JOHN EDMUND MCGREGOR, B.S. A Study of the $^3\text{He},\alpha$ Reaction with Several s-d Shell Nuclei. (Under the direction of Dr. EDWARD J. LUDWIG.)

ABSTRACT

Using the Duke University 4 MeV Van de Graaf accelerator to produce an 8 MeV beam of doubly-ionized $^3\text{He}$, angular distributions of $^3\text{He}$ particles elastically scattered from the nuclei $^{24}\text{Mg}$, $^{26}\text{Mg}$, $^{28}\text{Si}$, $^{30}\text{Si}$ and $^{32}\text{Si}$ were measured. Angular distributions were also obtained of the alpha particles resulting from the reactions $^{24}\text{Mg}(^3\text{He},\alpha)^{28}\text{Mg}$, $^{26}\text{Mg}(^3\text{He},\alpha)^{30}\text{Mg}$, $^{28}\text{Si}(^3\text{He},\alpha)^{32}\text{Si}$, $^{30}\text{Si}(^3\text{He},\alpha)^{34}\text{Si}$ and $^{32}\text{Si}(^3\text{He},\alpha)^{36}\text{Si}$. The Mg and Si targets were made from isotopically enriched oxides while an antimony sulfide target was used for sulfur. The scattered particles were collected in an array of 6 solid state surface barrier detectors and the pulses from these detectors were analyzed and processed by an on-line DDP-324 computer. Excitation curves were taken from 6.2 to 8.4 MeV for the $^3\text{He},\alpha$ reaction on $^{24}\text{Mg}$, $^{28}\text{Si}$ and $^{30}\text{Si}$. These data were analyzed using the theory of Ericson fluctuations which indicated that the reaction was predominately ($>80\%$) direct at angles less than 60 degrees. The elastic scattering data was fit with an optical model computer code and parameters for the entrance channel of a DWBA analysis were obtained. The DWBA calculations were made and compared to experiment and spectroscopic factors were extracted. Evidence was seen for a $J$-dependent
effect in the $l=2$ transfer processes which could not be produced in the theoretical calculations by the addition of a spin-orbit potential. The cross sections of the $^3\text{He}, \alpha$ reactions on $^{24}\text{Mg}$, $^{28}\text{Si}$ and $^{32}\text{S}$ were found to be considerably larger than those for $^{26}\text{Mg}$ and $^{32}\text{S}$. This is explained as being partially the result of the Q-value differences with the remaining difference attributed to reaction mechanisms other than the ordinary neutron pickup process.
A STUDY OF THE $^{3}\text{He}, \alpha$ REACTION
WITH SEVERAL S-D SHELL NUCLEI

by

John Edmund McQueen, Jr.

A thesis submitted to the faculty of the University of North Carolina at Chapel Hill in partial fulfillment of the requirements for the degree of Doctor of Philosophy in the Department of Physics

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__________________________
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CHAPTER I
INTRODUCTION

The purpose of this dissertation is to describe a systematic study that has been made of the \(^3\text{He, a}\) reaction on five nuclei in the s-d shell at an incident \(^3\text{He}\) energy of 8 MeV. These nuclei were \(^{24}\text{Mg}\), \(^{26}\text{Mg}\), \(^{28}\text{Si}\), \(^{30}\text{Si}\) and \(^{32}\text{S}\). A theoretical analysis of the DWBA theory as applied to \(^3\text{He, a}\) reactions has been made by Stock et al.\(^{1}\) in 1967. They have determined the absolute normalization constant for the ratio of theoretical to experimental cross sections to be approximately 23.

From a detailed examination of the approximations made in the DWBA theory, they conclude that the theory is most applicable if the optical model real well depth for the \(a\)-particle is approximately equal to the sum of the \(^3\text{He}\) and neutron well depths, where the depth of the neutron well is generally considered to be around 50 MeV. This is in contrast to the shallow (50-100 MeV) potentials often used in \(^3\text{He, a}\) reaction comparisons of theory to experiment \(^2\text{-7}\).

Previous \(^3\text{He, a}\) reactions on the nuclei studied in this experiment are described in reference 2) through 13). The neutron pickup reactions \((p, d)\) and \((d, t)\) on these nuclei are described in references 3, 5, 14 and 15).

Whereas the stripping reaction measures how empty a shell is,
the pickup reaction gives an indication of how full a shell is. Thus, oftentimes the ground state neutron configuration of the target and the single particle nature of the excited states in the residual nucleus can be inferred. Also, the \( ^3\text{He},d \) reaction provides a good way to study single neutron hole states.

For \( ^3\text{He},d \) reactions in general, the Q-values range from 5 to 10 MeV, thus giving the emergent alphas considerable energy above the peaks corresponding to elastic scattering. The only competing reaction is the \( ^3\text{He},p \) reaction where the high energy protons are not stopped in thin solid state detectors. The large Q-value also gives rise to the so-called angular momentum mismatch, which tends to enhance the cross section of those states excited with a higher angular momentum transfer. Therefore, the residual states reached via an \( l=2 \) transfer are abundantly populated. The high Q-value also intensifies the reactions leading to analogue states in the residual nuclei.

One of the chief aims of this experiment was to obtain a consistent set or sets of optical model parameters corresponding to the elastic scattering of \(^3\text{He}\) particles from the nuclei \(^{24}\text{Mg}, ^{26}\text{Mg}, ^{28}\text{Si}, ^{30}\text{Si}\) and \(^{32}\text{S}\). The shapes of these nuclei vary from very deformed (\(^{24}\text{Mg}\)) to nearly spherical (\(^{32}\text{S}\)). It was our intent to investigate the reaction mechanisms operative at the incident \(^3\text{He}\) energy of 8 MeV. It was found that at angles less than 60 degrees, the reaction was predominately direct (\( > 90\% \)) thus enabling a DWBA analysis assuming
neutron pickup. Spectroscopic factors can then be extracted and compared to those obtained from other neutron pickup reactions on these nuclei. These comparisons can yield information as to systematic trends as one moves up through the s-d shell. Using the spectroscopic factors, the ground state neutron configuration can sometimes be deduced.

In 1966, Schiffer et al.\textsuperscript{16} published a paper specifically treating J-dependence in the \( ^{28}\text{Si}(d,p)^{29}\text{Si} \) reaction. Other studies of J-dependence with different reactions have been done; namely with the \((p,d)\) reaction\textsuperscript{14} and the \((^3\text{He},d)\) reaction\textsuperscript{5,7,17,18}. One of the main purposes of this experiment is to search for a systematic J-dependence at 8 MeV and, if any is found, to compare it to that found in \((^3\text{He},d)\) reactions at other energies.
CHAPTER II
THEORETICAL CONSIDERATIONS

A. Fluctuation Theory

In a 1960 paper and in subsequent papers, Ericson has advanced a theory to explain the occurrence of certain fluctuations in reaction yields when the bombarding energy is smoothly varied. For the same entrance channel parameters, these fluctuations are seen to be uncorrelated between the different exit channels although a large number of intermediate states are excited. These fluctuations have consequently been designated as "Ericson fluctuations." In order for these fluctuations to be observable, the average energy width $\Gamma$ of the intermediate compound state must be much larger than the average level spacing $D$; i.e.

$$\Gamma \gg D.$$  

(2-1)

This is generally true at energies 3 to 4 MeV above the neutron binding energy. Another condition is that the experimental energy resolution must be smaller than $\Gamma$; otherwise the fluctuations will be averaged and the scattering described by Hauser-Feshbach theory.

If the initial state is labeled $|\alpha\rangle$, the final state $|\alpha'\rangle$, and the intermediate compound state $|i\rangle$, then the reaction is described by a scattering matrix $S_{\alpha i}$ which can be separated into two distinct parts:
$S_a$, which represents scattering from the initial state $|i\rangle$ into the compound state $|a\rangle$; and $S_a'$, which represents scattering from $|i\rangle$ into the final state $|a'\rangle$. For a given excitation $E$ in the compound nucleus, the probability of the intermediate state $|i\rangle$ at energy $E_i$ being formed is given by $f(E, E_i)$, which is proportional to

$$f(E, E_i) \propto \frac{1}{(E-E_i) + i \Gamma/2} \quad (2-2)$$

where $\Gamma$ is generally dependent on $i$. However, at high excitation energies (3 or 4 MeV above the neutron binding energy), $\Gamma$ is a sum of numerous partial widths and essentially becomes independent of the state $i$.

By virtue of the width of the compound state, all of the levels within a region $\Gamma$ are excited simultaneously and must be treated coherently. Thus the cross section can be written as

$$\sigma_{aa'}(E) \propto \Sigma_i \left( |\langle a \mid S_a \mid i \rangle \rangle f(E, E_i) \langle i \mid S_a' \rangle \langle a' \rangle \right)^2 \quad (2-3)$$

Now since this is compound scattering, $|\langle a \mid S_a \mid i \rangle \rangle$ and $\langle i \mid S_a' \rangle \langle a' \rangle$ have random phases with respect to each other. Also, since $\Gamma \gg D$, the number of states $i$ within $\Gamma$ is large and $|\langle a \mid S_a \mid i \rangle \rangle$ and $\langle i \mid S_a' \rangle \langle a' \rangle$ become random numbers, the real and imaginary parts of which have Gaussian distributions. If the transitions $|\langle a \mid a' \rangle \rangle$ and $|\langle a \mid a'' \rangle \rangle$ are compared under otherwise identical conditions, the matrix elements $\langle i \mid S_a' \rangle \langle a' \rangle$ and $\langle i \mid S_a' \rangle \langle a'' \rangle$ are independently random and thus result in no correlation in the final cross sections. This explains the uncorrelated fluctuations of the different exit channels for the same entrance channel.
Suppose one now defines an average cross section
\[ \langle \sigma \rangle = \frac{1}{m} \sum_{i=1}^{m} \sigma(E_i) \]
and an auto-correlation function
\[ C(\epsilon) = \frac{\langle \sigma(E+\epsilon) \sigma(E) \rangle}{\langle \sigma^2 \rangle} - 1 \]
where \( \langle f \rangle \) denotes an energy average. Note that
\[ C(0) = \frac{\langle \sigma^2 \rangle - \langle \sigma \rangle^2}{\langle \sigma \rangle^2} \]
which is called the normalized variance. \( C(0) \) can easily be determined from an excitation curve provided there are enough points to constitute a statistically reliable sample. It has been shown by von Witsch et al.\textsuperscript{22) that if the fluctuation cross section is independent of the channel spins, then
\[ C_{\text{experimental}}(\epsilon) = C_{\text{theoretical}}(0)(1-y_D^2) \frac{\Gamma^2}{\Gamma^2 + \epsilon^2} \]
where \( C_{\text{theoretical}}(0) \) can be expressed as
\[ C_{\text{theoretical}}(0) = \frac{1}{N_{\text{max}}} = \frac{2}{(2\Gamma+1)(2\Gamma+1)(2\Gamma'+1)(2\Gamma'+1)} \]
and \( y_D \) is the percentage of direct reaction. Here \( N_{\text{max}} \) is the number of contributing degrees of freedom and is determined by the spins \( I, i, I', \) and \( i' \) of the target, incident, residual and product nuclei, respectively. \( \Gamma \) is referred to as the coherence energy. If, in this region of strongly overlapping levels, \( \Gamma \) is independent of the spin \( J \), then from equation (2-7),
\[ C_{\text{Experimental}}(\varepsilon) = \frac{\Gamma^2}{\Gamma^2 + \varepsilon^2} C_{\text{Experimental}}^{(0)}. \quad (2-9) \]

This equation can be used to determine the value of \( \Gamma \), with which the percentage of direct reaction can be determined with the aid of equation (2-7). Ericson \(^{23}\) and Brink et al. \(^{24}\) have also derived a formula for the probability distribution of the differential cross section. This is expressed as

\[
P_{N, Y_D}(Y) = \left(\frac{-N}{1 - Y_D}\right)^N Y^{N-1} \exp\left(-\frac{N(Y+Y_D)}{1 - Y_D}\right) \frac{I_{N-1}(\frac{1}{1 + Y_D})}{\frac{1}{1 + Y_D}(1 - Y_D)^{-1}},
\]

\[
I_{N-1} = \frac{2iNY_D^{\frac{1}{2}}(1-Y_D)^{-\frac{1}{2}}}{1 - Y_D^2}
\]

(2-10)

Where \( Y = \sigma(\theta)/\langle\sigma(\theta)\rangle \), \( I_{N-1} \) is the modified Bessel function of order \( N-1 \) and \( N \) is usually taken to be between \( \frac{1}{2} N_{\text{max}} \) and \( N_{\text{max}}^{25} \). Assuming a value for \( N \), \( P(Y) \) can be calculated for different values of \( Y_D \) and compared to experimental histograms. This gives a measure of the percentage of compound reaction contributing to the cross section.

B. The DWBA for Direct Reactions

A direct reaction is considered to be a process in which the incident particle interacts with but a few of the target nucleons in a time interval of approximately \( 10^{-22} \) seconds. On the other hand, the formation and subsequent decay of the compound nucleus involves many interactions in which the energy of the incident particle is shared among many of the target nucleons over a time period of \( 10^{-16} - 10^{-17} \) seconds.
It is thought that, with the exception of isolated resonances, the direct reaction is the predominant process occurring at incident energies above about 10 MeV. As the energy is decreased below this value there is an increasingly larger contribution to the cross section from scattering through a compound nucleus. The direct reaction will be the mechanism described in the discussion below.

The DWBA description of the pickup process will be based mainly on a paper by Stock et al.\(^1\) If one considers the pickup process characterized by

\[
a + (B + n) \rightarrow B + (a + n),
\]

where "a" is the incident particle, "B" is the nuclear core, and "n" is the transferred particle, then the transition matrix can be written as

\[
T = \left\langle \phi_{a+n}^{-}, B \right| V_{aB} + V_{an}^{\text{opt}} + U_{a}^a \phi_{B+n} \phi_{B+n} \right| \chi_{a}^{+} \rightangle,
\]

where \(\phi^{-}\) is the solution to the total Hamiltonian

\[
H = H_{B} + H_{a+n} + T_{B,a+n} + V_{aB} + V_{Bn},
\]

and is subject to the boundary condition that \(a+n\) is outgoing. The \(\phi_i\) represent the internal wave functions of \(i\), that is \(\phi_B\) is the wave function for the core. \(U^a\) is the optical potential in the entrance channel and \(\chi_a^+\) is the solution to the Schrodinger equation with optical potential \(U^a\) for the relative motion of \(a\) and \(B+n\). \(V_{ij}\) represents the interaction between particles "i" and "j".

The formal solution of the Lippmann-Schwinger equation for \(\psi^{-}\) is
\[ \left| \psi_{a+n}^{-} \right> = \left| \chi_{a+n}^{-} \Phi_{a+n}^{+} \right> + (E^{-} - H)^{-1} (V_{aB} + V_{En} - U^{a+n}) \left| \chi_{a+n}^{-} \Phi_{a+n}^{+} \right> . \]  

(2-14)

If this is substituted into (2-12), one obtains

\[ T = \left< \Phi_{B}^{+} a+n \chi_{a+n}^{-} \left| \left( 1 + (V_{aB} + V_{En} - U^{a+n})(E^{+} - H)^{-1} \right) (V_{aB} + V_{an} - U^{a}) \right| \Phi_{a}^{+} B+n \chi_{a}^{-} \right> , \]

(2-15)

where \( U^{a+c} \) and \( \chi_{a+c}^{-} \) are the exit channel optical potential and corresponding distorted wave. The \( X_i \) satisfy the Schrödinger equation

\[ \left[ \nabla^2 + k_{ij}^2 - \frac{2M_{ij}}{\hbar^2} U(z_i, z_j, r) \right] X_{ij} = 0 \]  

(2-16)

where \( k_{ij} \) and \( M_{ij} \) are the wave number and reduced mass, respectively, and \( z_i \) and \( z_j \) are the charge numbers of particles \( i \) and \( j \). \( U(z_i, z_j, r) \) is the optical potential with the appropriate coulomb potential included.

The Born approximation consists of neglecting

\[ (V_{aB} + V_{bn} - U^{a+n}) (E^{+} - H)^{-1} . \]  

(2-17a)

Also the approximation is made that

\[ V_{aB} - U^{a} = 0 . \]  

(2-17b)

This gives

\[ T = \left< \Phi_{B}^{+} a+n \chi_{a+n}^{-} \left| V_{an} \right| \Phi_{a}^{+} B+n \chi_{a}^{+} \right> . \]  

(2-18)

Now some form must be assumed for the nuclei "B+n" and "a+n".

Let \( J_B, M_B; \vec{s}, u \) and \( J_a, M_a \) be the spin quantum numbers of particles "B", "n" and "a", respectively. Assume the nucleon "n" moves in a shell model orbit described by quantum numbers \( \vec{t}, m \) and \( \vec{j}, m_j \), where \( \vec{t} \) is the orbital angular momentum and \( \vec{j} = \vec{t} + \vec{s} \) is the total angular momentum. Then
\[ \overline{J_B} + j = \overline{J_B+n}, \]  
(2.19)

which implies
\[ |J_{B+n} - J_B| - s \leq \ell \leq J_B + J_{B+n} + s, \]  
(2.20a)

and parity
\[ \pi_B(-1)^\ell = \pi_{B+n} \]  
(2.20b)

Expanding \( \phi_{B+n} \) in terms of shell model wave functions gives, using

Clebsch-Gordan coefficients,
\[ \phi_{B+n} = \sum_{\ell, j} \frac{S^{1/2}}{\ell} \sum_{m, \mu} \frac{\Sigma^m \mu}{\ell} \left\langle J_{B+n} M_{B+n} \right| \left\langle J_B M_B \right| \right\rangle \]
\[ u_j(r_{B+n}) Y^m_{\ell}(\Omega_{B+n}) \psi_{B+n} \psi_{B B}, \]  
(2.21)

where \( \psi_{B+n} \) and \( \psi_{B B} \) are the internal wave functions of "n" and "B", and \( u_j(r_{B+n}) \) \( Y^m_{\ell} \) is the familiar shell model wave function. The expansion coefficient \( S_{\ell j} \) is called the spectroscopic factor and is a measure of the single particle character of the wave function of nucleon "n".

Assuming the relative motion of "a" and "n" in the exit channel to be an s state, one obtains
\[ \phi_{a+n} = \frac{1}{\sqrt{4\pi}} \left\langle J_a S M_a \mu \right| \left\langle J_{a+n} M_{a+n} \right| \right\rangle u_0(r_{an}) \psi_a \psi_{a+n}, \]  
(2.22)

here the \( \frac{1}{\sqrt{4\pi}} \) results from the zero order spherical harmonic.

Substituting (2.21) and (2.22) into (2.18) and integrating over the internal coordinates yields
\[ T = \sum_{\ell, j} \frac{1}{\sqrt{4\pi}} \left\langle J_a S M_a \mu \right| \right\rangle \left\langle J_{a+n} M_{a+n} \right| \right\rangle \left\langle t_{\ell \mu} \right| \left\langle J_B M_B \right| \right\rangle \left\langle J_{B+n} M_{B+n} \right| \right\rangle \left\langle u_0(r_{an}) Y_{a+n} \right| \right\rangle \left\langle V_{an} \right| \right\rangle \left\langle u_j(r_{B+n}) Y^m_{\ell}(\Omega_{B+n}) \right| \right\rangle \]
\[ (2.23) \]

For pickup, the cross section is given by
\[
\frac{d\sigma}{d\Omega} = \frac{M_{a,B+n}M_{B,a+n}}{2\pi \hbar^2} \frac{k_{a+n}}{k_a} \sum_{J} \frac{T_f^2}{(2J_a+1)(2J_B+n+1)}, \tag{2-24}
\]

Where \(M_{a,B}\) is the reduced mass of particles "a" and "B" and \(k_{\gamma}\) is the wave number of particle "\(\gamma\)". Substituting (2-23) into (2-24) and summing over the magnetic substates by means of Racah\(^{26}\) algebra, one obtains

\[
\frac{d\sigma}{d\Omega} = \frac{M_{a,B+n}M_{B,a+n}}{2\pi \hbar^2} \frac{k_{a+n}}{k_a} \frac{(2J_{a+n}+1)}{(2J_a+1)} \sum_{f,j,m} \frac{U_{fjm}^2}{(4\pi)^2(2f+1)}, \tag{2-25}
\]

where \(U_{fjm}\) is defined as

\[
U_{fjm} = \left\langle u_0(r_{an}) \chi^+_{a+n} \mid V_{an}(r_{an}) \mid u_j(r_{Bn}) \chi^m_{Bn}\right\rangle. \tag{2-26}
\]

Reducing the six-fold integration to three-fold by making the zero-range approximation and expanding the \(\chi\)'s in partial waves\(^{27}\), equation (2-25) becomes

\[
\frac{d\sigma}{d\Omega} = V_0^2 \sum_{f,j} \frac{S_{fj}}{S_f} \sigma_{fj}(\theta, E, Q), \tag{2-27}
\]

where

\[
\sigma_{fj} = \text{constant} \cdot \sum_m \left| \Gamma_{L_aL_{a+n}}^{L_f} \frac{\Gamma_{L_aL_{a+n}}^{L_f}}{L_aL_{a+n}} \frac{P_m^{L_aL_{a+n}}}{P_m^{L_aL_{a+n}}} \right|^2 \tag{2-28}
\]

\(P_m^{L_aL_{a+n}}(\cos \theta)\) is the associated Legendre polynomial of order \(L_a+n\),

\(\Gamma_{L_aL_{a+n}}^{L_f}\) is a product of Clebsch-Gordon coefficients given explicitly
\[
\int_{L_a L_{a+n}} f^2_{L_a L_{a+n}} = \int_0^\infty X_{L_a} (k_{a+n} r) X_{L_{a+n}} \left( \frac{M_B}{M_{B+n}} k_a r \right) u_{L_f} (r) \, dr .
\] (2-29)

If the total spin \( I \) of either the target or the residual nucleus is zero, then by virtue of the restrictions imposed by (2-20a) and (2-20b), the sum over \( f \) and \( j \) in (2-27) reduces to one term

\[
\frac{d\gamma}{d\alpha} = V_0^2 s_{f j} \sigma_{f j} (\theta, E, Q) .
\] (2-30)

An estimate of the magnitude of \( V_0 \) can be made if it is assumed that \( V_{an} (r_{an}) \) is the potential that binds particles "a" and "n" together. Then the Schrödinger equation becomes

\[
\left[ -\frac{\hbar^2}{2m} \nabla^2 + V_{an} (r_{an}) \right] u_0 (r_{an}) = -\frac{\hbar^2}{2m} a^2 u_0 (r_{an}) ,
\] (2-31)

where \(-\frac{\hbar^2}{2m} a^2\) is the binding energy of "n" to "a". This can be written

\[
\frac{\hbar^2}{2m} (V^2 - a^2) u_0 = Vu_0 = V_0 \delta (r_{an}) ,
\] (2-32)

or

\[
\int V_0 \delta (r_{an}) \, dr_{an} = V_0 = \frac{\hbar^2}{2m} \left[ \int \nabla^2 u_0 \, dr - a^2 \int u_0 (r) \, dr \right] .
\] (2-33)

The first integral is zero by virtue of Green's theorem. Therefore

\[
V_0 = -\frac{\hbar^2 a^2}{2m} \int u_0 (r_{an}) \, dr_{an} .
\] (2-34)

If \( u_0 (r_{an}) = \frac{\sqrt{\omega}}{2\pi} \frac{e^{-\omega r_{an}}}{r_{an}} \), which is correct asymptotically, then

\[
V_0^2 = \frac{2\pi \hbar^4 a}{m^2} .
\] (2-35)
The program used for the DWBA calculations is called DWUCK and was written by Dr. P. D. Kunz of the University of Colorado. His program allowed selection of optical model potentials, form factor, length and stepsize of integration and the number of partial waves used for each channel.

The potential used for calculation of the distorted waves is given by

$$ U(z_i, z_j, r) = V(r) + U_C(z_i, z_j, r), \quad (2-36) $$

where

$$ U_C(z_i, z_j, r) = \frac{z_i z_j e^2}{2R_C} \left( 3 - \frac{r^2}{R_C^2} \right) \quad r \leq R_C $$

$$ U_C(z_i, z_j, r) = \frac{z_i z_j e^2}{r} \quad r > R_C $$

$$ R_C = r_{0C} A^{1/3} $$

$V(r)$ is a standard optical model potential given by

$$ V(r) = - V f(r, r_R, a_R) - i W f(r, r_R, a_R) $$

$$ + V_{so} 2 \left( \frac{h}{m_{\pi}} \right)^2 \frac{1}{2} \frac{d}{dr} f(r, r_{so}, a_{so}) \overline{L} \cdot \overline{S} \quad (2-38) $$

where $m_{\pi}$ is the mass of the pion and $f(r, r_R, a_R)$ has the shape of the standard Wood-Saxon well

$$ f(r, r_R, a_R) = \left[ 1 + \exp \left( \frac{r - r_R A^{1/3}}{a_R} \right) \right]^{-1} \quad (2-39) $$

and

$$ \overline{L} \cdot \overline{S} = \begin{cases} \ell/2 & \text{for } j = \ell + 1/2 \\ -(\ell + 1)/2 & \text{for } j = \ell - 1/2 \end{cases} \quad (2-40) $$
The motion of particle "n" around "B" is described by a radial wave function \( \mu_{ij}(r) \), which is a solution to the Schrödinger equation with a potential of the form \( V_{\text{real}} f(r, r_R, a_R) + U_C \). Here \( U_C \) has the form of (2-37) and \( f(r, r_R, a_R) \) is given by (2-39). In DWUCK, the depth of \( V_{\text{real}} \) is adjusted such that the solution to the Schrödinger equation for \( \mu_{ij}(r) \), yields the correct binding energy for "n" bound to "B".

The potential parameters for input into DWUCK were determined from an optical model analysis of elastic scattering of "a" on the target nucleus and of "a+n" on the residual nucleus. This was done using the program JIP written by Dr. F. G. Percy of Oak Ridge National Laboratories. JIP calculates elastic scattering cross sections from potentials of the form given by (2-36). The calculated cross section is compared to the experimental cross section and a measure of agreement is obtained by calculation of \( \chi^2 \), given by

\[
\chi^2 = \sum_i \left( \frac{\sigma_{\text{TH}}(\theta_i) - \sigma_{\text{ex}}(\theta_i)}{\Delta \sigma_{\text{ex}}(\theta_i)} \right)^2,
\]

where \( \sigma_{\text{TH}}(\theta_i) \), \( \sigma_{\text{ex}}(\theta_i) \) and \( \Delta \sigma_{\text{ex}}(\theta_i) \) are, respectively, the theoretical cross section prediction, the experimental cross section value, and the experimental error, and each value of \( i \) represents one data point. The parameters of the calculation, i.e., the real and imaginary potentials, radii, and diffusenesses, are varied in such a manner so as to minimize the function \( \chi^2 \). The sets of parameters obtained are in general not
unique and the real potential $V$ and the real radius $r$ are related by $V r^n = \text{constant}$, where "$n$" is estimated$^{28}$ to be approximately $3/2$.

Now one should investigate the validity and consequences of the approximations made in the development of the DWBA theory. By disregarding the term (2-17a), one is essentially neglecting to consider formation of intermediate states in the "aB" and "nB" system. The substitution is also made that the interaction of a composite particle "a+n" with a core "B" and is written by a sum of two terms; one representing the interaction of "a" with "B", the other representing the interaction of "n" with "B". The potential $U^a$ in (2-17B) is determined by elastic scattering of "a" with "B+n". By setting $V_{aB} = U^a$, the possibility of core excitation resulting from the interaction "aB" is neglected.

In the calculation of the partial waves, only a few with $L$ around some $L_0$ make an appreciable contribution to the elastic scattering and thus these few are the only ones essentially determined. Since

$$\tilde{l} = \tilde{L}_{a+n} - \tilde{L}_a,$$

this means that the cross section is largest if

$$l = L_0^{a+n} - L_0^a.$$

(2-42)

This can be represented classically by

$$(k^{a+n} - k^a) R_0 = l$$

(2-43)

where $R_0$ is some approximate nuclear radius. Therefore, the cross section is expected to diminish if the Q-value of the reaction is such as to cause an angular momentum mismatch. If there is such a mis-
match, as is characteristic of \(^3\text{He}, \alpha\) reactions, then a considerable contribution to the scattering comes from the interior of the nucleus which has been neglected in the Born approximation.

To lessen the detrimental effects of approximation (2-17a), if one uses (2-17b) and replaces \(V_{nB}\) by \(U^{nB}\), one can write

\[
U^{a+n, B} = U^{a, B} + U^{n, B}.
\]

(2-44)

By following this criteria the neglect of term (2-17a) can be partially justified. The foregoing has been discussed in detail for \(^3\text{He}, \alpha\) reactions in Stock et al. 1).

Finally, a few comments should be made about two other direct reactions, knockout and heavy particle pickup. The knockout and the heavy particle pickup both assume that the target is composed of a core plus a cluster corresponding to the exit particle, in this case, the alpha particle. This is to be contrasted to the representation of the target as a core plus a neutron for the pickup case. A knockout reaction assumes that the incident \(^3\text{He}\) particle interacts with the alpha particle, knocking it out preferentially in the forward direction, with the \(^3\text{He}\) subsequently captured by the core to form the residual nucleus. On the other hand, the heavy particle pickup assumes that the incident \(^3\text{He}\) particle interacts with the core, picking it up, with this composite emerging preferentially in the forward direction. This results in the residual alpha's being mainly scattered at the back angles.
C. The Shell Model

Of all of the various models existing for the nucleus, the shell model in general explains with greater success most of the basic properties of a wide range of nuclei. Evidence for the existence of shell structure in nuclei manifests itself in the so-called magic numbers; 2, (6), 8, (14), 20, 28, 50, 82, and 126. These are the numbers of both neutrons and protons in a nucleus which form unusually stable configurations. The bracketed numbers are semi-magic in that a configuration of that many nucleons of a given type has a binding energy which is not as high as it is for magic number configurations.

The basic assumption of the shell model is that the potential for the $i^{th}$ particle $V_i = \Sigma_j \nabla V_{ij}(r_{ij})$ can be replaced with a central potential of the form $V_i = V(r_i)$, where $r_i$ is the coordinate of particle $i$ relative to the center of mass of the nucleus. Thus, the particle $i$ is considered to move independently of the motion of all of the other nucleons. In order to produce the shells closing at the correct numbers, a spin-orbit interaction must be included. The Schrodinger equation with spin-orbit coupling is

$$\frac{\hbar^2}{2\mu} \nabla^2 + V(r) + V_{SO}(r) \vec{I} \cdot \vec{s} \quad \psi(r) = E \psi(r). \quad (2-45)$$

The parity of the state is determined by $\pi = (-1)^f$. Equation (2-45) can be separated into a radial part and an angular part which is completely
independent of the form of the potentials. The solution to the angular part is simply a linear combination of spherical harmonics and spin functions. The choice of the potentials is important in order to produce the correct ordering of the shells. The harmonic oscillator potential with spin-orbit coupling is popular because it can be solved explicitly and it predicts the shells closing at the proper numbers. However, it fails to reproduce the level order in several places where the levels are very close together. A potential which does both is the Ross-Mark-Lawson potential

\[ V(r) = V_c f(r) - \frac{\lambda}{2Mc^2} \frac{1}{r} \frac{d}{dr} f(r) \bar{l} \cdot \bar{s}, \tag{2-46} \]

where \( f(r) \) is the Wood-Saxon form factor given by

\[ f(r) = \left[ 1 + \exp \frac{r-R}{a} \right]^{-1} \tag{2-47} \]

and

\[ V_c = -42.8 \text{ MeV} \]
\[ \lambda = 39.5 \]
\[ R = 1.3A^{1/3} \text{ fm} \]
\[ a = 0.69 \text{ fm} \]

For a potential such as the above, the radial equation must be solved numerically. The quantum numbers which specify the states are the principal quantum number \( n \), the angular momentum \( l \) and the total angular momentum \( j \). The \( \bar{l} \cdot \bar{s} \) term removes the degeneracy in \( l \) and the \( l + \frac{1}{2} \) states lie lower in energy than the \( l - \frac{1}{2} \) states. A typical calculation of shell model states is illustrated in the Nilsson level diagram in Figure 1. The states given at deformation \( \eta = 0 \) are shell model
states calculated using a harmonic oscillator potential with a spin-orbit term plus a phenomenological term of the form $T^2$ in order to produce the correct level ordering. The $1\, s_{1/2}$ state has been truncated from the bottom of the graph. It can be seen that at zero deformation, there are large energy gaps between certain states or groups of states. Since each shell model state can hold $2j + 1$ nucleons, it is easily seen that these gaps correspond to the magic numbers, which are circled in the diagram. For an odd-$A$ nucleus, the spin of the ground state is determined by the state into which the last nucleon goes, which is the lowest energy state available. The ground state spins of even-even nuclei are zero because of the pairing interaction. However, the last two paired nucleons of an even-even nucleus do not necessarily spend all of their time in the lowest available state. In other words the wave function of these two nucleons may be, and probably is, an admixture of several shell model states lying fairly close to one another in energy. However, both particles must be in the same shell model state in order for the nucleus to have zero spin. This distribution among several shell model orbits is evidenced by the pickup of neutrons from shell model states not expected to be occupied in the target ground state.
D. The Collective Model

1. Rotational-Particle Formulation

In the following section the collective formalism will be presented for a nuclear system consisting of an even-even core plus one particle moving in the potential field of the core. The particular model discussed here is due to Nilsson. Other articles treating this problem are Bohr and Mottelson, Mottelson and Nilsson, and Davidson.

The core is assumed to have a permanent average deformation with frequency low enough so that the particle can follow adiabatically. The vector coupling is illustrated in the figure at right. \( \vec{J} \) is the total particle angular momentum and is the sum of the orbital and spin angular momentums \( \vec{f} \) and \( \vec{s} \) respectively. \( \vec{J} \) is the total angular momentum of the particle plus core and is the sum of \( \vec{j} \) and \( \vec{L} \) where \( \vec{L} \) is the angular momentum of the core. \( z \) is an axis fixed in the core and \( z' \) is a space fixed axis. The relations between the various angular momenta are given by

\[
\vec{J} = \vec{L} + \vec{J}, \quad \vec{J} = \vec{j} + \vec{s}.
\]

(2-48)

By popular convention,

\[
J_Z = K, \quad J_{z'} = M, \quad j_z = \Omega.
\]

(2-49)

The total Hamiltonian is thus a sum of a particle Hamiltonian and a
core Hamiltonian, that is,

\[ H_{\text{total}} = H_{\text{core}} + H_{\text{particle}} \]  

or

\[ H = \frac{\hbar^2}{2I_0} \left( \frac{\tilde{T}_z^2}{I} \right) + \frac{p^2}{2m} + V(\mathbf{r}, \mathbf{r}, \mathbf{s}) \]  

where \( I \) is the effective moment of inertia tensor in the body fixed system, \( p \) is the momentum of the particle, and \( V \) is the potential the particle experiences due to the core. For a deformed core, \( V(\mathbf{r}, \mathbf{r}, \mathbf{s}) \) is non-spherical and therefore \( \tilde{J} \) is not a constant of motion. However, the core is usually assumed to be spheroidal with \( z \) the symmetry axis and consequently \( K \) remains a good quantum number. For the lower lying rotational levels of an even-even deformed core, it can be assumed\(^33\) that \( L_z = 0 \). Thus \( \tilde{J}_z = J_z = K = \Omega \) and, using \( \tilde{L} = \tilde{J} - \tilde{j} \), equation (2-51) can be written as

\[ H = \frac{\hbar^2}{2I_0} (\tilde{J}_z^2 - j_z^2) + \frac{\hbar^2}{2L} \frac{\tilde{j}_z^2}{I} + \frac{p^2}{2m} + V(\mathbf{r}, \mathbf{r}, \mathbf{s}) - \frac{\hbar^2}{2I_0} (j_x j_x + j_y j_y) \]  

where \( I_x = I_y = I_0 \) because of axial symmetry. Defining the raising and lowering operators as \( t = t_x + i t_y \), (2-52) can be written as

\[ H = H_R + H_p + H_{\text{RPC}} \]  

where

\[ H_R = \frac{\hbar^2}{2I_0} (\tilde{J}_z^2 - j_z^2) \]  

\[ H_p = \frac{p^2}{2m} + V(\mathbf{r}, \mathbf{r}, \mathbf{s}) + \frac{\hbar^2}{2I_0} \tilde{j}_z^2 \]  

and

\[ H_{\text{RPC}} = -\frac{\hbar^2}{2I_0} (j_+ j_- + j_- j_+) \]
\( H_R \) is thus the Hamiltonian of a symmetric rotator and \( H_p \) is the particle Hamiltonian. \( H_{RPC} \) is called the "rotational particle coupling" term (RPC) and represents the interaction between the core and the particle.

Assuming the particle to be tightly bound to the core, the particle energy levels are widely separated compared to the rotational energy levels and \( H_{RPC} \) can be neglected compared to \( H_R \) and \( H_p \). Then the properly symmetrized wave function for the system can be written

\[
\begin{aligned}
|EJKLM\rangle &= |JMK\rangle |K\rangle + (-1)^{J-j} |JM-K\rangle |-K\rangle,
\end{aligned}
\]

(2-55)

where \( |JMK\rangle = D_j^{MK} \) is the solution corresponding to \( H_R \) and is thus the familiar "symmetric top" wave function and \( |K\rangle \) is the solution corresponding to \( H_p \). The energy eigenvalues are given by

\[
E_j^K = \frac{\hbar^2}{2I_0} (J(J+1) - 2K^2) + E_p^K, \quad J \geq K,
\]

(2-56)

where \( E_p^K \) is the particle energy. Hence, the energy levels of an odd-\( A \) nucleus can be interpreted as a series of rotational bands; each band based on a single particle state characterized by a particular value of \( K \) with \( J \) ranging from \( J = K \) on up. The corrections to the energy eigenvalues and eigenfunctions due to the RPC term have been calculated and are given by Davidson 33).

2. The Single Particle Formulation of Nilsson

For the complete specification of the single particle Hamiltonian \( H_p \), all that is required is to specify the form of \( V(r,\ell,s) \). The most commonly used form is due to Nilsson 30). He uses a spheroidal potential of the form
\[ V(r, \ell, s) = \frac{m}{2} \left[ \omega_x^2 (x^2 + y^2) + \omega_z^2 z^2 \right] + C \ell \cdot \vec{s} + D \ell^2 . \] (2-57)

He also neglects the \( \ell^2 \) term and thus equation (2-54b) becomes
\[ H_p = H_0 + C \ell \cdot \vec{s} + D \ell^2 . \] (2-58)

where
\[ H_0 = -\frac{\hbar}{2m} \nabla^2 + \frac{m}{2} \left[ \omega_x^2 (x^2 + y^2) + \omega_z^2 z^2 \right] . \] (2-59)

This is in the form of an anisotropic harmonic oscillator with a spin-orbit and \( \ell^2 \) term appended. The \( \ell^2 \) term is a phenomenological term added to give the well more of a square well shape for higher \( A \), thereby depressing the higher angular momentum states. The constants \( C \) and \( D \) are usually chosen so as to reproduce the shell model-orbits at zero deformation. The effect of neglecting the \( \ell^2 \) term has been investigated by Chi and Davidson \(^{34}\) and, for the \( N = 2 \) oscillator shell, appears to have little effect.

A single deformation parameter \( \delta \) is introduced by
\[ \frac{2}{\omega_x} = \frac{2}{\omega_0} \left( 1 + \frac{2}{3} \delta \right) . \] (2-60a)
\[ \frac{2}{\omega_z} = \frac{2}{\omega_0} \left( 1 - \frac{4}{3} \delta \right) . \] (2-60b)

Requiring the nucleus to be of constant volume leads to
\[ \frac{2}{\omega_x} \omega_z = \text{constant} . \] (2-61)

This along with (2-60a) and (2-60b) defines \( \omega_0 \) as
\[ \omega_0(\delta) = \frac{2}{\omega_0} \left( 1 - \frac{4}{3} \delta - \frac{16}{27} \delta^2 \right)^{-1/6} , \] (2-62)

where \( \omega_0 = \omega_0(\delta = 0) \). The parameter \( \delta \) is related to the \( \beta \) of Bohr and
Mottelson in the first order by
\[
\delta = 0.95 \beta , \quad (2-63)
\]
Introducing the change of variable
\[
r = \left( \frac{m \omega_0}{\hbar} \right)^{1/2} x , \quad (2-64)
\]
\(H_0\) becomes
\[
H_0 = H_0^0 + H_0^\delta , \quad (2-65a)
\]
where
\[
H_0^\delta = \frac{\hbar \omega_0}{2} (-\nabla^2 + r^2) , \quad (2-65b)
\]
is the spherically symmetric part and
\[
H_0^\delta = -\frac{4\hbar \omega_0^5}{3} (\pi/3)^{1/2} r^2 Y_{20} , \quad (2-65c)
\]
is the deformed part.

Nilsson chose a basis representation in which \(H_0\) is diagonal along with \(\ell^2, \ell_z\) and \(s_z\). Now \(\ell^2, \ell_z\) and \(s_z\) all commute with \(H_0\) and the corresponding quantum numbers are \(\ell, \Lambda\) and \(\Sigma\), respectively. Since \(j_z = \ell_z + s_z = \Omega\) and \(\Omega\) is a good quantum number for the total Hamiltonian, the states for a given value of \(\Omega\) will consist of linear combinations of a set of basis vectors denoted by \(|N\ell \Lambda \Sigma\rangle\) with the restriction that \(\Omega = \Lambda + \Sigma\). The quantum number \(N\) represents the total number of oscillator quanta, i.e.,
\[
H_0 \left| N\ell \Lambda \Sigma \right\rangle = (N + \frac{3}{2}) \hbar \omega_0 \left| N\ell \Lambda \Sigma \right\rangle . \quad (2-66)
\]
In configuration space
\[
\left\langle \vec{r} \left| N\ell \Lambda \Sigma \right\rangle = r^\ell \exp(-\frac{\vec{r}^2}{2}) F(-n, \ell + \frac{3}{2}, r^2) Y_{\ell \Lambda \Sigma} \right\rangle , \quad (2-67)
\]
where \(2n + l = N\) and \(F\) is the confluent hypergeometric function.

A superficial examination is made of the coupling caused by \(H_0\) and this yields the selection rules

\[
\Omega = \Omega', \quad l = \frac{l}{l + 2}, \quad N = \frac{N}{N + 2}.
\] (2-68)

The approximation is made that the \(\Delta N = 2\) terms are neglected and, therefore, matrix elements are only considered between base vectors \(|Nl\Lambda \Sigma\rangle\) with the same \(N\) and \(\Omega = \Lambda + \Sigma\).

Introducing new parameters

\[
\chi = -\frac{1}{\bar{C}} \frac{C}{\hbar \omega_0},
\] (2-69a)

\[
\mu = \frac{2D}{\bar{C}},
\] (2-69b)

\[
\eta = \frac{8 \bar{C}}{X} \left[1 - \frac{4}{3} \delta - \frac{16}{27} \delta^3\right]^{-1/6},
\] (2-69c)

one can write

\[
H_0 = \chi h \omega_0 \eta \bar{U},
\] (2-70)

where

\[
\bar{U} = -\frac{4}{3} \left(\mu/\bar{C}\right)^{1/2} r^2 Y_{20}
\] (2-71)

Then

\[
H_0 = H_0 = \chi h \omega_0 R,
\] (2-72)

where

\[
R = \eta \bar{U} - 2\hat{l} \cdot \hat{s} - \mu \hat{l}^2.
\] (2-73)

Since \(H_0\) is diagonal in the \(|Nl\Lambda \Sigma\rangle\) representation,
\[ E_a^{N \Omega} = (N_a + \frac{3}{2}) \hbar \omega_0 (5) + \chi \hbar \omega_0 r_a^{N \Omega}, \]  

(2-74)

where \( r_a^{N \Omega} \) is the eigenvalue obtained by the diagonalization of \( R \) in the representation \( |N \Lambda \Sigma \rangle \) subject to the requirement that \( N' = N \) and \( \Omega = \Omega \).

Nilsson has performed this diagonalization numerically for a range of values of \( \eta \). For the parameter values he chose \( \chi = 0.05 \) and, for \( N = 0, 1 \) and \( 2, \mu = 0.0 \). In Nilsson's paper the eigenvalues \( r_a^{N \Omega} \) and the associated eigenvectors are tabulated. Also a plot of \( E_a^{N \Omega}/\hbar \omega_0 \) versus \( \eta \) is given for each particle orbit. A copy of this graph depicting the lowest particle states is shown in Figure 1.
Fig. 1. Nilsson diagram
CHAPTER III

EXPERIMENTAL APPARATUS AND PROCEDURE

The experiment utilized the Duke University 4 MeV Van de Graaff accelerator which is now part of the Triangle Universities Nuclear Laboratory (TUNL). An ion source capable of producing 0.5 namp of doubly-ionized $^3$He was used. The accelerated beam was analyzed by a switching magnet which separated the singly- and doubly-charged $^3$He. The angular positioning of the beam ports from this magnet was such that the accelerator could be controlled with the feedback from a slit system on the more intense singly-ionized component, while allowing the use of all of the doubly-ionized component at 60 degrees for the experiment. The beam energy was monitored by a precision generating voltmeter which determined the energy to within ± 0.25%. The absolute calibration of the generating voltmeter was done as follows. One point on the calibration curve was furnished by the $^6$Li(p,n)$^6$Be threshold at 1.88 MeV. Other points were obtained by calibrating a multi-channel analyzer with α-sources and measuring the energy of the peak from elastic scattering from gold at a forward angle. From the energy of the scattered particle the energy of the incident particle could be deduced from kinematic calculations. An $^{241}$Am source gave a calibration point at 5.48 MeV and a $^{231}$Pa source yielded a range of alpha-
particle energies from 4.72 - 7.36 MeV. Using the above calibration procedures, the dome voltage was determined to within ±10 keV, thereby determining the $^3\text{He}^{++}$ energy to within ±20 keV. The consistency was checked periodically with $^{24}\text{Mg}(^3\text{He}, d)^{23}\text{Mg}$ excitation curves.

The target chamber used is described in detail by Gerke\(^{36,37}\). A simple schematic is shown in Figure 2. Essentially, it is 30.5 cm in diameter and 7.6 cm in depth and has upper and lower rotatable scattering tables, each of which can support up to six detectors. The beam was defined by two 1/16" diameter tantalum collimators placed 15" apart. Approximately 2" after the last collimator an anti-scattering collimator of diameter slightly greater than 1/16" was placed. This arrangement defined a beam spot on target of approximately 3/32" diameter. The beam was integrated in a Faraday cup lined with tantalum and equipped with an electron suppressor biased at -300 volts. A vacuum of $5 \times 10^{-5}$ mm of Hg was maintained in the chamber by an oil diffusion pump trapped at dry ice temperature.

The angular distributions were taken with an array of either 4 or 6 detectors mounted on the scattering tables. In addition, a monitor detector was mounted on the wall of the chamber at approximately 65.5°. The solid angles were defined by a 3/32" x 5/16" slit for back angles and a 1/8" diameter hole at the forward angles. These were positioned at a distance of about 3.75" from the target. This determined an angular acceptance of approximately 1 1/2 degrees, which for the nuclei
studied corresponded to an energy spread of approximately 30 keV. Solid-state silicon surface-barrier detectors were used with a depletion depth of approximately 300 microns. In use, however, the detectors were biased so that only the alpha groups were completely stopped in the depleted volume, thereby reducing the charge collected for the other particle groups seen in this experiment. The detectors were normalized to the one at the most forward angle by overlapping the range of angles covered by each. The normalizations obtained with this method were compared with those obtained from geometrical calculations in order to minimize the chances of error.

Figures 3, 4 and 5 are block diagrams of the electronics used for the experiment. When six detectors were used, signals from two of the back angle detectors were stored in a 400 channel analyzer, allowing 200 channels per detector. The electronics used is shown in Figure 3a. The monitor detector at 65.5 degrees used the electronics setup shown in Figure 3b. The output from the other four detectors were stored in an on-line DDP-224 computer.

The output from the two forward angle detectors were stored in the computer using the electronic setup as shown in Figure 4. The pileup rejector was needed because of the excessively high count rate at forward angles due to Rutherford scattering of the incident beam particles. The pulses were frequently so close together that they overlapped to varying degrees in the main amplifier. This would create background counts in the range extending from the elastic peak to double the energy of the
Fig. 3. Block diagram of the electronics used for (a) storing 2 detectors in the 400-channel analyzer (b) the monitor detector
Fig. 4. Block diagram of the electronics used for the forward angle detectors
Fig. 5. Block diagram of the electronics used for fanning 2 detectors into 1 ADC
elastic peak. A time pickoff unit supplied a timing signal with the width of the output pulse adjusted to 10 nsec. This signal was then fed into a discriminator clipped to 12 nsec. The resulting output signal was then large enough to trigger the pileup rejector. The pileup rejector put out a pulse whenever two detector pulses arrived within a time $\Delta t$ of each other, where $\Delta t$ was adjusted to 0.5 $\mu$sec. This pileup pulse was used as the gate pulse to a linear gate operating in the inhibit mode. Thus the linear signal from the biased amplifier was blocked whenever it was accompanied by a pulse from the pileup rejector indicating that the linear signal was the sum of two pulses separated by less than 0.5 $\mu$sec. A pulser signal was split, with one branch going directly to a scaler and the other going through the experimental electronics. The pulser peak was positioned at the top of the spectrum out of the way of the particle groups. By comparing the pulser counts in the scaler with those of the pulser peak in the spectrum, a measure of the dead time of the analyzing system was obtained. One of the outputs of the main amplifier was fed into an oscilloscope while the scope was triggered by the output from the time pickoff. Thus, the discriminator level on the time pickoff could be adjusted so that it triggered on the prominent elastic group but not on the baseline noise, therefore eliminating considerable dead time. The circuit just described thus kept the spectrum relatively clean of background from pileup except at a position corresponding to exactly twice the energy of the elastics, which resulted from two pulses separated by less than 10 nsec (the time pickoff gives out only one signal in this case).
The electronics for the two back angle detectors stored in the computer is shown in Figure 5. These two detectors were mixed since there were only three ADC's for four detectors and their count rate was lower than for the two forward angle detectors. The 12 bits of the ADC were split up, with 10 being used for the linear signal and 2 used to identify which detector corresponded to the linear signal.

The on-line DDP-224 computer was made by Computer Control Company, a division of Honeywell. It had associated with it a 16K memory, 2 tape decks, a line printer, card reader, paper tape reader and punch. I/O capabilities consisted of a parallel channel, a fully-buffered channel, numerous sense lines and 16 priority interrupts. The ADC's, each with a given priority interrupt, were fanned into the computer through the parallel channel. Data accumulation and analysis was performed by use of 30 switches programed into 5 sense lines providing a control for the computer. An oscilloscope and light pen were used for data display and analysis.

The program used to accumulate data in this experiment is called MARS. After an ADC had digitized a pulse, it interrupted a monitor program which transferred control to MARS. MARS then processed the digitized pulse, stored it in the appropriate data block, and then reset the ADC. Control was then passed back to the monitor program. The computer clock also sampled each ADC, noting what percentage of the time it was busy. Thus a measure of dead time introduced by the ADC was incorporated into the program. MARS contained eight 512
channel data blocks. Data was stored in 4 of these for each run while
the other 4 containing the spectra of the previous run could be summed
with background subtraction by using the light pen. After each run, the
data was stored on magnetic tape in binary form and also printed out
on the line printer.

Two other programs were used for data analysis: MOLA and
PLT. MOLA read binary data from magnetic tape into the computer
and allowed light pen reduction. PLT read binary data from magnetic
tape into the computer and then plotted it out in logarithmic form on
the line printer. These were invaluable aids in identifying spectra and
unfolding overlapping peaks.

All of the targets were prepared by evaporation. $^{24}\text{Mg}$ targets
were made by evaporating a mixture of Ta and isotopically enriched
MgO onto a clean glass slide. These were floated off in water and
picked up on stainless steel target rings. The $^{26}\text{Mg}$ targets were made
similarly except that they were evaporated onto glass slides containing
either a 10 or 20 $\mu$gm/cm$^2$ layer of carbon. The $^{28}\text{Si}$ and $^{30}\text{Si}$ targets
were made by evaporating isotopically enriched SiO$_2$ with an electro-
static electron gun onto a glass slide which had been dipped in a strong
soap solution and then allowed to dry. The evaporated targets were
floated off in water and mounted on the target rings. The $^{32}\text{S}$ targets
were prepared by evaporating Sb$_2$S$_3$ from a tantalum boat onto carbon-
coated (10 - 20 $\mu$gm/cm$^2$) glass slides. The average total thickness
and $^3\text{He}$ energy loss of these targets is shown in Table 1. The energy
loss is for 8.0 MeV $^3$He particles.

<table>
<thead>
<tr>
<th>Isotope</th>
<th>Average Total Thickness (μg/m/cm$^2$)</th>
<th>Average $^3$He Energy Loss (keV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{24}$Mg</td>
<td>130</td>
<td>50</td>
</tr>
<tr>
<td>$^{26}$Mg</td>
<td>100</td>
<td>40</td>
</tr>
<tr>
<td>$^{28}$Si</td>
<td>130</td>
<td>55</td>
</tr>
<tr>
<td>$^{30}$Si</td>
<td>160</td>
<td>70</td>
</tr>
<tr>
<td>$^{32}$S</td>
<td>130</td>
<td>35</td>
</tr>
</tbody>
</table>

Table 1

The target chamber alignment was checked with a zero-angle measurement. This was done by using a counter system to measure the yield of a rapidly varying elastic peak at ±15, ±26, and ±36 degrees for the same number of incident particles. The chamber was found to be within 0.2 degrees of perfect alignment.

In the data analysis, the different runs were normalized to the monitor counts to allow for effective target thickness changes due to beam spot wandering and evaporation. For the extreme forward angles 12 micron Ni foils were placed in front of the detectors. These foils slowed down the elastic and α-particle groups an amount such that the double energy elastic peak fell below the alpha groups of interest. This was possible because of the considerable difference in rate of energy loss between low energy $^3$He and higher energy α's. At times
the $^{241}$Am and $^{231}$Pa α-sources were also used to calibrate the electronics and thus aid in spectrum identification.
CHAPTER IV
DATA ANALYSIS AND RESULTS

A. Fluctuation Analysis

Figures 6 and 7 are excitation curves taken in 50 keV steps at several angles for several states in $^{23}$Mg, $^{27}$Si and $^{29}$Si. The curves are drawn as an aid to the eye. In studying this set of curves, one notices that the fluctuations in the excitation curves for the different states at the same angle are basically uncorrelated. This indicates that there are no large, overbearing resonances present in this region of excitation in the compound nucleus. For the reactions considered here, the intermediate compound states which are excited are at an energy of approximately 20 MeV with respect to the ground state. At this excitation energy it is calculated from the prescription of Gadioli and Zetta\textsuperscript{38}) that the average level spacing ranges from about 10 to 150 electron volts. Using equation (2-5), calculations of the auto-correlation function $C(\epsilon)$ have been made. From equation (2-7), it is readily seen that if $\epsilon = \Gamma$, then $C_{\text{exp}}(\Gamma) = \frac{1}{2} C_{\text{exp}}(0)$. Therefore, $\Gamma$ is the half-maximum width and can be obtained from plots of $C(\epsilon)$.

Several of these are plotted in Figures 8, 9 and 10 and it is apparent that $\Gamma_{\text{average}} \approx 75$ keV. This value of $\Gamma$ agrees very well with values obtained from a graph plotted by Ericson and Mayer-Kuckuk\textsuperscript{21}). Hence for this experiment, $\Gamma \approx 75$ keV $\gg$ D $\approx$ 10 - 150 ev and since the
Fig. 7. $^{30}\text{Si}$ excitation curves
Fig. 8. $^{24}$Mg correlation and probability graphs
Fig. 9. $^{28}\text{Si}$ correlation and probability graphs
Fig. 10. $^{30}\text{Si}$ correlation and probability graphs
experimental resolution is of the same order of magnitude or less than 
1, Ericson's theory of fluctuations should apply. Assuming that Ericson's 
theory is applicable, it is easily deduced from equations (2-7) and (2-8) 
that

\[ y_D = \sqrt{1 - N^c \exp(0)} \]

where \( N \) is usually taken to be anywhere from \( \frac{1}{2} N_{\text{max}} \) to \( N_{\text{max}} \).

For the reactions considered here, the target spin is zero and \( N_{\text{max}} \) 
is equal to \( 2I + 1 \), where \( I \) is the spin of the final state in the residual 
nucleus. In Figure 8 are shown plots for \(^{24}\text{Mg}(^3\text{He}, \alpha)^{23}\text{Mg}\) of the auto-
correlation function \( C(\alpha) \) and of the probability distribution \( P(Y) \), given 
explicitly by equation (2-10). In column 3 of Table 2 the results of 
calculations of the percentage of direct reaction using equation (4-1) 
are listed. In column 4 of the same table are listed

\(^{23}\text{Mg}^*\)

<table>
<thead>
<tr>
<th>State</th>
<th>( \theta )</th>
<th>( y_D )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.451</td>
<td>45°</td>
<td>90%</td>
</tr>
<tr>
<td>g. s.</td>
<td>135°</td>
<td>50%</td>
</tr>
<tr>
<td>0.451</td>
<td>135°</td>
<td>65%</td>
</tr>
</tbody>
</table>

\* Used \( N \approx 2/3 N_{\text{max}} \)

Table 2

the values of percentage direct reaction estimated from the plots of

\( P(Y) \) in Figure 8. In Tables 3 and 4, the analogous results for

\(^{28}\text{Si}(^3\text{He}, \alpha)^{27}\text{Si} \) and \(^{30}\text{Si}(^3\text{He}, \alpha)^{29}\text{Si} \) are listed with the corresponding
graphs shown in Figures 9 and 10. It is seen that the two methods of estimating $y_D$, the percentage of direct reaction, give roughly comparable results. Since a DWBA analysis is performed, it is especially

### $^{27}\text{Si}$ *

<table>
<thead>
<tr>
<th>State</th>
<th>$\theta$</th>
<th>$y_D$ Equation (4-1)</th>
<th>$y_D$ P($Y$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>g.s.</td>
<td>$45^\circ$</td>
<td>90%</td>
<td>$\sim 80%$</td>
</tr>
<tr>
<td>0.73</td>
<td>$45^\circ$</td>
<td>80%</td>
<td>$\sim 80%$</td>
</tr>
<tr>
<td>g.s.</td>
<td>$110^\circ$</td>
<td>70%</td>
<td>$\sim 60%$</td>
</tr>
</tbody>
</table>

* Used $N \approx 2/3$ $N_{\text{max}}$

Table 3

### $^{28}\text{Si}$ *

<table>
<thead>
<tr>
<th>State</th>
<th>$\theta$</th>
<th>$y_D$ Equation (4-1)</th>
<th>$y_D$ P($Y$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.273</td>
<td>$45^\circ$</td>
<td>90%</td>
<td>$\sim 80%$</td>
</tr>
<tr>
<td>1.273</td>
<td>$70^\circ$</td>
<td>90%</td>
<td>$\sim 80%$</td>
</tr>
<tr>
<td>1.273</td>
<td>$165^\circ$</td>
<td>20%</td>
<td>$\leq 20%$</td>
</tr>
</tbody>
</table>

* Used $N \approx 2/3$ $N_{\text{max}}$

Table 4

encouraging to note that all of the reactions are predominately direct at the forward angles. They do, however, become substantially less direct at the backward angles.
B. Absolute Cross Section Determination

Because of $^{12}$C, $^{16}$O and $^{181}$Ta contaminants in the targets, the $^3$He particles elastically scattered from the nuclei of interest could not be separated from the contaminant elastic scattering at angles more forward than 25 or 30 degrees. Since the contaminant peaks were of appreciable size, no attempt was made to separate these peaks from the peaks of interest any further forward than 25 or 30 degrees. This presented a handicap since the usual method of obtaining absolute cross sections was to compare the yield of scattered particles to the yield expected from Rutherford scattering and extract the product of solid angle and target thickness. However, at this energy and these angles, the cross section is not expected to be completely Rutherford scattering. This problem was circumvented by making the comparison between the elastic cross section and that predicted by the optical model code for a reasonable set of parameters. This gave a value of more than 90% Rutherford scattering at the considered angles and this percentage varied little with different sets of parameters. However, because of the uncertainty in the target thickness-solid angle determination and the uncertainties in the data accumulation and analysis, there is an uncertainty of approximately 20% in the absolute cross sections given in the following sections.
C. $^{24}\text{Mg}(^3\text{He},\alpha)^{23}\text{Mg}$ Analysis

Recent neutron pickup experiments on $^{24}\text{Mg}$ have been done by Dubois and Earwaker ($^3\text{He},\alpha)^2$), Haun ($^3\text{He},\alpha)^8$), Joyce et al. ($^3\text{He},\alpha)^9$) and Kozub (p,d) $^{14}$. These works contain extensive references to the development of the existing knowledge of $^{25}\text{Mg}$ and $^{24}\text{Mg}$. In Figure 11 is shown a level diagram of $^{25}\text{Mg}$, which is known to be prolate with a deformation $\epsilon \approx 0.2$. It is easily interpreted as having a rotational structure based on the Nilsson model. A $K^T = 3/2^+$ rotational band based on Nilsson orbit 7 consists of the ground, 0.451 MeV, 2.048 MeV and 2.712 MeV states with the ground state being the band head. A $K^T = 1/2^+$ rotational band based on Nilsson orbit 9 consists of the 2.356 MeV, 2.904 MeV and 3.968 MeV states with the 2.356 MeV state being the band head. These bands are illustrated in the energy level diagram in Figure 11.

In Figure 12 an 8 MeV spectrum at $\theta_{\text{lab}} = 45$ degrees is shown. The total resolution here is approximately 60 keV. The Q-value for the $^{24}\text{Mg}(^3\text{He},\alpha)^{23}\text{Mg}$ reaction is 4.024 MeV and this restricted the number of observable states since those near and below the groups corresponding to elastic scattering were swamped by groups from other reactions.

$^3\text{He}$ optical model parameters for DWBA calculations were obtained by fitting the $^3\text{He}$ elastic scattering with the computer search code JIB. This program varied specified optical parameters so as to
Fig. 11. 23 Mg and 25 Mg level diagram
Fig. 12. $^{24}\text{Mg}(^{3}\text{He},\alpha)^{23}\text{Mg}$ spectrum at 8 MeV
minimize \( \chi^2 \), where

\[
\chi^2 = \Sigma_i \frac{(\sigma_i^{\text{calc}} - \sigma_i^{\text{exper}})^2}{(\Delta \sigma_i^{\text{exper}})^2}
\]

(4-2)

In Figure 13 a plot is shown of the \( ^{24}\text{Mg} + ^3\text{He}, \ ^3\text{He} + ^{24}\text{Mg} \) elastic scattering at 8 MeV along with the optical model fit to the data. The corresponding \(^3\text{He}\) parameters are listed in Table 5.

<table>
<thead>
<tr>
<th>Optical Model Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>( V )</td>
</tr>
<tr>
<td>--------------------------</td>
</tr>
<tr>
<td>( ^{24}\text{Mg} )</td>
</tr>
<tr>
<td>(^3\text{He})</td>
</tr>
<tr>
<td>a</td>
</tr>
<tr>
<td>( ^{26}\text{Mg} )</td>
</tr>
<tr>
<td>(^3\text{He})</td>
</tr>
<tr>
<td>a</td>
</tr>
<tr>
<td>( ^{28}\text{Si} )</td>
</tr>
<tr>
<td>(^3\text{He})</td>
</tr>
<tr>
<td>a</td>
</tr>
<tr>
<td>( ^{8}\text{MeV} )</td>
</tr>
<tr>
<td>(^3\text{He})</td>
</tr>
<tr>
<td>a</td>
</tr>
<tr>
<td>( ^{30}\text{Si} )</td>
</tr>
<tr>
<td>(^3\text{He})</td>
</tr>
<tr>
<td>a</td>
</tr>
<tr>
<td>( ^{7}\text{MeV} )</td>
</tr>
<tr>
<td>(^3\text{He})</td>
</tr>
<tr>
<td>a</td>
</tr>
<tr>
<td>( ^{32}\text{S} )</td>
</tr>
<tr>
<td>(^3\text{He})</td>
</tr>
<tr>
<td>a</td>
</tr>
</tbody>
</table>

Bound state neutron parameters \( R_p = 1.2 \text{fm}, A_r = 0.65 \text{fm} \), 100 s.o.
Fig. 13. Angular distributions corresponding to the $^{24}\text{Mg}(^3\text{He},^3\text{He})^{24}\text{Mg}$ and $^{24}\text{Mg}(^3\text{He},\alpha)^{23}\text{Mg}$ reactions at 8 MeV.
The DWBA calculations were made using the computer code DWUCK. The optical parameters used for the incident and exit channels are given in Table 5. The $\alpha$-parameters are from McFadden and Satchler's analysis of 27 MeV $\alpha$-scattering on Mg and Al. The imaginary well has been increased to 30 MeV in order to obtain closer agreement with the data. Also shown in Figure 13 are plots of the cross section resulting from the ($^3$He, $\alpha$) reaction leading to the ground and first three excited states of $^{23}$Mg. The plotted curves represent the DWBA predictions for the various states. An $l = 2$, $3/2^+$ prediction is compared to the ground state. The 0.451 MeV state is shown compared to an $l = 2$, $5/2^+$ prediction. The 2.048 MeV state is not expected to be excited via the pickup mechanism, however an $l = 3$ (solid curve) and $l = 4$ (dashed curve) are shown for comparison. The 2.356 MeV state is shown compared with an $l = 0$, $1/2^+$ prediction. The agreements with the distributions corresponding to the ground, 0.451 MeV and 2.350 MeV states, which are believed to be excited by a pickup mechanism, are acceptable. The 2.048 MeV state has a spin-parity assignment of $7/2^+$ which would require a $g_{7/2}$ admixture in the ground state of $^{24}$Mg. This is very improbable from binding energy considerations.

Using the relation

$$\frac{d\sigma_{\text{exp}}}{d\Omega} = 23 \cdot S \cdot \frac{2s+1}{2(2J+1)} \frac{d\sigma_{\text{DWUCK}}}{d\Omega}, \quad (4-3)$$

where $S$ is commonly called the spectroscopic factor, $s$ is the spin of the transferred particle and $J$ is the total angular momentum of the...
transferred particle, spectroscopic factors were extracted for the reactions studied. The factor 23.0 comes from the paper by Stock et al.\textsuperscript{1)} For $^{24}\text{Mg}(^{3}\text{He},\alpha)^{23}\text{Mg}$, the extracted spectroscopic factors are shown in Table 6. These are compared with similar factors from the $(^{3}\text{He},\alpha)$ reaction of Joyce et al.\textsuperscript{9)} and the $(p,d)$ results of Kozub\textsuperscript{14)}. Several sets of parameters giving comparable fits to the elastic scattering were used in the DWUCK calculations for the $(^{3}\text{He},\alpha)$ reaction. Although the data comparisons to these predictions were of equal quality the spectroscopic factors extracted differed in absolute value. However, the ratios were fairly constant. It is seen that relative ratios between the 8 MeV spectroscopic factors and those at 15 MeV are similar while the 5/2$^+$/3/2$^+$ ratio from the $(p,d)$ work is 45\% larger than that from the 8 MeV $(^{3}\text{He},\alpha)$ work.

$^{23}\text{Mg}$ Spectroscopic Factors

<table>
<thead>
<tr>
<th>State</th>
<th>$J$</th>
<th>$I$</th>
<th>8 MeV $(^{3}\text{He},\alpha)$</th>
<th>15 MeV$^a$ $(^{3}\text{He},\alpha)$</th>
<th>33.6 MeV$^b$ $(p,d)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>g. s.</td>
<td>2</td>
<td>3/2</td>
<td>0.87</td>
<td>1.6</td>
<td>0.72</td>
</tr>
<tr>
<td>0.451</td>
<td>2</td>
<td>5/2</td>
<td>4.40</td>
<td>7.8</td>
<td>5.66</td>
</tr>
<tr>
<td>2.356</td>
<td>0</td>
<td>1/2</td>
<td>0.24</td>
<td>0.57</td>
<td>0.20</td>
</tr>
</tbody>
</table>

$^a$ reference 9  $^b$ reference 14

Table 6

It is interesting to compare the experimental ratio of the spectroscopic factors of the 3/2$^+$ ground state and the 5/2$^+$ 0.451 MeV
state with the theoretical ratio calculated using the Nilsson model. The experimental ratio is \( S_{3/2} (0.0 \text{ MeV}) / S_{5/2} (0.451 \text{ MeV}) = 1/5 \). If one assumes that the last two neutrons are in Nilsson orbit 7 with no band mixing, then using the prescription of Satchler\(^{41}\) one obtains \( S_{3/2} (0.0 \text{ MeV}) / S_{5/2} (0.451 \text{ MeV}) = 1/24 \). Kelson and Levinson\(^{42}\) have made calculations assuming mixing between bands based on Nilsson orbits 5, 7 and 9. Again, using Satchler's prescription for band-mixed spectroscopic factors\(^{41}\), one obtains a theoretical ratio of 1/2.3. This is somewhat more in line with the experimental ratio of 1/5 but is obviously nowhere near agreement. Thus the only conclusion that can be drawn from the foregoing is that the data is consistent with some degree of band mixing, assuming a pickup process is mainly responsible for the reaction.

In summarizing, it was unfortunate that the low Q-value and the other reaction groups prevented the observation of the higher excited members of the band. However, in examining the spectrum it looks as if the \( K^m = 3/2^+ \) band has 80\% of the pickup strength. Also, the presence of the relatively strong \( 7/2^+ \) \( 2.048 \text{ MeV} \) state indicates a multiple excitation process similar to the one assumed for the \( 1.61 \text{ MeV} \) \( 7/2^+ \) state in \( ^{25}\text{Mg} \) excited by neutron pickup\(^{43}\) and the presence of a \( K^m = 1/2^+ \) band implies an admixture of Nilsson orbit 9 in the \( ^{24}\text{Mg} \) ground state.
D. \( ^{26}\text{Mg}(^3\text{He},\alpha)^{25}\text{Mg} \) Analysis

\(^{25}\text{Mg} \) has been widely studied and is considered to be the nucleus in the s-d shell most amenable to interpretation in terms of the Nilsson model. The initial collective interpretation of \(^{25}\text{Mg} \) was done by Litherland et al. \(^{44} \) in 1953 using the then relatively new Nilsson model. Experiments utilizing the \((d,p) \) reaction have been performed by Buck and Hodgson \(^{45} \), by Middleton and Hinds \(^{46} \), and by Cujec \(^{47} \). Dehnard and Yntema \(^3 \), using the neutron pickup reactions \(^{26}\text{Mg}(d,p)^{25}\text{Mg} \) and \(^{26}\text{Mg}(^3\text{He},\alpha)^{25}\text{Mg} \), have extracted considerable spectroscopic information and have compared this information with predictions resulting from their collective model analysis of \(^{25}\text{Mg} \). Shown in Figure 11 is an energy level diagram of \(^{25}\text{Mg} \), which has a deformation somewhere between \( \delta = 0.2 \) and \( \delta = 0.3 \). In terms of the collective model, \(^{25}\text{Mg} \) is thought to consist of a \( K^T = 5/2^+ \) band based on Nilsson orbit 5 containing the ground, 1.614 MeV and 3.400 MeV states with the ground state being the band head. A \( K^T = 1/2^+ \) band based on Nilsson orbit 9 contains the 0.585 MeV, 0.975 MeV, 1.960 MeV and 2.738 MeV states with the 0.585 MeV state being the band head. Also, a \( K^T = 1/2^+ \) band is based on Nilsson orbit 11 and contains the 2.562 MeV, 2.801 MeV and 3.905 MeV states with the band head being the 2.562 MeV state. Of particular interest is the noticeable absence of a \( K^T = 3/2^+ \) hole-state rotational band based on Nilsson orbit 7, the band head of which is expected to be at approximately 4.2 MeV \(^3 \). The preceding bands
are illustrated in the energy level diagram of Figure 11.

Figure 14 is a particle spectrum taken at a lab angle of 40 degrees with the incident $^3$He energy equal to 8 MeV. The overall resolution here is approximately 90 keV. The state at 7.82 MeV is thought to be the $T = 3/2$ analog of the $^{26}$Na ground state.

In Figure 15 is a graph of the $^3$He elastic scattering from $^{26}$Mg at 8 MeV along with two optical model fits (dotted-set A, solid-set B). The corresponding parameters are listed in Table 5. Also shown in Figure 15 are plots of the absolute cross sections from the $^{26}$Mg($^3$He,$^6$He)$^{25}$Mg reaction leading to the ground and first 4 excited states of $^{25}$Mg. Superimposed on this data are the DWBA predictions (dotted-set A, solid-set B) calculated using DWUCK. The optical model parameters used for the calculations are shown in Table 5. The $\alpha$-parameters are again McFadden and Satchler's parameters with the real well depth for set B increased slightly to 202 MeV to obtain somewhat better agreement with the data. The agreement between theoretical prediction and experimental data is fairly good with the exception of the 0.978 MeV state. The dip at so forward an angle precludes a fit with an $l = 2$ calculation. Also, the prediction for the 1.611 MeV state is shown only for comparison since this state is not believed to be excited by a pickup mechanism. The reason for this is that the ground state of $^{25}$Mg would not be expected to contain any $g_{7/2}$ characteristics since the shell model energy of this state is considerably far removed from the other shell model states being filled in this mass range.
Fig. 14. $^{26}\text{Mg}(^3\text{He},\alpha)^{25}\text{Mg}$ spectrum at 8 MeV
Fig. 15. Angular distributions corresponding to the 
$^{26}\text{Mg}(^3\text{He}, ^3\text{He})^{26}\text{Mg}$ and $^{26}\text{Mg}(^3\text{He}, a)^{25}\text{Mg}$ 
reactions at 8 MeV.
Listed in Table 7 are the spectroscopic factors obtained from the previously mentioned comparisons. Also listed are the spectroscopic factors of other neutron pickup experiments on $^{26}\text{Mg}$. Since no fit was obtained for the 0.975 MeV state, little weight can be given

$^{25}\text{Mg}$ Spectroscopic Factors

<table>
<thead>
<tr>
<th>State</th>
<th>$I$</th>
<th>$J$</th>
<th>$^{8}\text{He},a$ (MeV)</th>
<th>$^{8}\text{He},a$ (MeV)</th>
<th>$^{15}\text{He},a$ (MeV)</th>
<th>$^{33}\text{He},a$ (MeV)</th>
<th>$^{40}\text{He},a$ (MeV)</th>
<th>$^{3}\text{He},a$ (MeV)</th>
<th>$^{d}\text{He},a$ (MeV)</th>
<th>$^{p}\text{He},a$ (MeV)</th>
<th>$^{s}\text{He},a$ (MeV)</th>
<th>Theory</th>
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<td></td>
</tr>
<tr>
<td>0.000</td>
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<td>5/2</td>
<td>1.56</td>
<td>1.82</td>
<td>1.90</td>
<td>2.50</td>
<td>2.50d</td>
<td>2.50d</td>
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<td>0.585</td>
<td>0</td>
<td>1/2</td>
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<td>0.23</td>
<td>0.07</td>
<td>0.18</td>
<td>0.17</td>
<td>0.17</td>
<td>0.42</td>
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<td></td>
<td></td>
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<tr>
<td>0.975</td>
<td>2</td>
<td>3/2</td>
<td>(0.31) (0.53)</td>
<td>0.07</td>
<td>0.12</td>
<td>0.11</td>
<td>0.11</td>
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<td>7/2</td>
<td>0.16</td>
<td>0.25</td>
<td>---</td>
<td>0.24</td>
<td>0.25</td>
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<td>0.00</td>
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</tr>
<tr>
<td>1.962</td>
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<td>0.14</td>
<td>0.22</td>
<td>0.56</td>
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<td></td>
</tr>
</tbody>
</table>

a) present work  b) reference 9  c) reference 3  d) Normalized to ground state of ($d,t$)

Table 7

to its spectroscopic factor. It is seen that general agreement is obtained between the 3.0 MeV ($^{3}\text{He},a$) results and those obtained elsewhere with the exception of the 15 MeV ($^{3}\text{He},a$), where the $K^\pi=1/2^+$ band appears to be considerably less strongly excited. In the last column of Table 7 is a list of theoretical calculations I have made using a deformation $\delta = 0.25$, assuming no band mixing, and assuming that Nilsson orbits 5 and 9 are 100% occupied when in actual fact they share
2 neutrons between them. Consequently, only interband ratios are significant. If the ratios of the spectroscopic factors between the three states of the $K^\pi = 1/2^+$ band are calculated, it is seen that the experimental and theoretical values agree within 30%. Dehnhard and Yntema\(^3\) have performed calculations without and with band mixing for \(^{26}\text{Mg}\). For the band mixed calculations, they assumed that Nilsson orbits 6 and 7 were completely filled and the remaining 2 particles were distributed over the orbits 5, 9, 8 and 11. For the spectroscopic factors, the inclusion of band mixing gave only a slight improvement in the comparison of experiment and theory.

The spectroscopic factors quoted here have additional substantiation by the fact that the experimentally extracted factors using the two optical model parameter sets A and B agree within 30%. The relatively strong excitation of the 1.611 MeV, 7/2\(^+\) level in \(^{25}\text{Mg}\) by the \(^3\text{He},\alpha\) reaction is similar to the strong excitation of the 2.048 MeV, 7/2\(^+\) level in \(^{23}\text{Mg}\) by the same reaction. In this experiment, the resolution and numerous proton groups at the lower energies prevented the observation of all of the sizable particle groups in each band, hence good population parameters for the ground state of \(^{26}\text{Mg}\) could not be obtained. However, a rough comparison of the relative spectroscopic strength of the two bands indicates that the 8.0 MeV \(^3\text{He},\alpha\) results are consistent with Dehnhard and Yntema's\(^3\) assertion that Nilsson orbit 5 is about 70\% filled.
The nuclei $^{27}\text{Si}$ and $^{28}\text{Si}$ have been widely studied by use of reactions involving pickup of a single neutron. The $(^3\text{He},\alpha)$ reaction at various energies has been done by Wildenthal and Glaudemans\textsuperscript{4}) at 10 MeV; Bray and Nurzynski\textsuperscript{10}) at 12 MeV; and Swenson, Zurmuhle and Fou\textsuperscript{11}) at 15 MeV. The (p,d) reaction has been studied at 27.6 MeV by Jones, Johnson and Griffiths\textsuperscript{15}) and at 33.6 MeV by Kozub\textsuperscript{14}).

An energy level diagram of $^{27}\text{Si}$ is shown in Figure 16. It is thought that the deformation of nuclei in the s-d shell changes from positive (prolate) to negative (oblate) around mass number $A \approx 28$. Therefore, the deformations of $^{27}\text{Si}$ and $^{28}\text{Si}$ may be close to zero and they are not expected to be rotational in nature. Since $^{27}\text{Si}$ consists of 14 protons and 13 neutrons the shell model formalism requires that the protons form a closed shell and that the ground state of $^{27}\text{Si}$ have 5 neutrons and a hole in the $1d_{5/2}$ shell. The shell model therefore predicts a ground state spin of $5/2^+$ in accordance with experiment. Adding the extra neutron to form $^{28}\text{Si}$ should close the $1d_{5/2}$ shell, but the presence of low-lying $1/2^+$ and $3/2^+$ states in $^{27}\text{Si}$ populated by neutron pickup reactions implies admixture of the $2s_{1/2}$ and the $1d_{3/2}$ neutron strength in the $^{28}\text{Si}$ ground state.

Shown in Figure 17 is a $^{28}\text{Si}(^3\text{He},\alpha)^{27}\text{Si}$ spectrum taken at 8 MeV at a lab angle of 45 degrees. The overall resolution is roughly 80 keV. Since the Q-value of the reaction is only 3.403 MeV, only a
Fig. 16. $^{27}\text{Si}$ and $^{29}\text{Si}$ level diagram
Fig. 17. $^{28}\text{Si}(^3\text{He}, \alpha)^{27}\text{Si}$ spectrum at 8 MeV
few of the states, namely those above the groups corresponding to elastic scattering could be analyzed. Note the tantalum contaminant which severely limited back angle measurements of the differential cross sections from several states.

Shown in Figure 18 is a plot of the 8 MeV elastic scattering data along with the optical model fit obtained. The $^3\text{He}$ optical model parameters resulting from the fit are listed in Table 3. Also shown in Figure 18 are the angular distributions resulting from the $^{28}\text{Si}(^3\text{He}, \alpha)^{27}\text{Si}$ reaction leading to the ground and first four excited states of $^{27}\text{Si}$ along with the DWBA prediction for each state. The a-parameters used are again McFadden and Satchler's parameters modified slightly in an attempt to obtain better agreement between experiment and theory. The 2.17 MeV state is not a state normally expected to be excited by a neutron pickup process and thus no theoretical curve is shown. It is seen that the predictions do not agree very well with the experimental data.

In spite of the fact that generally poor agreement was obtained between theory and experiment, spectroscopic factors were extracted from the predictions providing the best comparisons. These factors are listed in Table 8 along with those obtained from $(p, \alpha)$ and other $(^3\text{He}, \alpha)$ work. One immediately notices that the 8 MeV $(^3\text{He}, \alpha)$ factors are approximately twice as large as those obtained from the other neutron pickup experiments. However the relative values of the 8 MeV $(^3\text{He}, \alpha)$ factors are in line with the relative values of the other spectro-
Fig. 18. Angular distributions corresponding to the $^{28}{\text{Si}}(^3\text{He}, ^3\text{He})^{28}{\text{Si}}$ and $^{28}{\text{Si}}(^3\text{He}, a)^{27}{\text{Si}}$ reactions at 8 MeV.
scopic factors.

\[ ^{27}\text{Si Spectroscopic Factors} \]

<table>
<thead>
<tr>
<th>State (MeV)</th>
<th>(^{3}\text{He, a})_a^\text{a}</th>
<th>(^{3}\text{He, a})_b^\text{b}</th>
<th>(^{3}\text{He, a})_c^\text{c}</th>
<th>(^{3}\text{He, a})_d^\text{d}</th>
<th>(p, d)_e^\text{e}</th>
<th>(p, d)_f^\text{f}</th>
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<td>0.950</td>
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<tr>
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<td>2 5/2</td>
<td>2.34</td>
<td>1.30</td>
<td>0.72</td>
<td>0.76</td>
<td>0.23</td>
</tr>
</tbody>
</table>

\(a)\) present work \(b)\) reference 4 \(c)\) reference 10 \n\(d)\) reference 11 \(e)\) reference 15 \(f)\) reference 14

Table 8

When the optical model search on the \(^{3}\text{He} \) elastic scattering was done, three sets of parameters which gave roughly comparable fits to the data were obtained. These three sets were used in DWBA calculations using approximately the same \(a\)-parameters and spectroscopic factors were extracted for each case. The set used for the predictions shown in Figure 18 is the \(^{3}\text{He} \) set which is most in line with the \(^{3}\text{He} \) parameters obtained for the other nuclei and, perhaps significantly, is also the set which yielded spectroscopic factors most in agreement with those obtained from the work of others. However, since the agreement of the DWBA predictions with the data was no better for one set than for the others and since for the three sets the spectroscopic factors and their ratios varied by more than a factor of two, the 8 MeV \(^{3}\text{He, a} \)
spectroscopic factors listed in Table 8 have considerable uncertainty. Consequently, no calculations of neutron occupation numbers were made. However, Bray and Nurzynski\textsuperscript{10} have made such calculations which indicate that neutron orbits 1d\textsubscript{5/2}, 2s\textsubscript{1/2} and 1d\textsubscript{3/2} in the ground state of \textsuperscript{28}Si are filled to approximately 60\%, 30\% and 30\%, respectively.

F. \textsuperscript{30}Si(\textsuperscript{3}He,\alpha)\textsuperscript{29}Si Analysis

The nucleus \textsuperscript{29}Si has been widely studied since it can be easily reached with both stripping and pickup reactions. Using the (d, p) reaction to produce excited levels in \textsuperscript{29}Si, Baker and Segel\textsuperscript{48} measured the lifetime of the first four excited levels and Becker et al.\textsuperscript{49} used the (d, p) reaction to measure spins and parities with the method of angular correlation. Dehnhard and Yntema\textsuperscript{50} have used the (\textsuperscript{3}He,\alpha) and (d, t) reactions on \textsuperscript{30}Si in order to obtain spectroscopic information about the levels of \textsuperscript{29}Si and the ground state neutron configuration of \textsuperscript{30}Si. Bromley et al.\textsuperscript{50} first interpreted \textsuperscript{29}Si in terms of the collective model and found that the most probable shape was oblate with a deformation parameter of \(\delta = -0.15\). The 14 protons and 14 neutrons fill through the 1d\textsubscript{5/2} shell leaving one neutron which goes in Nilsson orbit 9 since the ground state spin of \textsuperscript{29}Si is 1/2\textsuperscript{+}. According to the interpretation of Bromley et al.\textsuperscript{50}, there is a \(K^\pi = 1/2^+\) band based on Nilsson orbit 9 consisting of the ground state, 2.632 MeV and 2.427 MeV states with the ground state being the band head. A \(K^\pi = 3/2^+\)
band based on Nilsson orbit 8 contains the 1.273 MeV and 3.069 MeV states with the 1.273 MeV state the band head. They found that in order to explain the branching ratio of the 2.032 MeV state, the bands had to be strongly mixed. It is not clear, however, that this will correctly predict the observed γ-ray intensities and lifetimes. The shell model interpretation of $^{29}$Si by Baker and Segel represents the ground state as a closed shell plus a single $2s_{1/2}$ neutron, the 1.273 MeV state as a closed shell plus a single $1d_{3/2}$ neutron, the 2.032 MeV state as approximately equal parts $(1d_{5/2})^{-1}(2s_{1/2})^2$ and $(1d_{5/2})^{-1}(1d_{3/2})^2$, and the 3.621 MeV state as a single particle $1f_{7/2}$ state. However, they note that this model does not give correct predictions for the M1 electromagnetic transitions. As for $^{30}$Si, the presence of a low-lying $3/2^+$ level strongly excited in neutron pickup reactions infers a considerable admixture of the $1d_{3/2}$ single particle state with the $2s_{1/2}$ single particle state in the $^{30}$Si ground state wave function. The $5/2^+$ states are excited by the pickup of a $1d_{5/2}$ neutron and are therefore called hole states.

Shown in Figure 19 is a spectrum of the $^{30}$Si($^3$He, d)$^{29}$Si reaction at $\theta_{\text{lab}} = 70$ degrees taken with 8 MeV $^3$He particles. The overall experimental resolution is approximately 100 keV. The Q-value of 9.9629 MeV for the reaction puts a substantial number of excited states up above the groups corresponding to elastic $^3$He scattering, as can be seen. The state at 8.320 MeV is the $T = 3/2$ analog of the $^{29}$Al ground state. Also note the presence of a state at 4.078 MeV, which Dehnhard
Fig. 19. $^{30}\text{Si}(^{3}\text{He},\alpha)^{29}\text{Si}$ spectrum at 8 MeV
and Yntema\textsuperscript{5}) failed to see with the same reaction using 33 MeV \(^3\text{He}\) particles. Shown in Figure 20 is a plot of the 8 MeV elastic scattering cross section along with two optical model fits to the data. The two sets of optical model parameters used to fit the data are listed in Table 5. The solid curve is the fit using \(V = 161\) MeV and the dotted curve represents the \(V = 145\) MeV set. Both sets gave almost identical fits to the elastic scattering data although at back angles the 145 MeV potential curve follows the data points a little more closely.

Shown also in Figures 20 and 21 are the experimental cross sections corresponding to the \(\text{(^3He, d)}\) reaction leaving the residual nucleus in the ground and first six excited states along with two DWBA predictions for each. The optical model parameters are given in Table 5. The \(\alpha\)-parameters are those of McFadden and Satchler\textsuperscript{29}). The dotted curve corresponds to the \(V_{\text{He}} = 145\) MeV, \(V_{\alpha} = 197.1\) MeV set while the solid curve corresponds to the \(V_{\text{He}} = 161.9\) MeV, \(V_{\alpha} = 204.1\) MeV set. For the ground state, note that the 145 MeV curve is more in phase with the experimental data than the 161 MeV curve. For the 2,032 MeV state, there is obviously no fit but the predictions are matched according to the general slope of the data points. This is done in order to obtain a rough estimation of the spectroscopic factor. Also, there is no fit for the 2,427 MeV state but it appears that there may be a considerable compound reaction contribution past 90 degrees. For the 3,062 MeV state, the fit is fairly good out to 90 degrees but the compound contribution offsets the normal falloff past 90 degrees. The
Fig. 20. Angular distributions corresponding to the $^{30}\text{Si}(^3\text{He}, ^3\text{He})^{30}\text{Si}$ and $^{30}\text{Si}(^3\text{He}, ^2\alpha)^{29}\text{Si}$ reactions at 8 MeV
Fig. 21. Angular distributions corresponding to the $^{30}\text{Si}^{3}\text{He},a^{29}\text{Si}$ reactions at 8 MeV.
size of the cross section for the 3.621 MeV state indicates a large admixture of a \((1f_{7/2})^2\) neutron configuration in the ground state of \(^{30}\text{Si}\). This state is not expected to be excited by a pickup process on the basis of a single particle shell model. The 4.078 MeV state is shown with \(l = 4, 7/2^+\) predictions although close comparisons are not expected since this state is believed to be excited by a multiple excitation process.

Shown in Figure 22 is a plot of the 7 MeV elastic scattering along with the optical model fit obtained. The parameters are listed in Table 5. Shown in Figures 22 and 23 are the 7 MeV \(^3\text{He},a\) angular distributions leading to the ground and first six excited states of \(^{29}\text{Si}\) along with the DWBA prediction for each state. The entrance and exit channel parameters used in the calculations are listed in Table 5. As can be seen, the comparisons aren’t as close as those for 8 MeV which is to be expected since there appears to be a greater compound contribution to the cross section at this lower energy. No prediction is shown with the 4.079 MeV state since it is supposedly not populated by a pickup process. The spectroscopic factors extracted for the foregoing fits are given in Table 9. Also listed are the spectroscopic factors obtained from the \((\text{d},t)\) and \((^3\text{He},a)\) work of Deichardt and Yntema.\(^5\). The spectroscopic factors extracted for the 7 MeV \(^3\text{He},a\) were larger than those obtained from the 8 MeV \(^3\text{He},a\), suggesting an increased compound nucleus contribution. Therefore, the 7 MeV factors were normalized to the spectroscopic factor for the ground state obtained from the \((\text{d},t)\) data in order to obtain some relative values for comparison. On comparing the 7 and 8
Fig. 22. Angular distributions corresponding to the $^{30}\text{Si}(^3\text{He}, ^3\text{He})^{30}\text{Si}$ and $^{30}\text{Si}(^3\text{He}, ^2\text{He})^{29}\text{Si}$ reactions at 7 MeV
Fig. 23. Angular distributions corresponding to the $^{30}\text{Si}(^{3}\text{He}, \alpha)^{28}\text{Si}$ reactions at 7 MeV
MeV ($^3\text{He}, \alpha$) spectroscopic factors with those from the 33 MeV (d, t), it is seen that relatively good agreement is obtained.

\[
\begin{array}{|c|c|c|c|c|c|c|c|}
\hline
\text{State} & (^3\text{He}, \alpha)_{8 \text{ MeV}} & (^3\text{He}, \alpha)_{7 \text{ MeV}} & (^3\text{He}, \alpha)_{22 \text{ MeV}} & (d, t)_{\text{Nilsson Model}} \\
(\text{MeV}) & I & J & (145) & (161) & (171) & (0.7) & (0.7) & K^\pi=1/2^+ \\
\hline
0.000 & 0 & 1/2 & 0.70 & 0.80 & 0.8^b & 0.7 & 0.35 \\
1.273 & 2 & 3/2 & 1.46 & 1.50 & 1.24 & 1.2 & 0.7 \\
2.032 & 2 & 5/2 & (1.3) & (1.38) & (0.89) & 1.7 & 1.7^c \\
2.427 & 2 & 3/2 & (0.35) & (0.25) & 0.21 & 0.17 & 0.20 \\
3.069 & 2 & 5/2 & 0.31 & 0.34 & 0.44 & 0.18 & 0.10 \\
3.623 & 3 & 7/2 & 0.17 & 0.19 & 0.11 & 0.11 & 0.08 \\
\hline
\end{array}
\]

a) Reference 5  
b) Norm to (d, t), N=8  
c) Norm to (d, t), N=13  

Spectroscopic factors in parentheses are based on an overall estimate of the ratio of the experimental cross section to the DWBA prediction in spite of poor agreement.

Table 9

By summing the spectroscopic factors for their 5/2\(^+\) states, Dehnhard and Yntema\(^5\) concluded that on the basis of the shell model the 1d\(_{5/2}\) shell is filled for the ground state of \(^{30}\text{Si}\). They concluded that the two neutrons remaining outside the \(^{28}\text{Si}\) core are almost equally distributed between the 2s\(_{1/2}\) and 1d\(_{3/2}\) shell model states. From the spectroscopic factors obtained from the 7 and 8 MeV ($^3\text{He}, \alpha$) data, it is concluded that the two particles spend considerably more time in the 1d\(_{3/2}\) state than in the 2s\(_{1/2}\) state.

On the other hand, the spectroscopic factors can be interpreted...
in terms of the Nilsson model. Following Bromley et al. and assuming that the ground, 2.032 MeV and 2.427 MeV states comprise a \( K^\pi = 1/2^+ \) band based on Nilsson orbit 9 and that the 1.273 MeV and 3.069 MeV states comprise a \( K^\pi = 3/2^+ \) band based on Nilsson orbit 8, the spectroscopic factors can be summed by bands thus giving, for the ground state of \(^{30}\text{Si}\), the distribution of the two neutrons between Nilsson orbits 8 and 9.

Also, calculations were made of the spectroscopic factors using the prescription of Satchler assuming no band mixing. The spectroscopic factors were calculated for deformations of \( \delta = -0.1 \) and \( \delta = -0.2 \) from the coefficients of the wave functions given by Chil.

Then the spectroscopic factors for \( \delta = -0.15 \) were obtained by interpolation. These are shown in the last two columns of Table 9, where on the basis of the present work and that of Dehnard and Yntema, the \( K^\pi = 1/2^+ \) and the \( K^\pi = 3/2^+ \) bands have each been assumed to be 50% filled. The experimental values are generally larger than those predicted indicating that band mixing may be necessary in order to produce the correct magnitudes. However, the ratios of the theoretical spectroscopic factors between members of the same band are close to the corresponding experimental ratios.

It should be noted that there is good agreement between the 8 MeV spectroscopic factors obtained using the two different sets of \(^{3}\text{He}\) optical model parameters, both of which yielded comparable fits to the experimental data.
C. \(^{32}\text{S}(^{3}\text{He}, \alpha)^{31}\text{S}\) Analysis

Compared with the other nuclei in this experiment, \(^{31}\text{S}\) has been the least studied of all. Level identifications and level energies have been determined using the \((^{3}\text{He}, \alpha)\) reaction by Ajzenberg-Selove and Wiza\(^{12}\), by Roobard et al.\(^{13}\), and by Graue\(^{6}\). A \(^{32}\text{S}(p, d)^{31}\text{S}\) experiment was performed by Kozub\(^{14}\) to test for J-dependence and to obtain spectroscopic information. Also, spectroscopic factors were obtained and a model analysis was made using the results of a \(^{32}\text{S}(^{3}\text{He}, \alpha)^{31}\text{S}\) experiment at 15 MeV by Fou and Zurmühle\(^{7}\). They also made a few observations about J-dependence for \((^{3}\text{He}, \alpha)\) reactions. An energy level diagram of \(^{31}\text{S}\) is shown in Figure 24.

The nucleus \(^{31}\text{S}\) consists of 16 protons and 15 neutrons so that, in the shell model formalism, the protons fill up through the \(2s_{1/2}\) shell and the neutrons fill up through the \(1d_{5/2}\) shell with one neutron in the \(2s_{1/2}\) shell. Thus the ground state spin of \(^{31}\text{S}\) is 1/2\(^+\). From the energy level diagram of \(^{31}\text{S}\), one sees that the three lowest states have spins of 1/2\(^+\), 3/2\(^+\) and 5/2\(^+\). In a \((^{3}\text{He}, \alpha)\) reaction, the 1/2\(^+\) and 5/2\(^+\) states would be populated by pickup of a \(2s_{1/2}\) neutron and a \(1d_{5/2}\) neutron respectively, assuming that the neutrons in the ground state of \(^{32}\text{S}\) filled through the \(2s_{1/2}\) shell. However, the presence of the 3/2\(^+\) state at 1.22 MeV indicates that the ground state neutron configuration of \(^{32}\text{S}\) contains an admixture of the \(2s_{1/2}\) and \(1d_{3/2}\) states.

Shown in Figure 25 is a \(^{32}\text{S}(^{3}\text{He}, \alpha)^{31}\text{S}\) spectrum taken at
Fig. 24. $^{31}$S level diagram
$^{32}\text{S}(^{3}\text{He},a)^{31}\text{S}$

$E_{^{3}\text{He}} = 8.0\text{ MeV}$

$\theta_{\text{lab}} = 55^\circ$

Fig. 25. $^{32}\text{S}(^{3}\text{He},a)^{31}\text{S}$ spectrum at 8 MeV
$E_{3^\text{He}} = 8 \text{ MeV}$ and $\theta_{\text{lab}} = 55 \text{ degrees}$. The overall resolution is approximately 100 keV. Because the Q-value is only 5.490 MeV, only a few states, those above the groups corresponding to elastic scattering, could be observed since the Sb elastic group and other elastic groups dominated the lower end of the spectrum.

A plot of the 8 MeV elastic scattering cross section is shown in Figure 26 along with the optical model fit from JIB. The parameters are listed in Table 5. Also shown in Figure 26 are the angular distributions for the ground, 1.22 MeV and 2.21 MeV states, respectively, along with their DWBA predictions. The optical parameters are listed in Table 5. As can be seen, the experimental data and the theoretical predictions do not agree very well. The vertical positions of the predictions were adjusted so as to try to match the general slope of the experimental data, thus giving some idea of the magnitude of the spectroscopic factors. These are listed in Table 10 along with spectroscopic factors obtained from a 15 MeV ($^3\text{He}, \alpha$) experiment.

$$\begin{tabular}{|c|c|c|c|c|c|}
\hline
State & $I$ & $J$ & $(^3\text{He}, \alpha)_8 \text{ MeV}$ & $(^3\text{He}, \alpha)_b^{15} \text{ MeV}$ & $(p, \alpha)_b^{33.6} \text{ MeV}$ \\
(MeV) & & & & & \\
\hline
0.00 & 0 & 1/2 & 4.44 & 0.9 & 1.04 \\
1.24 & 2 & 3/2 & 3.83 & 1.1 & 0.94 \\
2.23 & 2 & 5/2 & 7.80 & 2.9 & 2.77 \\
\hline
\end{tabular}$$

a) reference 7 \hspace{1cm} b) reference 14

Table 10
Fig. 26. Angular distributions corresponding to the $^{32}\text{S}(^3\text{He}, ^3\text{He})^{32}\text{S}$ and $^{32}\text{S}(^3\text{He}, ^3\text{He})^{31}\text{S}$ reactions at 8 MeV.
and a 33.6 MeV (p,d) experiment. As can be seen, the 8 MeV 
(3He,α) spectroscopic factors are considerably larger than those ob-
tained from the others listed. The comparison of relative spectro-
scopic factors is, however, fairly good. However, due to the low 
quality of the fits, these spectroscopic factors are quite uncertain. 
Since factors were extracted only for the lowest three states, no calcu-
lations of neutron configurations were made.

H. Discussion

On comparing the relative magnitudes of the various cross 
sections, one notices that the cross sections corresponding to the strong 
states of the 4n nuclei (24Mg, 28Si, 32S) are a factor of from 5 to 10 
times larger than those corresponding to states in 26Mg and 30Si. 
This can partially be accounted for by the fact that the Q-values corre-
sponding to 26Mg and 30Si are approximately 5 MeV larger than those 
corresponding to the 4n nuclei. For the required l-value transfers of 
0 and 2, the higher Q-values result in a mismatch between the contrib-
uting partial waves in the entrance and exit channels. This results in 
a lowered cross section. Shown in Figure 27 are distorted wave calcula-
tions made with the usual Q-values for the reaction and with the 
Q-values artificially decreased by 5 MeV. It is seen that for the 
l = 0 and l = 2 transfers, the predicted cross sections are increased by 
a factor of from 2 to 3 while the effect for the l = 3 transfer is to change
Fig. 27. Dependence of the DWBA cross sections on the Q-value
the shape so that no direct comparison can be made. Stock et al.\textsuperscript{1)} obtained a cross section difference of about 2 for an $t = 1$
$54\text{Cr}(^{3}\text{He,}\alpha)^{58}\text{Cr}$ calculation at 18 MeV by changing Q by 10 MeV.

The fact that the 4n nuclei can be represented by an integral number of α-particles raises the possibility of α-particle clustering. Alpha particles existing within a nucleus will enhance the probability of an alpha particle knockout or heavy particle pickup reaction. This amplitude would then be added to the ordinary pickup amplitude and increase the observed cross section. Thus, an addition of a knockout process may explain the remaining discrepancy between the cross sections corresponding to $^{24}\text{Mg}$, $^{28}\text{Si}$ and $^{32}\text{S}$ and the $^{28}\text{Mg}$ and $^{30}\text{Si}$ cross sections. It would also explain the difficulty in fitting the $^{24}\text{Mg}$, $^{28}\text{Si}$ and $^{32}\text{S}$ angular distributions by calculations assuming a neutron pickup mechanism only. In addition, it might explain the excitation of many of the states not expected to be populated by a neutron pickup process and the difficulties regarding the ratios of the spectroscopic factors for several states. Additional calculations are certainly needed to see what agreement might be obtained assuming a knockout mechanism.

A Hauser-Feshbach calculation of the compound nucleus contribution could not be made since the beam-target energy spread was approximately equal to the coherence energy and thus to the average level width in the compound nucleus. The aforementioned theory is invalid unless the experimental resolution averages over many coherence widths, i.e., $\Delta E \gg \Gamma$. However, the Ericson fluctuation
analysis indicated that the reaction mechanism was predominately di-
rect (\(> 80\%\)) for angles less than 60 degrees with the compound nucleus
contributing increasingly as one moves to the back angles. Assuming
a compound nucleus contribution of less than 20\%, a Q-value effect and
that the \(^{26}\text{Mg}\) and \(^{30}\text{Si}\) reaction are almost all proceeding by a neutron
pickup (as evidenced by the agreement with \((p, d)\) and \((d, t)\) reactions),
the increased cross sections for the 4n nuclei might indicate an exchange
contribution equal to or somewhat greater than the neutron pickup con-
tribution. This, of course, neglects all possible interference effects
on the cross section.

In the optical model analysis of the nuclei, it was found that very
similar sets of parameters with \(V_{3\text{He}} = 168 \pm 10\ \text{MeV}\) and \(R_r = 1.15 \pm .04\)
fm gave reasonable fits to all of the elastic scattering data. For these
sets, the product \(V \cdot R_r^{3/2}\) varied by less than 8\%. The \(V_{3\text{He}} = 155.6\)
MeV set for \(^{26}\text{Mg}\) and the \(V_{3\text{He}} = 145\ \text{MeV}\) set for \(^{30}\text{Si}\) are exceptions.
The elastic scattering data corresponding to the 4n nuclei could not be
fit with a set containing \(V_{3\text{He}} = 150\ \text{MeV}\).

In the DWBA analysis, a neutron pickup direct reaction was as-
sumed for angles less than 60 degrees and the DWBA predictions were
mainly matched to the experimental data at these forward angles in
order to extract the spectroscopic factors. The general quality of the
DWBA fits to the experimental data is poorer for the 4n nuclei than for
\(^{26}\text{Mg}\) and \(^{30}\text{Si}\). For each of the nuclei studied, spectroscopic factors
were extracted from DWBA calculations using several different sets of
optical model parameters, all of which fit the corresponding elastic scattering to a comparable degree. However, the sets of $^3\text{He}$ optical parameters for $^{24}\text{Mg}$, $^{28}\text{Si}$ and $^{32}\text{S}$ were determined less decisively than those for $^{26}\text{Mg}$ and $^{30}\text{Si}$. In comparing the spectroscopic factors obtained using the different parameter sets, those for the $4n$ nuclei varied appreciably more than those of $^{26}\text{Mg}$ and $^{30}\text{Si}$. Consequently, the spectroscopic factors for $^{26}\text{Mg}$ and $^{30}\text{Si}$ are probably accurate to within 30% while those for $^{24}\text{Mg}$, $^{28}\text{Si}$ and $^{32}\text{S}$ are considerably more uncertain. Also, the spectroscopic factors for the $4n$ nuclei range from 2 to 3 times larger than those obtained from most other ($^3\text{He},\alpha$) reactions on the same nuclei while the 3 MeV cross sections are comparable or less than the other cross sections. This is not true for $^{26}\text{Mg}$ and $^{30}\text{Si}$.

In this experiment, a systematic $J$-dependence was seen for $l = 2$ transfers leading to either $3/2^+$ or $5/2^+$ final states. These are shown for comparison in Figure 28. Comparing the $l = 2$ angular distributions within each nucleus it is seen that, with the notable exceptions of the 2.65 MeV $5/2^+$ state in $^{27}\text{Si}$ and the 3.06 MeV $5/2^+$ state in $^{29}\text{Si}$, there exists in the $5/2^+$ distributions a dip in the cross section around 20 to 30 degrees which is absent in the $5/2^+$ distributions. The same effect is seen in the 15 MeV $^{32}\text{S}(^3\text{He},\alpha)^{31}\text{S}$ work of Fou and Zurmühlé 7) although in the 15 MeV ($^3\text{He},\alpha$) work on $^{24}\text{Mg}$ and $^{26}\text{Mg}$ by Joyce et al. 9) this effect is absent. From his 33.6 MeV $(p,d)$ results, Kozub 14) concluded that the $J$-dependence in the $(p,d)$ reaction varied systematically
Fig. 28. Comparison of angular distributions for J-dependence
as one considered different mass regions of the s-d shell. He suggested that this might infer some correlation between J-dependence and deformation. An attempt was made to reproduce the 8 MeV J-dependence with the inclusion of a spin-orbit force but this was to no avail. Thus the dip in the $3/2^+$ distributions occurring further than for the $5/2^+$ distributions can only be considered as an empirical rule.
CHAPTER V
CONCLUSIONS

In summarizing the results of this experiment, one must first mention the reaction mechanism. It was found that at as low an incident $^3$He energy as 8.0 MeV the direct reaction is still the dominant process at forward angles, thus allowing a DWBA analysis to be performed. It was found that the $^3$He elastic scattering angular distributions corresponding to all of the nuclei studied could be fit with varying degrees of success using very similar sets of optical model parameters having $V_r \approx 168$ MeV and $R_r \approx 1.15$ fm. The DWBA analysis and the extraction of spectroscopic factors were more successful for $^{26}$Mg and $^{30}$Si than for $^{24}$Mg, $^{28}$Si and $^{32}$S although most states were excited more strongly with the latter nuclei. It might be possible to attribute this to the fact that for the 4n nuclei, there is an enhanced probability of a substantial exchange contribution to the reaction amplitude. There was also found to be a greater uncertainty in the optical model parameters obtained from elastic scattering fits which could also produce the poorer agreement of the DWBA predictions with experiment for the 4n nuclei.

With regard to the work by Stock et al. 1), it was found that the best agreement between theory and experiment was obtained when their
suggestions concerning optical potentials were adhered to. When good DWBA comparisons were obtained, the spectroscopic factors extracted using their absolute normalization constant of 23 compared well with those resulting from other neutron pickup work. As for the observed J-dependence, the two exceptions to the rule and the fact that the type of J-dependence appears to differ for different energies precludes the advancement of any reliable empirical measure for spin determination. However, this rule coupled with other related information may be helpful in making spin assignments with a higher degree of probability.
REFERENCES


