15 \) and 0.087, the latter value seems more probable.

In \(^{23}\text{Na}\), the decay of the 3.68-MeV seventh excited state is very similar to the decay of the \(^{23}\text{Mg}\) 3.792-MeV level with the exception of a 10% branch of the 3.68-MeV level through the second excited state. If such a transition existed for the \(^{23}\text{Mg}\) 3.792-MeV level, then there should be 1.60- and 1.74-MeV \( \gamma \) rays in the decay spectrum. Gamma rays with energies of 1.60 and 1.92 MeV were observed in the decay spectrum but were thought to arise from the decay of the 3.968-MeV level through the 2.048-MeV level. Dubois (56) has measured the \( a_7 \) angular distribution from the \(^{22}\text{Ne}(3\text{He},d)^{23}\text{Na}\) reaction and reports \( \lambda_p = 1 \) for this distribution which implies \( J^\pi = (1/2, 3/2)^- \) for the \(^{23}\text{Na}\) 3.68-MeV level. In view of the observed decay of this level the spin-parity is most likely 3/2\(^-\) since for a 1/2\(^-\) assignment the M2 transition rates would completely dominate the E1 rates. The \(^{23}\text{Na}\) 3.68-MeV level and the \(^{23}\text{Mg}\) 3.792-MeV level thus appear to be mirror levels, both with \( J^\pi = 3/2^- \).

9. The 3.968-MeV Level

The 3.968-MeV ninth-excited state was not preferentially excited at either \( ^3\text{He} \) energies. At an incident energy of 8 MeV the thin-
target a-particle spectrum (Fig. 3) indicated that it would be difficult to extract this state from the strongly populated seventh excited state. At a $^3\text{He}$ energy of 6.37 MeV this latter state was not excited so strongly and an attempt was made to extract the $\alpha^9 - \gamma$ angular correlation. In obtaining this correlation only a small portion of the low-energy portion of the composite $\alpha^7,8,9$ peak was taken as corresponding to the population of the 3.968-MeV level. Initially, the $\gamma$-ray spectra corresponding to this narrow region of the a-particle axis were summed together, the sum extending over nineteen separate spectra. It was discovered later that due to a slight gain shift among a number of the runs that a weak transition had been missed. Figure 29 shows the 3.968-MeV level $\gamma$-ray spectrum summed over four different angles, the gain being the same for each of the spectra. A ground state transition can be identified along with the associated first and second escape peaks. The first escape peak for a 3.97-MeV $\gamma$ ray is expected to be larger than the photopeak from consideration of the spectral shape obtained from the $^{13}\text{C}(^3\text{He},\alpha^1 - \gamma_{4.43})^{12}\text{C}$ reaction. The small peak at 2.45 MeV can be identified with a transition through the 2.904-MeV level which was shown to decay 60% to the ground state and 40% to the 0.451-MeV level (see Fig. 10). The 2.90-MeV transition would lie under the second escape peak (2.95-MeV) of the 3.97-MeV transition. The 1.09-MeV 9 $\rightarrow$ 6 transition would lie under the first escape peak (1.09) and Compton distribution of the strong 1.60-MeV peak. The strong peaks at 1.92-MeV and 1.60 MeV could result from a transition through the 2.048-
Figure 29. The 3.968-MeV Level Gamma-Ray Decay Spectrum. This figure illustrates the spectrum of $\gamma$-rays in coincidence with, primarily, the $\alpha$ particles leading to the $^{23}\text{Mg}$ 3.968-MeV level in the $^{24}\text{Mg}(^3\text{He},\alpha\gamma)^{23}\text{Mg}$ reaction at a $^3\text{He}$ energy of 6.37 MeV. This spectrum is the sum of four spectra taken at four different angles. The $\gamma$-ray peaks correspond to the transitions indicated in the decay scheme.

The transition through the 2.904-MeV level was identified by the presence of the peak at 2.45 MeV which has been shown to be present in the decay of this level. The positions of the 1.09-MeV peak leading to the 2.904-MeV level, the 2.90-MeV ground state branch, and the 2.06-MeV ground state branch are indicated by arrows. Each of these transitions is masked by close-lying stronger peaks. The branching ratios, given in parentheses, have been estimated in the manner described in the text. Accidental coincidences have been subtracted.
3.968-MeV LEVEL \(\gamma\)-RAY SPECTRUM

\(\times \frac{1}{3}\)

0.45

1.09

1.60

1.92

2.05

2.90

3.97

2.45

\(\frac{23}{\text{Mg}}\)

0

1st escape

2nd escape

CHANNEL NUMBER

COUNTS / CHANNEL
MeV level or through the 2.356-MeV level. Recall that the 2.048-MeV level decays 85% to the 0.451-MeV level and 15% to the ground state yielding 1.80- and 2.05-MeV γ-rays whereas the 2.356-MeV level decays 29% to the ground state and 71% to the 0.451-MeV level resulting in 2.36- and 1.91-MeV γ-rays. If the 3.963-MeV level decays through the 2.356-MeV level resulting in the 1.92-MeV γ-ray of the observed intensity, there should be a strong γ-ray peak in the spectrum close to 2.36 MeV. As this peak is not evident the 1.80- and 1.92-MeV peaks probably result from a branch through the 2.048-MeV level, the latter peak being the $9 \rightarrow 2$ transition. The 15% 2.05-MeV transition to the ground state would lie under the strong 1.92-MeV peak. From the summed spectrum of Fig. 29 the number of counts in the 3.97 and 1.92 MeV peaks was estimated. The number of 2.90-MeV counts was calculated using the estimated number of 2.45-MeV counts and the observed branching of the 2.904-MeV level. These intensities, when corrected for γ-ray efficiencies and photoefficiencies, resulted in branches of 39%, 49%, and 12% to the ground state, 2.048-MeV level, and 2.904-MeV level, respectively. Errors are not given for these branching ratios due to the uncertainty in obtaining these numbers. The ratio 49/39 was estimated from the initial summed γ-ray spectrum for which the statistical uncertainty was less.

The angular correlation of the 3.97-MeV γ-ray was extracted using only the photopeak and was characterized by $a_2 a_0 = -0.20 \pm 0.04$ and $a_4 / a_0 = 0.40 \pm 0.06$. When the $\chi^2$ test was applied to this correla-
tion with $J$ ranging from $1/2$ to $9/2$, a clear preference for $J = 5/2$ with $x_{90} = \tan (-77 \pm 11)^{0} (= -4.33 \pm 0.19)$ was found. These results are shown in Figure 30 along with the experimental data. The large value of $x_{90}$ implies an $E2 + M1$ mixture which would lead to a $\pi = +$ assignment to the 3.968-MeV level. Since an $\ell_n$ assignment has not been reported for this level and in view of the difficulty in obtaining the correlation it is not felt that a firm $5/2$ assignment can be made for this level although $5/2^{+}$ seems most likely.

The analog of the $^{23}$Mg 3.968-MeV level in $^{23}$Na appears, on the basis of branching, to be the 3.85-MeV level which decays 50% to the ground state and 50% to the 2.08-MeV second excited state. (47) This level has been assigned $\pi = +$ (47) which is consistent with an implied $\pi = -$ assignment to the $^{23}$Mg 3.968-MeV level. With this correspondence the $^{23}$Mg 3.85-MeV level and the $^{23}$Na 3.92-MeV level are most likely mirror levels leading to an inversion of levels as was observed for the two levels in these two nuclei near 2.70 MeV. Evidence for the spins of the $^{23}$Na 3.85- and 3.92-MeV levels come from the study of the $^{22}$Ne($^3$He, d)$^{23}$Na reaction by Dubois (56) who reports favored $\ell_p$ values of 3 and 2, respectively, for these two levels. The $\ell_p = 3$ assignment to the $^{23}$Na 3.85-MeV level leads to $J^\pi = (5/2, 7/2)^-$ for this level which is in conflict with the above arguments. However, these $\ell_p$ values are not firmly established and from inspection of the angular distributions it is not inconceivable that the assignments could be reversed. If the $^{23}$Na 3.92-MeV level has
Figure 30. $E_\gamma = 3.97$-MeV Angular Correlation and $\chi^2$ versus Arctan $x_{go}$ for the 3.968-MeV Level. The data, shown at the top of the figure, were fitted assuming $J$, the spin of the 3.968-MeV level, to take on values from $1/2$ to $9/2$ resulting in the illustrated curves. The fits reject all spins at the 0.1% confidence level except for $J = 5/2$ with $x_{go} = \tan (-77)^\circ$. The theoretical correlation for this solution is shown by the solid line through these data. The dashed line through these data is the fit for $J = 5/2$ with $x_{go} = \tan (1)^\circ$ and shows the extent to which a large mixing ratio is required to fit the data.
\( l_p = 3 \) instead of 2, then a \( J^\pi \) assignment of \( 5/2^- \) is most probable for this state in view of the observed branching. Dubois and Earwaker (24) have found that the \( a_y \) angular distribution from the weakly excited \(^{23}\text{Mg} \) 3.856-MeV level can be reasonably well fitted with \( l_n = 3 \) which strengthens the hypothesis that the \(^{22}\text{Na} \) 3.92-MeV level and the \(^{23}\text{Mg} \) 3.856-MeV level are mirror states, both probably with \( J^\pi = 5/2^- \). More experimental work is clearly needed in this region of the \(^{23}\text{Mg} - ^{23}\text{Na} \) spectrum to remove the conflicts discussed above.

10. The 3.856-MeV Level

The 3.856-MeV level could not be resolved from the close-lying 3.792- and 3.968-MeV levels. It appeared from the thin-target \( \alpha \)-particle spectrum that this level was weakly excited at the bombarding energies of 6.37 and 8.05 MeV. Dubois and Earwaker (24), who were able to resolve this level, found it to be weakly excited at a \(^3\text{He} \) energy of 12 MeV. Gamma radiation from the decay of this level was neither observed in the present study nor in the work of Dubois and Earwaker.


The 4.353-MeV tenth-excited state was studied at \(^3\text{He} \) energies of 6.37 and 8.05 MeV, the excitation being approximately the same at both energies. The summed \( \gamma \)-ray spectrum is shown in Figure 31. From this spectrum the level was found to decay (96 ± 5)\% to the ground state and (4 ± 3)\% to the 2.356-MeV level. In order to determine if the strong peak
Figure 31. The 4.353-MeV Level Gamma-Ray Decay Spectrum. This figure illustrates the spectrum of $\gamma$ rays in coincidence with the $\alpha$-group leading to the $^{23}$Mg 4.353-MeV level in the $^{24}$Mg($^3$He, $\gamma$)$^{23}$Mg reaction at a $^3$He energy of 6.37 MeV. This spectrum is the sum of four spectra taken at four different angles. The $\gamma$-ray peaks correspond to the transitions shown in the decay scheme. A 4.35-MeV spectral shape is shown sketched in. Accidental coincidences have been subtracted.
at 3.84-MeV could be due to a transition through the 0.451-MeV level, a spectral shape was taken for the 100% transition from the $^{12}$C 4.43-MeV level using the $^{13}$C($^3$He, $a_1 \gamma$)$^{12}$C reaction. It was found from this spectrum that the three highest energy peaks in Fig. 31 could be identified with the photopeak and two escape peaks from the ground state decay of the 4.353-MeV level. The spectral shape for the 4.35-MeV $\gamma$ ray is indicated by the dashed line in the figure. A small peak was observed at 2.36 MeV and a larger one at close to 2 MeV. The first peak can be identified with the ground state decay of the 2.356-MeV level and the second one would be composed of the 10 → 3 and 3 → 1 transitions resulting in $\gamma$ rays with energies of 1.99 and 1.91 MeV, respectively. Another possible interpretation of these peaks could be a 10 → 2 transition resulting in a 2.35-MeV $\gamma$ ray and a 2.05-MeV $\gamma$ ray from the decay of the 2.048-MeV level. For this decay mode there should be a strong peak at 1.60 MeV resulting from the 86% 2 → 1 transition. Such a transition is not observed and it is concluded that the 4.353-MeV level decays through the 2.356-MeV level.

The $\gamma$ rays from the decay of this level were observed with the Ge detector. The energy of the double escape peak of the ground state decay was $3.330 \pm 0.006$ MeV which when combined with $2m_0c^2 = 1.022$ MeV leads to an energy of $4.352 \pm 0.008$ MeV for the tenth-excited state in agreement with the value $4.353 \pm 0.015$ MeV given in Ref. 8. The energies of the less intense first escape peak and photopeak were estimated to be $3.824 \pm 0.008$ MeV and $4.327 \pm 0.010$ MeV, respectively, in fair agree-
ment with the Ref. 8 result. The weak decay through the 2.356-MeV level could not be identified.

The angular correlation of the 4.35-MeV photopeak was characterized by \( a_2/a_0 = -0.02 \pm 0.03 \) and \( a_4/a_0 = -0.01 \pm 0.04 \). From the \( \chi^2 \) test this correlation could be fitted with \( J = 1/2 \) with \( x_{10} \) undetermined, \( J = 3/2 \) with \( x_{10} = \tan (-15 \pm 5)^0 = -0.268 \pm 0.087 \) or \( \leq \tan (-84)^0 \leq -9.51 \), and \( J = 5/2 \) with \( x_{10} = \tan (11 \pm 3)^0 = 0.194 \pm 0.052 \). The \( \chi^2 \) fits and the experimental correlation are shown in Figure 32.

Discrimination among the three possible solutions is provided by the work of Kozub (18) who found \( \ell_n = 0 \) for the \( d_{10} \) angular distribution from the \( ^{24}\text{Mg}(p,d)^{23}\text{Mg} \) reaction and also by Dubois and Earwaker (24) who found that the \( a_{10} \) angular distribution exhibited an \( \ell_n = 0 \) pickup pattern. From these results \( J^\pi = 1/2^+ \) for the \( ^{23}\text{Mg} \) 4.353-MeV level.

The analog level in \( ^{23}\text{Na} \) is assumed to be at 4.43 MeV. Metzger (60) has studied this level using a resonance fluorescence method and found that this level decays \((35 \pm 3)\% \) to the ground state, \((5 \pm 3)\% \) to the 2.39-MeV third excited state, and has spin 1/2. A negative parity assignment was indicated for this state on the basis of electron scattering data although this result appeared speculative. Dubois (56) has studied the \( d_{10} \) angular distribution in the \( ^{22}\text{Ne}(^3\text{He},d)^{23}\text{Na} \) and found what appeared to be forward peaking of the distribution which suggests \( \ell_p = 0 \) and \( J^\pi = 1/2^+ \) for the 4.43-MeV level. In view of the firm 1/2\(^+ \) assignment to the \( ^{23}\text{Mg} \) 4.353-MeV level the spin of the \( ^{23}\text{Na} \) 4.43-MeV level is
Figure 32. \( E_\gamma = 4.35\text{-MeV Angular Correlation and } \chi^2 \text{ versus} \)

\[
\arctan x_{100}
\]

for the 4.353-MeV Level. The experimental
data, shown at the top of the figure, were fitted assuming
\( J \), the spin of the 4.353-MeV level, to assume values
from 1/2 to 3/2 resulting in the five curves illustrated in the
figure. Solutions are found for \( J = 1/2 \) with \( x_{100} \) unde-
termined and for \( J = 3/2 \) and 5/2, both for specific mixing
ratios. The solid line through the data is the fit for
\( J = 1/2 \).
most probably 1/2\(^+\) so that there is a one-to-one correspondence as regards spin and branching between these two levels.

12. Summary of Experimental Results

The experimental branching ratios and spin-parity assignments deduced from this work in conjunction with the known \( \lambda_n \) assignments for levels in \(^{23}\)Mg up to 4.353 MeV are summarized in Figure 33. The level scheme is from Ref. 8. The 3/2\(^+\) assignment to the ground state is based on beta-decay measurements (9) and is consistent with an \( \lambda_n = 2 \) assignment to this level as found from both (p, d) (18, 19) and \(^3\)He, \( \alpha \) (10, 16, 24) reactions on \(^{24}\)Mg. The 5/2\(^+\) assignment to the 0.451-MeV level results from the analysis of the 0.45-MeV cascade transition from the decay of the 2.904-MeV level in conjunction with the \( \alpha_1 - \gamma_{0.45} \) angular correlation. This result is supported by the J-dependence of the (p, d) reaction on \(^{24}\)Mg (51), the firm 5/2\(^+\) assignment to the mirror level in \(^{23}\)Na, and is consistent with the \( \lambda_n = 2 \) assignment to this level found from both the (p, d) (18, 19) and \(^3\)He, \( \alpha \) (10, 16, 24) reactions on \(^{24}\)Mg. The analysis of the 0.45-MeV cascade correlation from the 2.048-MeV level leads to a spin assignment of 7/2 for this level although the 2.05-MeV correlation can be fitted with 3/2 as well as 7/2. The probable positive parity assignment, given in parentheses in the figure, is based on the small mixing ratio \( x_{20} = 0.06 \pm 0.05 \) (23) obtained for the \( 2 \rightarrow 0 \) transition with \( J = 7/2 \) which implies an M3 + E2 mixture for the 2.05-MeV radiation. The isotropy of the angular correlations associated with the 2.356-MeV level is consistent
Figure 33. $^{23}$Mg Energy Level Diagram. This figure illustrates the $^{23}$Mg energy level diagram taken from Ref. 8. The branching ratios were determined in the present work as were the spins when combined with the known $l_n$ values.
with the \( \lambda_n = 0 \) and \( J^{\pi} = 1/2^+ \) assignment to this level as found from both (p, d) \((17, 18)\) and \( ^3\text{He}, \alpha \)(16, 24) reactions on \(^{24}\text{Mg}\). The 2.712-MeV level was found to have \( J = 9/2 \) or \( 5/2 \) with \( 9/2 \) favored both from the analysis of the 2.26-MeV correlation and the observed branching. The probable positive parity assignment is based on the strong branching to the \( 5/2^+ 0.451\)-MeV level. A \( 9/2^- \) assignment to the 2.712-MeV level would lead to an improbably large \( M2 \) component in the 2.26-MeV radiation. The correlation of the 100% decay of the 2.768-MeV level was isotropic and when coupled with the \( \lambda_n = 1 \) assignment to this level (24) leads to \( J^{\pi} = (1/2, 3/2)^- \) for this level. The \( 1/2^- \) assignment is favored if the transition is pure \( E1 \) as expected although \( 3/2^- \) can not be rigorously excluded. The correlations from the decay of the 2.904-MeV level when combined with the \( \lambda_n = 2 \) assignment to this state (24) lead to \( J^{\pi} = (3/2, 5/2)^+ \) for this level. The \( 3/2^+ \) assignment is favored on the basis of the approximate rule for \( E2/M1 \) mixing ratios in mirror nuclei discussed earlier. The 3.792-MeV level has \( \lambda_n = 1 \)(24) and with this result the anisotropy of the correlations from this level leads to a \( J^{\pi} = 3/2^- \) assignment. No spectroscopic information was obtained for the weakly excited 3.856-MeV level. The 3.968-MeV level was found to have \( J = 5/2 \) with a probable positive parity assignment based on the large mixing ratio found for the ground state decay. As the \( \lambda_n \) value is not known for this level the \( 5/2 \) spin should not be regarded as a firm assignment. The isotropic correlation of the ground state decay of the 4.353-MeV level is consistent
with the $l_n = 0$ and $J^\pi = 1/2^+$ assignment to this level as deduced from both the (p, d) (18) and (3He, a) (24) reactions on $^{23}$Mg.

The least squares angular distribution coefficients are listed in Table 1 along with the $\chi^2$ for each fit. The experimental branching ratios determined in this work along with those reported in Ref. 23 and 24 are given in Table 2. A weighted average (49) of these branching ratios is also given assuming the error in each of the ratios to be simply a standard deviation. The mixing ratios determined in this work and those of Ref. 23 and 24 are presented in Table 3 along with the weighted average of the measurements. It is gratifying to see that the results of the three experiments agree quite well. Figure 34 shows the level schemes of $^{23}$Na and $^{23}$Mg along with spin-parity assignments, branching ratios, and selected mixing ratios. The branching ratios and mixing ratios for $^{23}$Mg are the weighted average values given in Tables 2 and 3. The $^{23}$Na branching ratios and level scheme are from Ref. 47, the spin-parity assignments from Ref. 7 and 56, the $l_p$ values from Ref. 56, and the mixing ratios from Ref. 7. It should be pointed out that the $^{23}$Na 2.71- and 3.85-MeV levels have been assigned a positive parity in Ref. 47. This assignment conflicts with the $l_p = 3$ value favored for the 3.85-MeV level. (56)
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<th>( a_4/a_0 )</th>
<th>( \chi^2 )</th>
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* The parentheses indicate some uncertainty in the enclosed numbers.
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* The phases given conform to Convention II of Ref. 7.
# The averages given are weighted averages.
♂ The 2.356- and 4.353-MeV levels with J^π = 1/2^+ are not included in this table.
Figure 34. $^{23}$Na and $^{23}$Mg Energy Level Diagrams. Shown in this figure are the level schemes for the $^{23}$Na-$^{23}$Mg mirror pair along with selected spectroscopic information. The sources of the data are given in the text.
Chapter IV

COLLECTIVE MODEL ANALYSIS

A. General Formalism

The initial success in correlating the experimental data of the $^{21}$Ne, $^{21}$Na, and $^{23}$Na nuclear systems with the static- and dynamic-level predictions of a strong-coupling model suggest that such an approach should be applicable in attempting to interpret the level properties of the $^{23}$Mg nucleus. The most useful of these models is that due to Nilsson. (3) Good discussions of this formulation are given in Reference 3, the text by Preston (61), the review article by Davidson (62), and the dissertation of Webb. (63) Only the essential features of the model will be presented here.

1. Basic Considerations

The strong-coupling collective model is one form of the unified model which is an attempt to combine the successful aspects of both the hydrodynamic, or collective, picture and the single particle picture of nuclear behavior. The simplest form of the unified model in the strong-
coupling limit for an odd-\(A\) nucleus is that of a single particle coupled to an even-even core formed by all of the other nucleons. The potential well generated by the core is assumed to have a permanent average deformation with any oscillations of shape being of sufficiently low frequency to permit the particle to follow adiabatically.

The total angular momentum of the nucleus, \(\vec{J}\), a constant of the motion, may be written as \(\vec{J} = \vec{L} + \vec{j}\) where \(\vec{L}\) is the angular momentum of the core and \(\vec{j}\) is the angular momentum of the odd nucleon and consists of an orbital part, \(\vec{J}\), and an intrinsic part, \(\vec{S}\). The projection of \(\vec{J}\) and \(\vec{j}\) along the body-fixed \(z\) axis is taken to be \(K\) and \(\Omega\), respectively, while the projection of \(\vec{J}\) along the space-fixed \(z'\) axis is \(M\). These angular momenta are illustrated in the accompanying diagram.

In general, the potential well in which the odd particle moves is neither spherical nor axially symmetric so that \(\vec{j}\), \(\Omega\), and \(K\) are not constants of the motion. However, if the body-fixed \(z\) axis is assumed to be an
axis of symmetry, then $L_z = 0$ so that $K = J_z = \Omega$ are constants of the motion. The assumption of axial symmetry has been shown to be a valid one, at least for low-lying levels. (62)

The total Hamiltonian will consist of two terms, one being the kinetic energy of rotation of the core, while the other is the Hamiltonian for the extra-core particle:

$$H = \left(\hbar^2/2\right) \sum_k \left( L_k^2 / \partial_k \right) + p^2/2m + V(r, l, s).$$

(13)

Here the $\partial_k$ are the components of the moment of inertia tensor in the principle axis system, $p^2/2m$ is the kinetic energy of the odd particle, and $V(r, l, s)$ is the deformed potential generated by the core.

For axial symmetry, i.e., $\partial_1 = \partial_2 \equiv \partial_0$, Eq. 13 can be put in the form

$$H = H_R + H_p + H_{RPC}$$

(14)

where

$$H_R = \left(\hbar^2/2\partial_0\right) \left[ \overrightarrow{J}^2 - 2J_z^2 \right] ,$$

(15)

$$H_p = p^2/2m + V(r, l, s) + \left(\hbar^2/2\partial_0\right) \overrightarrow{J}^2$$

(16)

and

$$H_{RPC} = -\left(\hbar^2/2\partial_0\right) \left[ J_x j_x + J_y j_y \right] .$$

(17)

Using the raising and lowering operators, defined by $t_\pm = t_x \pm i t_y$,

Eq. 17 can be rewritten in the form

$$H_{RPC} = -\left(\hbar^2/2\partial_0\right) \left[ J_+ j_- + J_- j_+ \right] .$$

(18)
The term $H_{RPC}$ has been called the "rotation-particle coupling" term and is responsible for the interaction between single particle and collective motion.

When the particle is tightly bound to the core, the particle levels in the potential $V(r, \vec{L}, \vec{S})$ are widely separated compared with the spacing of the rotational levels of the core. Under this circumstance the $H_{RPC}$ term is neglected with respect to the other two terms, $H_R$ and $H_p$, and the resulting energy eigenvalues of the system for a particular value of $K$ are given by

$$E^K_J = \left( h^2 / 2 \mathfrak{D}_0 \right) \left[ J(J+1) - 2K^2 \right] + E^K_p, \quad J \geq K \quad (19)$$

where $E^K_p$ is an eigenvalue of $H_p$ defined in Eq. 16. The energy level sequence of a symmetric, deformed, odd-\(A\) nucleus is interpreted as a $K$-band of rotational states based on the single particle state of energy $E^K_p$. The total wave function in the approximation that $H_{RPC}$ can be neglected is $|EJMK\rangle = |JMK\rangle \cdot |\Omega = K\rangle$ where the $|JMK\rangle$ and $|\Omega\rangle$ are the eigenfunctions of $H_R$ and $H_p$, respectively.

The $H_{RPC}$ term connects states diagonal in $K = 1/2$, the pertinent matrix element being

$$\langle EJM1/2 | H_{RPC} | EJM 1/2 \rangle = (-1)^{J+1/2} \left( h^2 / 2 \mathfrak{D}_0 \right)(J+1/2)a \quad (20)$$

where $a$ is the decoupling parameter determined by the exact form of the single particle Hamiltonian $H_p$. The relative energies in a rotational band are then given, with the addition of the diagonal term, by
\[ E^K_J = \left( \frac{\hbar^2}{2 I_o} \right) \left[ J(J+1) + \delta_K, \frac{1}{2} a(-1)^J + \frac{1}{2} (J + 1/2) \right]. \] (21)

Since the collective angular momentum has no component along the z axis, K is due entirely to the particle angular momentum and is constant throughout the band. All higher values of \( J \geq K \) are allowed and when \( K \neq 1/2 \) the lowest state in the band has \( K = J \) and successive states have \( J = K + 1, \ K + 2, \ldots \). When \( K = 1/2 \) the level order is determined by the value of \( a \) as shown in the accompanying diagram.

\[ \begin{align*}
E^K_J & \quad \frac{5}{2} \quad \frac{7}{2} \\
\frac{1}{2} & \quad \frac{3}{2} \\
\text{LOWEST STATE} & \quad 0 \quad 2 \quad 4 \quad 6 \\
\text{DECOUPLING PARAMETER, } a
\end{align*} \]

For example, for \( a \) between -2 and -3, the level order in a \( K = 1/2 \) band is 3/2, 1/2, 7/2, 5/2, 11/2, 9/2, etc.

Turning now to the case where the particle is loosely bound to the core, the particle states will be close together causing the rotational bands to overlap strongly. For this case the corrections due to \( H_{RPC} \)
are important and it is found that this interaction connects levels $E^K_J$ and $E^{K+1}_J$ which have the same spin and parity but belong to K-bands with $\Delta K = 1$. In general, it is also possible for $H_{RPC}$ to connect states of equal $J$ which have $K = 1/2$. The matrix elements of $H_{RPC}$ can be conveniently written in the form

$$\langle K+1 | H_{RPC} | K \rangle = -\left(\frac{r^2}{2 c_o} \right) A_{\beta, \beta'} [(J-K)(J+K+1)]^{1/2}$$  \hspace{1cm} (22)$$

where the mixing parameter $A_{\beta, \beta'}$ is determined by the exact form of the eigenfunctions of $H_p$. The indices $\beta$ and $\beta'$ refer to two different single particle states which are mixed by $H_{RPC}$. A second order perturbation calculation of the $H_{RPC}$ interaction gives

$$E^{K'}_J = E^K_J - \Delta$$

and

$$E^{K'+1}_J = E^{K+1}_J + \Delta$$  \hspace{1cm} (23)$$

where

$$\Delta = \left| \frac{\langle JM K+1 | H_{RPC} | JMK \rangle}{E^{K+1}_J - E^K_J} \right|^2,$$

and the primes indicate the corrected energies. Band mixing thus leads to mixing of the wave functions for the unperturbed levels and an effective "repulsion" of these levels. Both of these effects are determined by the difference of the unperturbed energy eigenvalues and the coupling

$$\left(\frac{r^2}{2 c_o} \right) A_{\beta, \beta'}.$$
2. Single Particle States in a Deformed Potential

The most useful solution of the single particle problem defined in Eq. 16 (neglecting the $\hat{j}^2$ term (62)) is that due to Nilsson. The interaction of a nucleon with an axially symmetric, deformed potential is described in this work by the Hamiltonian

$$ H_p = H_o + C \hat{l} \cdot \vec{s} + D \hat{l}^2 $$

(24)

where

$$ H_o = \frac{p^2}{2m} + \frac{m/2}{2} \left[ \omega_x^2 (x^2 + y^2) + \omega_z^2 z^2 \right] $$

(25)

is the Hamiltonian of a particle in an anisotropic harmonic oscillator potential, $C \hat{l} \cdot \vec{s}$ is a spin-orbit potential and $D \hat{l}^2$ is a term which will decrease the energy of the higher angular-momentum states from their oscillator values. A deformation parameter $\delta$ is introduced in terms of the oscillator frequencies $\omega_x = \omega_y$ (for axial symmetry) and $\omega_z$:

$$ \omega_x^2 = \omega_o^2 (1 + 2\delta/3), \quad \omega_z^2 = \omega_o^2 (1 - 4\delta/3). $$

(26)

The strengths $C$ and $D$ are chosen so that for zero deformation, i.e., $\delta = 0$, the level sequence reproduces that of the shell model. The condition for constant nuclear volume leads to the equation

$$ \omega_o (\delta) = \hat{\omega}_o \left( 1 - 4 \delta^2/3 - 16 \delta^3/27 \right)^{-1/6} $$

(27)

where $\hat{\omega}_o$ is the value for $\omega_o (\delta)$ at zero deformation.

By first neglecting the $\hat{l} \cdot \vec{s}$ and $\hat{l}^2$ terms, $H_p$ can be split
into a spherically symmetric term, \( \hat{H}_o \), and a term \( H_\delta \) which represents
the coupling of the particle to the axis of symmetry. Introducing a change
of variables defined by \( \rho_1 = (m \omega_o / \hbar)^{1/2} x \), etc., \( H_p \) becomes
\[
H_p = H_o = \hat{H}_o + H_\delta
\]
where
\[
\hat{H}_o = (\pi \omega_o / 2) \left( -\nabla_\rho^2 + \rho^2 \right)
\]
and
\[
H_\delta = -\delta \hbar \omega_o \left( 4/3 \pi / 5 \right)^{1/2} \rho^2 Y_2^o .
\]
Choosing new parameters
\[
\chi = -C/2 \pi \omega_o , \quad \mu = 2D/C
\]
and
\[
\gamma = (\delta / \chi) \omega_o / \omega_o^o
\]
the total Hamiltonian \( H_p \) can be written as
\[
H_p = \hat{H}_o + \chi \hbar \omega_o R
\]
where the operator \( R \) is given by
\[
R = \gamma U - 2 \hbar \cdot S - \mu \hbar \cdot \hbar / 2
\]
with
\[
U = -\left( 4/3 \pi / 5 \right)^{1/2} \rho^2 Y_2^o .
\]
The parameters of the Nilsson Hamiltonian are thus \( \chi \) which determines
the spin-orbit splitting, \( \gamma \) or \( \delta \) the core deformation, \( \mu \) which determines
the depression of higher \( \hbar \) levels, and \( \hbar \omega_o \) the spacing of harmonic
oscillator levels.
Nilsson chose as a representation to diagonalize $H_p$ one for which $H_0$ is already diagonal, the basis being formed from the commuting operators $H_0$, $l^2$, $l_z$, and $s_z$ with corresponding quantum numbers $N + 3/2$, $l$, $\Lambda$, and $\Sigma$, respectively. Here $N$, the principal quantum number, represents the total number of oscillator quanta. The base vectors are enumerated as $| N \lambda \Lambda \Sigma \rangle$ which corresponds to states for which $j_z = \Lambda + \Sigma$. Only the operator $R$ need be considered in the diagonalization since $H_0 | N \lambda \Lambda \Sigma \rangle = (N + 3/2) \hbar \omega | N \lambda \Lambda \Sigma \rangle$ and the term in $H_0$ can be removed from Eq. 33.

Nilsson diagonalized the operator $R$ with the basis given above obtaining the eigenvalues $r^N \Omega(\eta)$. The energy eigenvalues of $H_p$, for a single particle, are given by

$$E^N \Omega = (N + 3/2) \hbar \omega + \chi \hbar \omega^0 r^N \Omega \ . \quad (36)$$

B. Application to $^{23}\text{Mg}$

One of the first tests of the model is its prediction of the ground state spin and parity. This spin, according to the Nilsson model, should be determined by the orbit occupied by the last odd nucleon. With a knowledge of the equilibrium deformation, one could simply fill up the Nilsson orbits, allowing two protons and two neutrons in an orbit, and so determine the orbit of the unpaired nucleon which, in principle, would determine the ground state spin and parity.
The most direct method of determining this deformation is from the observed static electric-quadrupole moment. The intrinsic quadrupole moment, $Q_0$, is given to second order in $\delta$ by (3)

$$Q_0 \approx \frac{(4/5)Z R_Z}{\delta (1 + 2\delta/3)}$$

(37)

where $R_Z = 1.2 A^{1/3}$ is $Q_0$ related to the observed quadrupole moment $Q_S$ by

$$Q_0 = \frac{(J + 1)(2J + 3)}{3K^2 - J(J + 1)} Q_S$$

(38)

where $J$ and $K$ are, respectively, the ground state spin and its projection along the symmetry axis. Although $Q_S$ is not known for $^{23}$Mg, it has been measured for the mirror nucleus $^{23}$Na. With $Q_S = +0.101 \pm 0.011$ b (6) and $J = K = 3/2$ for the $^{23}$Na ground state, Eqs. 38, 37, and 32 lead to

$$\chi \eta \approx \delta \approx +0.40.$$  It is shown in Ref. 6 that a tenable choice for the spin-orbit parameter $\chi$ for N or $Z = 11$ nuclei is $\chi \approx +0.10$ which corresponds to a deformation of $\eta \approx +4.0$. With the reasonable assumption that, apart from a small $Z$ dependence, the quadrupole moment of $^{23}$Mg is the same as that for $^{23}$Na, the values of $\chi$ and $\eta$ just given for $^{23}$Na should apply to $^{23}$Mg.

The energy levels for a single particle in a Nilsson potential is plotted in Figure 35. This figure is from Ref. 6 with the energies calculated from an expression (Eq. 1 of Ref. 6) which is a refinement of Eq. 36. This calculation was made with $\chi = 0.10$ and $\mu = 0$. This figure shows the Nilsson orbits available to a single particle as a function of the nuclear deformation $\eta$. For prolate deformation of the core $\eta > 0$. 

Figure 35. Energy Levels of the Nilsson Model. This diagram illustrates the energy levels of a single particle in a Nilsson potential as a function of the deformation for the region of the 2s-1d shell pertinent to a discussion of $^{23}\text{Mg}$. This figure was taken from Ref. 6 where the energies were calculated with \( \chi = 0.10 \) and \( \mu = 0 \). The integers adjacent to the levels indicate the associated Nilsson orbit number. The levels are identified at zero deformation by the shell model notation \( \ell_j \) and at non-zero deformation by the values of \( K^\pi = \sum_{-j}^{j} \) for the orbit. A particle which occupies a state \( \ell_j \) in a spherical potential can occupy \((2j + 1)\) states with component angular momenta \( K = j, (j - 1), \ldots, -(j - 1), -j \) along the symmetry axis in a deformed potential. There is no energy difference between the states \( K \) and \(-K\) so that each shell-model level splits into only \((1/2)(2j + 1)\) levels in the deformed field. The solid lines in the figure represent the positive parity orbits and the dashed lines the negative parity orbits.
whereas for oblate deformation $\eta < 0$. For spherical symmetry $\eta = 0$
and the level order corresponds to the shell model order. Each single-
particle orbit is labeled with the appropriate value of $K^\pi = \Omega^\pi$.

From the viewpoint of the Nilsson model, the $^{23}\text{Mg}_{11}$ nucleus con-
sists of a core composed of 12 protons and 10 neutrons coupled to zero
total angular momentum with a single neutron outside of this core. If
$\eta \approx +4$ for $^{23}\text{Mg}$, it is seen in Fig. 35 that this suggests that the mini-
imum-energy configuration is $[12346]_7^4 [7]_7^2 7$, the abbreviation repre-
senting a core composed of two neutrons and two protons (in accordance
with the Pauli exclusion principle) in orbits 1, 2, 3, 4, and 6, two protons
in orbit 7 coupled to zero angular momentum, and with the unpaired neu-
tron also in orbit 7. For this configuration $\Omega^\pi = 3/2^+$ and for the lowest
state of a rotational band $J = K = \Omega$. The ground state spin and parity of
$^{23}\text{Mg}$ would be $J^\pi = 3/2^+$ as is observed. To show that this configuration
has the minimum energy the total nuclear binding energy may be deduced
from the single particle energies. Preston (61) gives this total nuclear
energy, neglecting Coulomb energy differences and residual interactions,
as
\begin{equation}
E = \left(\frac{1}{2}\right)(1 + \mu^*/2m) \sum_i E_i - \left(\frac{\mu^*}{4m}\right) \sum_i \left\langle \frac{\vec{l}_i \cdot \vec{s}_i + D \vec{l}_i^2}{2} \right\rangle
\end{equation}
(30)
where $E_i$ is the Nilsson single particle energy, $\mu^*$ is an effective mass
resulting from a momentum correction to the oscillator potential, and $C$
and $D$ are related to $K$ and $\mu$ through Eq. 31; the summation index $i$ runs
over all occupied levels. In addition, there should be a rotational energy
term of the form $\frac{\hbar^2}{2D} \frac{J^2}{2} \mathcal{D}$ where $D = B \eta^2$, $B$ being an empirical constant. (64) An estimation of the equilibrium shape for the $^{21}\text{Ne}$-$^{21}\text{Na}$ mirror pair has been carried out (6) by simply summing the energy eigenvalues, i.e., the $E_1$ of Eq. 39, of Fig. 35 for 21 nucleons, resulting in the energy curves of Figure 36. In this figure, taken from Ref. 6, the energy ordinate has been arbitrarily normalized to an excitation scale. In view of the manner in which these curves were obtained, they should be appropriate to a discussion of the $^{23}\text{Mg}$-$^{23}\text{Na}$ mirror pair. These curves indicate that the \[\begin{bmatrix} 12346 \end{bmatrix}^4 \begin{bmatrix} 7 \end{bmatrix}^2 \] configuration has the minimum energy with a prolate equilibrium deformation of $\eta \approx +4.8$.

In addition to single particle excitations formed by placing the unpaired nucleon in one of the unoccupied levels, it is also possible to form hole states or core-excited states by promoting a core nucleon into an orbit with the unpaired nucleon. Orbits 4 and 6 in Fig. 36 are expected to be the lowest-lying hole states for the $N = 11$ nuclei. It should be mentioned that the Nilsson model is not entirely consistent since it allows excitations of the last odd particle to energies higher than those needed to generate hole states. The possibility of hole states will, however, be considered in this discussion. It is also interesting to note from Fig. 36 that different equilibrium distortions arise from different configurations. Other quantities such as the "inverse moment of inertia", $\frac{\hbar^2}{2D_0}$, may also have slightly different values for different configurations, a result that is neither unexpected nor physically unreasonable.
Figure 36. Total Normalized Binding Energy for $A = 21$. These total nuclear binding energy curves for mass 21 nuclei are from Ref. 6 and were formed by summing the single particle energy eigenvalues shown in Fig. 35 for the two configurations shown. For example, the configuration $[12346]^4 B$ represents a core with four nucleons, two protons and neutrons, each, in orbits 1, 2, 3, 4, and 6, with the odd nucleon in the orbit indicated by the letter B. The solid curves in this figure represent single particle excitations, whereas the dashed curves represent hole or core excited states formed by promotion of a core nucleon from Nilsson orbits 6 ($K^\pi = 1/2^+$) and 4 ($K^\pi = 1/2^-$), respectively, into orbit 7. The minimum energy is associated with the $[12346]^4 7$ configuration for which $\eta \approx +4.8$. The other energy curves are presented as excitations above this minimum energy. With a slight change of scale, these curves are appropriate for a discussion of the $^{23}\text{Mg}$ and $^{23}\text{Na}$ mirror pair.
With the identification of the $^{23}$Mg ground state as the lowest member of a possible $K^\pi = 3/2^+$ rotational band, it is reasonable to assume that the levels at 0.451, 2.048, and 2.712 MeV with favored spins and parities of $5/2^+$, $7/2^+$, and $9/2^+$, respectively, are the second, third, and fourth members of this band. To test this assumption the branching ratios and mixing ratios measured for the transitions among these levels were compared with the predictions of the collective model assuming that each of the four levels belonged to an unperturbed $K^\pi = 3/2^+$ rotational band based on the orbit 7 ground state.

For intraband transitions ($K \neq 1/2$), the electromagnetic transition probabilities between states with spins $J$ and $J'$ and $K = K' = 3/2$ are given by (62)

$$T(M1) = \frac{1}{\pi} \left( \frac{e \gamma}{\hbar c} \right)^2 \left( \frac{\pi}{\hbar c} \right)^2 \langle J', 3/2, 0 | J, 1, J' | 3/2 \rangle^2$$

$$\cdot G_{M1}^2 \text{ sec}^{-1}$$

(40)

and

$$T(E2) = \frac{4 \pi}{75} \frac{1}{\pi} \left( \frac{e \gamma}{\hbar c} \right)^5 B(E2; J \rightarrow J') \text{ sec}^{-1}$$

(41)

where

$$B(E2; J \rightarrow J') = \frac{5}{16 \pi} e^2 Q_0^2 \langle J, 2, 3/2, 0 | J, 2, J', 3/2 \rangle^2$$

(42)

and

$$G_{M1} = g_S \left( a_{21}^2 - a_{22}^2 \right) - 3 g_R$$

(43)

Here $g_S$ and $g_R$ are, respectively, the neutron and core gyromagnetic ratios and $E_\gamma$ is the transition energy. For these calculations $g_R$ was
taken to be \( \frac{Z}{A} \), the hydrodynamic estimate, and also 0.30, a more realistic value for s-d shell nuclei. (6, 63) The \( a_{21} \) and \( a_{22} \) coefficients (functions of \( \eta \)) are the Nilsson normalized eigenfunctions appropriate to orbit 7. In calculating the intrinsic quadrupole moment, \( Q_0 \), from Eq. 37, the parameter \( \chi \) was taken to be 0.08 and 0.10 and \( \eta \) assumed the values 2, 4, and 6. The expression for \( B(E2) \) neglects the single-particle contribution as collective effects are anticipated to predominate for intraband transitions. Single-particle effects are also small here since the electromagnetic effects of a chargeless neutron are quite small. It should be mentioned that the \( B(E2) \) expression used here differs by apparently a factor of 4 from the collective contribution of the \( T(E2) \) expression used by Bromley et al., (65) Howard et al., (6) Dubois and Earwaker, (24), and by Poletti and Start. (7) This observation was pointed out by Da Silva et al. (23) The \(^{23}\text{Mg} \) calculations carried out by Poletti and Start appear to be overestimated using their \( T(E2) \) expression.

The transitions within the \( K^\pi = 3/2^+ \) band are assumed to be either \( E2-M1 \) mixtures or pure \( E2 \) in character. The probability for transitions from a state \( i \) to state \( j \) is expressed as the sum of the relevant partial probabilities (6):

\[
T_{ij} = T_{ij}^{(E2)} + T_{ij}^{(M1)}
\]

(44)

The square of the multipole mixing ratio is

\[
\chi_{ij}^2 = \frac{T_{ij}^{(E2)}}{T_{ij}^{(M1)}}
\]

(45)
For cases where branching occurs from a state $i$ to two lower-lying states $j$ and $j'$, respectively, the branching ratio is expressed as

$$BR = \frac{T_{ij'}}{T_{ij}} \quad .$$

(46)

The quantities $x_{10}^2$, $x_{21}^2$, $T_{20}/T_{21}$, and $T_{41}/T_{42}$ were calculated and compared with the weighted average experimental values given in Tables 2 and 3. These results are presented in Figure 37 where a nuclear deformation of approximately $+3 \leq \eta \leq +5$ is indicated, although a larger value is required to agree with the observed value of $T_{20}/T_{21}$. The experimental errors are substantial and $\eta$ cannot be further restricted.

Another empirical property which can be compared with the model is the sign of $x_{10}$ and $x_{21}$ which is predicted for the $K^\pi = 3/2^+$ band based on orbit 7 to be (62)

$$\text{sign } x = \text{sign } q_s \left( a_{21}^2 - a_{22}^2 \right) / q_R - q_{Q0} \quad .$$

(47)

This quantity is negative for both $x_{10}$ and $x_{21}$ in agreement with the experimental phases given in Table 3. In Table 4 are summarized the experimental quantities just considered and the corresponding theoretical predictions calculated with $\eta = +4$, $\chi = 0.10$, and $q_R = 0.30$. The rather good agreement between theory and experiment supports the initial assumption that the members of the $K^\pi = 3/2^+$ band are relatively unperturbed by mixing with other rotational bands.

The Nilsson model can be used to predict the $f_0 \tau$ values for beta
Figure 37. Gamma-Ray Transition Probabilities for the $^{23}\text{Mg}$ $K^\pi = 3/2^+$ Rotational Band. These diagrams illustrate the comparison of the experimental and calculated mixing and branching ratios for the $\gamma$-ray transitions between members of the $^{23}\text{Mg}$ ground state rotational band. The experimental values, shown shaded, are the weighted average results determined from the present study and Ref. 23 and 24 and are summarized in Tables 2 and 3. The dashed lines in these figures are the collective model calculations for $g_R = 0.30$ and $\chi = 0.10$ while the solid lines are the calculations for $g_R = Z/A = 0.52$ and $\chi = 0.08$. The calculated results for $g_R = 0.30$ with $\chi = 0.08$ and $g_R = 0.52$ with $\chi = 0.10$ are intermediate between the solid and dashed lines. The calculations were performed for $\gamma = 2$, 4, and 6. The mixing and branching ratios are defined in the text. The pertinent level scheme is shown at the bottom of the figure. The $J^\pi$ values are the ones favored in this study. Each of the levels is assumed to be a member of a pure $K^\pi = 3/2^+$ rotational band based on the Nilsson orbit 7, $J^\pi = 3/2^+$ ground state.
Table 4. Predicted and experimental mixing and branching ratios for the $K^{\pi} = 3/2^+$ rotational band

<table>
<thead>
<tr>
<th></th>
<th>Theory*</th>
<th>Experiment</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>This work</td>
</tr>
<tr>
<td>$X_{10}$</td>
<td>-0.074</td>
<td>-0.07 ± 0.052</td>
</tr>
<tr>
<td>$X_{21}$</td>
<td>-0.173</td>
<td>-0.182 ± 0.038</td>
</tr>
<tr>
<td>$T_{20}(E2)$</td>
<td>0.074</td>
<td>0.176 ± 0.036</td>
</tr>
<tr>
<td>$T_{21}(M1)+T_{21}(E2)$</td>
<td>0.074</td>
<td>0.176 ± 0.036</td>
</tr>
<tr>
<td>$T_{41}(E2)$</td>
<td>2.26</td>
<td>1.94 ± 0.28</td>
</tr>
<tr>
<td>$T_{42}(M1)+T_{42}(E2)$</td>
<td>2.26</td>
<td>1.94 ± 0.28</td>
</tr>
</tbody>
</table>

* Calculated with $\eta = 2$, $\chi = 0.10$, and $g_R = 0.30$.

† Weighted average of this work and Ref. 23 and 24.
decay and these results may be compared with the observed $\beta^+$ decay of the $^{23}\text{Mg}$ ground state. For Gamow-Teller allowed beta transitions with $\Delta J = 0, \pm 1$ and $\Delta \pi = +1$, $f_{\alpha t}$ is expressed as (3, 6)

$$f_{\alpha t} = B_g \left[ (1 - x) D_F(0) + x D_{GT}(0) \right]^{-1} \quad (48)$$

where $B_g = 2787 \pm 70$ and $x = 0.56 \pm 0.012$ are universal constants and $D_F(0)$ and $D_{GT}(0)$ are the respective Fermi and Gamow-Teller allowed transition matrix elements. For beta transitions between mirror nuclear nuclear states, $D_F(0) = 1$ whereas for Fermi-forbidden, Gamow-Teller allowed $\Delta J = \pm 1$ transitions, $D_F(0) = 0$. The matrix element $D_{GT}(0)$ can be expressed in terms of the $\gamma$-dependent Nilsson eigenfunctions. The calculation of $f_{\alpha t}$ is thus independent of all of the Nilsson parameters with the exception of the nuclear deformation. Comparison of these calculations with experiment should indicate this parameter more definitely than the $\gamma$-ray transition probabilities which are a function of $g_R$ and $\chi$, as well as $\gamma$. As mentioned previously, the $^{23}\text{Mg}$ ground state ($J^\pi = K^\pi = 3/2^+$) decays 91% to the $^{23}\text{Na}$ ground state ($J^\pi = K^\pi = 3/2^+$) with log ft $= 3.74 \pm 0.01$, and 9% to the $^{23}\text{Na}$ 0.439-MeV level ($J^\pi = 5/2^+$, $K^\pi = 3/2^+$) with log ft $= 4.45 \pm 0.03$. (6) For the 91% branch

$$f_{\alpha t} = 2787 \left[ 0.44 + 0.34 (a_{21}^2 - a_{22}^2) \right]^{-1} \quad (49)$$

and for the 9% branch

$$f_{\alpha t} = 1.246 \cdot 10^4 (a_{21}^2 - a_{22}^2)^{-1} \quad (50)$$

where $a_{21}$ and $a_{22}$ are the Nilsson normalized eigenfunctions for orbit 7.
Within the Nilsson model, the calculation of beta-ray transition probabilities can only be carried out between states having the same nuclear deformation. The calculations were performed taking $\eta = +4$ for both $^{23}\text{Na}$ and $^{23}\text{Mg}$ and assuming the levels involved in both nuclei to be members of unperturbed $K^\pi = 3/2^+$ rotational bands. For the 91% branch, $\log f_0 t = 3.60$, and for the 9% branch, $\log f_0 t = 4.22$. Both results are only approximately 5% lower than the observed values and support both the validity of the collective model and the existence of a nuclear deformation on the order of $\eta = +4$ for $^{23}\text{Na}$ and $^{23}\text{Mg}$.

Still another quantity which can be calculated within the context of the Nilsson model is the magnetic moment of the ground state. Taking the $^{23}\text{Mg}$ ground state to the bandhead of a pure $K^\pi = 3/2^+$ rotational band based on orbit 7, this moment is given as $(3,6)$

$$\mu = (3/5) \left[ g_s \left( \frac{a_{21}^2 - a_{22}^2}{2} \right) + g_R \right]. \quad (51)$$

Taking $\eta = +4$ and $g_R = 0.30$ leads to $\mu = -0.816$ nm. Although $\mu$ has not been experimentally determined for $^{23}\text{Mg}$, it is interesting to note that this calculated value is intermediate between the observed ground state magnetic moments of $^{21}\text{Ne}$ and $^{25}\text{Mg}$: $-0.662$ nm and $-0.86$ nm, respectively. \text{(66)}

Within the spirit of the Nilsson model, each of these nuclei has a neutron for the odd nucleon as is assumed for $^{23}\text{Mg}$. The negative sign for the magnetic moments of these two nuclei reflects the negative magnetic moment of a free neutron.
With the identification of a $K^\pi = 3/2^+$ rotational band it is encouraging to look for other bands in $^{23}$Mg. The sequence of levels at 2.356, 2.904, and 3.968 MeV with favored spins and parities of $1/2^+$, $3/2^+$, and $5/2^+$, respectively, suggests that these levels are members of a $K^\pi = 1/2^+$ rotational band with the bandhead at 2.356 MeV. From the Nilsson energy level diagram of Figure 35, it is seen that for a prolate deformation of $\eta \approx 4$ the $1/2^+$ level with the observed excitation would most likely be formed from a $[12346]^4[7]^2$ configuration or from a $[1234]^4[\beta]^3[7]^4$ configuration representing, respectively, a single-particle excitation and a core-excited state. Both configurations would leave the unpaired neutron in an orbit with $\Omega^\pi = 1/2^+$, and assuming the 2.356-MeV level to be the bandhead, the spin and parity of this state would be $J^\pi = K^\pi = \Omega^\pi = 1/2^+$ as is observed. The association of the 2.356-MeV level with orbit 9 or 6 can be approached in several ways. The total binding energy curves of Figure 36 suggests that the lowest lying $1/2^+$ configuration for N or Z = 11 nuclei should be generated by promotion of the unpaired particle into orbit 9. Using Eq. 36, the energy difference, $\Delta E_{\beta,\beta'}$, between two Nilsson orbits, $\beta$ and $\beta'$, within the same oscillator shell is

$$\Delta E_{\beta,\beta'} = \chi \eta^0 \Omega^0 \Delta r_{\beta,\beta'}(\eta)$$

(52)

where $\Delta r_{\beta,\beta'}(\eta)$ is the difference in eigenvalues between the two orbits. The parameter $\chi \eta^0 \Omega^0$ is taken to be $41 A^{-1/3}$ MeV, after Nilsson. (3) The energy difference between orbits 7 and 9 and 7 and 6 was calculated
for $\eta = 4, \chi$ in the range $0.05 \leq \chi \leq 0.13$, $\mu$ in the range $0 \leq \mu \leq 0.5$, and compared with the observed difference of 2.356 MeV. Nilsson suggested $\mu = 0$ for the 2s-1d shell model region although Bishop (37) found that in some nuclei only a non-zero value for $\mu$ gives a consistent interpretation of the spectra in terms of the Nilsson parameters. The results of these calculations, illustrated in Figure 38, show that the 2.356-MeV level is best identified with orbit 9 if $\chi \approx 0.10$. For this value of $\chi$ a value of $\mu \approx 0.1$ seems more appropriate than a zero value. The level can be identified with orbit 6 for $\chi = 0.05$, although this value is now considered unrealistic for N or Z = 11 nuclei.

Another approach in identifying the 2.356-MeV level comes from a determination of the decoupling parameter, $a$, for the $K^\pi = 1/2^+$ band assumed to be based on this state. Using Eq. 21 it is found that for $\hbar^2/2\mathcal{D}_0 = 170$ keV and $a = -0.28$, the 3/2 and 5/2 members should be at 2.905 and 3.968 MeV, respectively, in agreement with experiment. The inverse moment of inertia is consistent with the expectation that it should be smaller for an odd-mass nucleus than for the neighboring even nucleus. From the first $0^+$ and $2^+$ states of $^{24}$Mg it is calculated that $\hbar^2/2\mathcal{D}_0 = 228$ keV. The decoupling parameter for orbit 9 or 6 can be calculated from the coefficients, $a_\Lambda (\eta)$, of the eigenfunctions for these orbits (3):

$$a = a_{20}^2 + 2\sqrt{6} a_{20} a_{21} + a_{00}^2. \tag{53}$$

Bishop (67) has performed these calculations for various values of $\mu$ and these results are presented in Figure 38 where they are compared with the
Figure 38. Energy Difference $\Delta E_{7,9}$ and $\Delta E_{7,6}$ versus $\mu$ and Decoupling Parameter for Nilsson Orbits 6 and 9 versus $\eta$. The two figures at the top of the page are plots of the energy difference between orbits 7 and 9 and 7 and 6 calculated using Eq. 52, with $\eta = 4$ and presented as a function of $\mu$ for different values of $\chi$. The observed value in these plots is the 2.356-MeV energy difference between the ground state (orbit 7) and the third-excited state (orbit 9 or 6). The two figures at the bottom of the page are plots of the decoupling parameter, $\eta$, for orbits 9 and 6 as a function of $\eta$ for different values of $\mu$. These plots are from Ref. 67. The observed value of $\eta$, i.e., 0.26, was obtained from a fit of Eq. 21 to the levels at 2.356, 2.904, and 3.968 MeV, assumed to be members of a $K^\Pi = 1/2^+$ rotational band based on the 2.356-MeV level. These plots show that the 2.356-MeV bandhead is best identified with Nilsson orbit 9.
result \( \alpha = -0.26 \). It is apparent from these curves that for \( \gamma \approx +4 \) the 2.356-MeV level is to be identified with orbit 9 rather than orbit 6. The curves for orbit 9 also show that \( \mu = 0 \) cannot reproduce the observed value of \( \alpha \) whereas \( \mu \approx 0.16 \) leads to good agreement.

With the identification of the \( 3/2^+ \) and \( 5/2^+ \) members of the \( K^\pi = 1/2^+ \) band, the predicted \( 7/2^+ \) member would lie at 4.86 MeV. The observed level closest to this predicted level is at 4.63 MeV and may have \( J^\pi = 7/2^+ \). The corresponding level in \(^{23}\)Na at 4.78 MeV does, in fact, have a favored spin and parity of \( 7/2^+ \). (56)

The next lowest-lying \( 1/2^+ \) band in \(^{23}\)Mg would be expected to be based on orbit 6, representing a hole state. The bandhead could be the \( 1/2^+ \) state at 4.353 MeV. The energy difference \( \Delta E_{7,6} \) with \( \gamma_6 = 4 \), can be met for \( \gamma = 0.08 \) and \( \mu = 0 \) or for \( \gamma = 0.05 \) and \( \mu = 0.5 \), with the former set of parameters favored. For \( \mu = 0 \) and \( \gamma = 4 \), the calculated decoupling parameter is approximately 2.2. (57) The level order for this band, if unperturbed by band mixing, would be \( 1/2 \), \( 5/2 \), \( 3/2 \), \( 9/2 \), \( 7/2 \), etc., as can be seen from the pertinent diagram of Section A. The next level in \(^{23}\)Mg above 4.68 MeV is at 5.29 MeV. Dubois and Earwaker (24) have found \( \ell_n = 2 \) for this level making it a likely candidate for the \( 5/2^+ \) member of this proposed band.

The levels at 2.768 and 3.792 MeV with favored \( J^\pi \) values of \( 1/2^- \) and \( 3/2^- \), respectively, appear to be members of a \( K^\pi = 1/2^- \) band based on the 2.768-MeV level. From Figure 35 and 36 it is seen that the
lowest-lying negative parity configurations for $\eta \approx 4$ would be expected to be $[12346]^2 [7]^2 14$ and $[12367]^4 [4]^2 4$, the first representing a single-particle excitation and the second a core excitation. For both configurations the unpaired neutron would be in an orbit for which $\Omega^\pi = 1/2^-$, leading to $J^\pi = 1/2^-$ for the assumed bandhead at 2.768 MeV. If the energy differences $\Delta E_{7,4}$ and $\Delta E_{7,14}$ are calculated, it is found that it is not possible to meet the observed difference with any reasonable value of $\Omega$. The total binding energy curve suggests that the configuration based on orbit 14 lies lower than the one based on orbit 4. As pointed out by Nilsson, the Nilsson energy level diagrams are not expected to yield the exact level order or correct energy differences between the levels but should tell which level spins and parities are likely to appear in the lowest states of the spectrum.

The decoupling parameters calculated for orbits 4 and 14 with $\mu = 0$ and $\eta = 4$ are +0.63 and -3.32, respectively. For $a = 0.63$ the level sequence for a $K^\pi = 1/2^-$ band would be, if unperturbed, $1/2^-$, $3/2^-$, $5/2^-$, $7/2^-$, etc., with the $3/2^-$ and $5/2^-$ members lying close together, the $7/2^-$ and $9/2^-$ members, etc. For $a = -3.32$ the level order, if unperturbed, would be $3/2^-$ followed by two close-lying $7/2^-$ and $1/2^-$ levels as seen in the pertinent diagram of Section A. With $\hbar^2 / 2\mathcal{D}_o = 170$ keV and $a = 0.65$, the energies of the $3/2^-$ and $5/2^-$ members of a $K^\pi = 1/2^-$ band based on the 2.768-MeV level are predicted, from Eq. 21, to be 3.682 and 3.888 MeV, respectively. This calculation accounts fairly well for the
observed $3/2^-$ level at 3.792 MeV and suggests that the observed level at 3.856 MeV has $J^\pi = 5/2^-$. This assignment is in agreement with a tentative \( l_n = 3 \) assignment to this level. (24) The $7/2^-$ and $9/2^-$ members of the band would be expected at 5.94 and 6.37 MeV, respectively. It appears that the 2.768-MeV bandhead is best identified with orbit 4.

The $^{23}$Mg energy level diagram is shown again in Figure 39 where the levels have been assigned to the rotational bands just proposed.

All of the calculations thus far have been made assuming pure states and at least for the calculation of the $\gamma$-ray transition probabilities within the \( K^\pi = 3/2^+ \) band this seems to be a good assumption. However, it is not possible to fit the energy levels of the \( K^\pi = 3/2^+ \) band using the simple form \( E(J) = (\hbar / 2 \sqrt{ \alpha_0 }) \left[ J(J + 1) \right] \) as would be expected for a pure band. Dubois (56) has recently obtained remarkable fits to the energy levels of $^{23}$Na but only by considering the mixing of many bands. These calculations were made possible with spin and parity assignments to many high-lying levels in $^{23}$Na which led to the identification of a number of well-developed rotational bands. Such an approach will surely be necessary in order to fit the $^{23}$Mg level scheme but this can only be attempted after spin and parity assignments are made for higher levels in $^{23}$Mg. It will be interesting to see if the mixed wave functions which have led to an excellent fit to the $^{23}$Na level scheme, and presumably would for $^{23}$Mg, can in turn lead to the $\gamma$-ray transition probabilities for the intraband and interband transitions in these two mirrors with equal results.
Figure 39. Proposed Rotational Bands in $^{23}\text{Mg}$. 
OBSERVED

\[ E_x (\text{MeV}) \quad J^{\pi} \]

\[
\begin{align*}
4.353 & \quad \frac{3}{2}^+ \\
3.968 & \quad (\frac{5}{2}^+) \\
3.792 & \quad 3.856 \quad (\frac{5}{2}^+) \\
3.72 & \quad (\frac{3}{2}^+) \\
2.904 & \quad \frac{3}{2}^+ (\frac{5}{2}^+) \\
2.712 & \quad \frac{1}{2}^+ (\frac{3}{2}^-) \\
2.356 & \quad \frac{1}{2}^+ \\
2.048 & \quad \frac{1}{2}^+ (\frac{3}{2}^-) \\
0.451 & \quad \frac{5}{2}^+ \\
0 & \quad 2^{3}\text{Mg} \\
\end{align*}
\]

PROPOSED ROTATIONAL BANDS IN \( ^{23}\text{Mg} \)

\[
\begin{align*}
\text{Ex} (\text{MeV}) & \quad J^{\pi} \\
4.353 & \quad \frac{3}{2}^+ \\
3.792 & \quad 3.856 \quad (\frac{5}{2}^-) \\
2.904 & \quad \frac{3}{2}^+ (\frac{5}{2}^+) \\
2.712 & \quad \frac{1}{2}^+ (\frac{3}{2}^-) \\
2.356 & \quad \frac{1}{2}^+ \\
2.048 & \quad \frac{1}{2}^+ (\frac{3}{2}^-) \\
0.451 & \quad \frac{5}{2}^+ \\
0 & \quad (\frac{3}{2}^+) \\
\end{align*}
\]

\[ K^{\pi} = \frac{1}{2}^+ \]

\[ K^{\pi} = \frac{3}{2}^+ \]
Among the rotational bands proposed in the present work, including levels up to 4.353 MeV, there are several sources of band mixing. The \( K^\Pi = 3/2^+ \) ground state band can mix with the \( K^\Pi = 1/2^+ \) band based on the 2.356-MeV level and also with the proposed \( K^\Pi = 1/2^+ \) band based on the 4.353-MeV level. In addition, these two \( 1/2^+ \) bands can mix together.

In an initial attempt to estimate the effect of band mixing in \( ^{23}\text{Mg} \), it is instructive to try to predict the branching ratio of the 2.356-MeV level. This level, which is the bandhead of a \( K^\Pi = 1/2^+ \) band based on orbit 9, decays 69\% to the \( J^\Pi = 5/2^+ \), 0.451-MeV level and 31\% to the \( J^\Pi = 3/2^+ \) ground state, both of these levels having \( K^\Pi = 3/2^+ \). It is interesting to note that the mirror level in \( ^{23}\text{Na} \) at 2.39 MeV, which has \( J^\Pi = K^\Pi = 1/2^+ \), decays in the opposite fashion: 65\% to the \( J^\Pi = 3/2^+ \) ground state and 35\% to the \( J^\Pi = 5/2^+ \), 0.439-MeV level, both of these levels having \( K^\Pi = 3/2^+ \). Since the \( ^{23}\text{Mg} \) ground state band has no \( 1/2^+ \) member, the 2.356-MeV level is not affected by mixing with the \( K^\Pi = 3/2^+ \) band. This state could, however, be perturbed by the \( 1/2^+ \) level at 4.353 MeV, although this effect should be small considering the energy separation of the two levels (e.g., see Eq. 23).

If the 2.356-MeV level is assumed to be a pure state, then only single particle effects can contribute to \( B(E2) \) as collective components in E2 transitions only exist where \( K = K' \). (32) Another pertinent selection rule is that M1 transitions are forbidden between bands where \( K - K' = 2 \). (32) The electric effects of a chargeless neutron would be quite small.
The effect of band mixing is to introduce collective components to the T(E2) transition probabilities which will exceed the single particle contribution.

The $K^{\pi} = 1/2^+$ and $3/2^+$ bands were mixed using the procedure of Kerman. (68) The inverse moment of inertia was taken to be 170 keV for both bands, and for $\eta = 4$ and $\mu = 0.167$, the mixing parameter $A_{7,9}$ of Eq. 22 was calculated to be $-0.994$. Explicitly (69),

$$A_{7,9} = a_{21}^1 \bar{a}_{21} + \sqrt{6} \ a_{20}^1 \bar{a}_{21} + 2 \ a_{21}^1 \bar{a}_{22} ,$$

the primed and unprimed coefficients corresponding to orbits 9 and 7, respectively. Using these quantities the wave functions for the ground state and 2.356-MeV state were found to be

$$\psi_{3/2} (0 \text{ MeV}) = -0.10 \psi_{3/2, 1/2} + 0.995 \psi_{3/2, 3/2}$$

and

$$\psi_{1/2} (2.36 \text{ MeV}) = \psi_{1/2, 1/2}$$

(55)

where the $\psi_j$ are expressed in terms of the unperturbed wave functions $\psi_{J, K}$. Using Kerman's expression for $B(E2)$ and $B(M1)$ with $q_{R} = 0.30$ and $\chi = 0.10$, the branching ratio $T_{30} / T_{\text{total}} = T_{30} (E2) + T_{30} (M1) / T_{30} + T_{31} (E2)$ was calculated to be 100% whereas the observed value is 31%. This calculation was not sensitive to the E2 width since the M1 width completely dominates the E2 width. Agreement between the calculated and observed branching ratios can be improved by increasing the moment of inertia parameter although the amount required is much greater than expected. Another difficulty is that increasing this parameter leads to
larger discrepancies in the calculated energy levels. A similar situation was found by Bromley et al. (65) in mixing the $K^\pi = 3/2^+$ and $1/2^+$ bands in $^{28}$Si. The discrepancy between the calculated and experimental branching ratio could be removed if either $B(E2)$ was underestimated or $B(M1)$ was overestimated. Both effects occur frequently in s-d shell nuclei.

Poletti and Start (7) have suggested that the decay of the $^{23}$Na 2.39-MeV level could be understood if some nuclear structure effect inhibited the M1 transition to the ground state by a factor of at least $2 \times 10^3$.

The branching ratio and mixing ratio was calculated in a similar fashion for the decay of the 2.904-MeV level. This level, the assumed $3/2^+$ member of the $K^\pi = 1/2^+$ band, decays 64% to the ground state and 46% to the 0.451-MeV level. The mixed wave functions with $\gamma = 4$ and $(A_{7,9}) (\frac{3}{2}^+_{-} 2_{-}^1) = 200$ keV were calculated to be

$$\psi_{3/2}^{(2.90 \text{ MeV})} = 0.905 \psi_{3/2,1/2^+} + 0.117 \psi_{3/2,3/2}$$

and

$$\psi_{5/2}^{(0.45 \text{ MeV})} = -0.172 \psi_{5/2,1/2} + 0.985 \psi_{5/2,3/2} \ .$$

The calculated branching ratio $T_{60}/T_{\text{total}}$ was 46% in fair agreement with the observed value of 64%. The mixing ratios $x_{60}$ and $x_{61}$ were found to be 0.03 and -0.01, respectively, in comparison with the experimental values $x_{60} = -0.185 \pm 0.014$ and $x_{61} = -0.079 \pm 0.036$. These experimental results are the weighted averages from Table 3. Agreement between the calculated and experimental results could be improved, again, if either $B(E2)$ was underestimated or $B(M1)$ was overestimated, although
the discrepancy in the sign of $x_{60}$ is not trivial.

The band-mixing calculations for the 2.904-MeV level are encouraging and the inclusion of other bands could conceivably lead to better agreement between theory and experiment. A lifetime measurement for this level would be extremely useful since it would lead to a determination of the separate $E2$ and $M1$ widths. The inhibition or enhancement of these partial transition probabilities could then be tested.

The decay of the $1/2^+$ state in $^{23}$Na and $^{23}$Mg at 2.39 and 2.356 MeV, respectively, appears somewhat anomalous. The ground state transition in both nuclei, which competes favorably with the assumed pure $E2$ transition to the $5/2^+$ first excited state, implies a strongly inhibited $M1$ component in this radiation. As an inspection of Figure 34 will show, these two levels decay in opposite fashions whereas the decay of the other corresponding levels is quite similar. A determination of the partial widths $T_{30}(E2)$ and $T_{30}(M1)$ is needed here to test the implied $M1$ inhibition.
Chapter V

CONCLUSIONS

Most of the objectives set forth at the beginning of this work have been reached. Spin and parity assignments, in conjunction with known $l_n$ assignments, have been made to all of the excited states up to 4.353 MeV in $^{23}$Mg, with the exception of the weakly excited 3.856-MeV level. It was shown on the basis of several arguments that a $J^\pi = 5/2^-$ assignment is likely for this level. Gamma-ray branching ratios were obtained for nine excited states as were the multipole mixing ratios for most of these transitions. The measurements with the Ge(Li) detector led to the determination of the decay modes of the 2.712- and 2.768-MeV levels and also to the confirmation of the energies of most of the levels studied. The collective model calculations made, although not extensive, demonstrated that such a model is applicable to the $^{23}$Mg nucleus. A prolate deformation of $\eta \approx 4$ is indicated for this nucleus and all of the levels up to at least 4.353 MeV can be identified as members of rotational bands.

The method of axial symmetry has been seen to be a good method for obtaining nuclear spectroscopic information although additional (186)
information such as parities, lifetimes, etc., must be known in most cases
to make firm spin assignments. The Ge(Li) detector can be an invaluable
tool in an angular correlation experiment although coincidence measurements
with these counters is admittedly a very inefficient technique. The three-
parameter analysis method was seen to be a useful technique and it seems
that this is the best way to measure angular correlations of the type studied
in this work.

The results of the present study suggest further experiments and
work. The favored spin assignments to the 2.048-, 2.712-, and 2.904-MeV
levels could possibly be further substantiated by repeating the correlation
measurements using more angles with better statistics. Some type of peak-
unfolding computer program would be useful in extracting the 2.23-MeV γ
ray from the decay of the 2.712-MeV level. A computer program which
could perform simultaneous $\chi^2$ fits might be useful in analysing the corre-
lations measured in this work. Lifetime measurements for the levels of
$^{23}\text{Mg}$ are needed as such measurements would yield the total transition
probabilities which could be coupled with the measured mixing and branch-
ing ratios to determine the partial transition probabilities, e.g., $T_{21}(E2)$.
These quantities could then be compared with the model prediction. The
determination of the $\lambda_n$ value for the $^{23}\text{Mg}$ 3.968-MeV level is needed to
determine the parity of this level and the $\lambda_n = 1$ assignments to the 2.768-
and 3.792- MeV levels (24) should be confirmed using the $^{24}\text{Mg}(p, d)^{23}\text{Mg}$
reaction. In addition, the energies of the $^{23}\text{Mg}$ levels could be determined
with an error of approximately 1 keV or less using the Ge(Li) detector.
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