AVERAGE NEUTRON TOTAL CROSS SECTIONS
AND THE OPTICAL MODEL

by

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Date:

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H. W. Newson, Supervisor

A dissertation submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy in the Department of Physics in the Graduate School of Arts and Sciences of Duke University

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ABSTRACT

(Physics)

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Average neutron total cross sections were measured for samples of potassium chloride, scandium, zinc, arsenic, selenium, rubidium bromide, cesium iodide, cerium, neodymium, samarium, gadolinium, terbium, dysprosium, holmium, mercury, and the separated isotopes $^{144}$Nd, $^{146}$Nd, $^{148}$Nd, $^{149}$Sm, and $^{152}$Sm. These measurements were made with an improved geometry designed to minimize the inscattered and extraneous neutron background.

Cross sections averaged over the energy ranges of 100-650 keV and 350-650 keV were compared with optical model calculations made using both spherical and deformed potentials.

The agreement between the spherical optical model calculations and the averaged cross section data is only fair below $A=140$, where the nuclei are not statically deformed, and fall completely above $A=140$ where the nuclei are known to exhibit large static deformations. The agreement between the deformed optical model calculations and the averaged cross sections is somewhat better but discrepancies are still apparent near $A=100$ and above $A=140$. The effect of the simplified coupling schemes and of the known limits of error in
the quadrupole deformation parameters which were used in the
calculations are discussed at length.

$S$, $p$, and $d$-wave strength functions, the $s$-wave
scattering length, and the $p$-wave phase shift ratio were
extracted from the average neutron total cross sections data
using the Duke low resolution method. These parameters are
compared with optical model calculations made using deformed
potentials. The measurements of the $s$-wave strength functions
add to the overwhelming evidence that the $4s$ giant resonance is
split, and the measurements of the $s$-wave scattering length $R'$
add considerably to the details of its variation in the $4s$
giant resonance region.
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AVERAGE NEUTRON TOTAL CROSS SECTIONS
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CHAPTER 1
INTRODUCTION

The first application of the complex potential well model to low energy neutron total cross sections by Feshbach, Porter, and Weisskopf (1954) was inspired by the work of Barshall and his collaborators (1952), who had made a survey of average total neutron cross sections for many different elements and had reported that the data showed broad fluctuations which varied systematically with neutron energy and target mass. This sort of behavior suggests that the neutrons were being scattered by an average potential created by the target nucleons. Since potential well models had already been used to describe the bound nuclear excited states (Haxel, Jensen, and Suess, 1949; Mayer, 1949), and complex potentials had been used to describe high energy ($E \sim 100$ MeV) neutron scattering (Fernbach, Serber, and Taylor, 1949), it seemed natural that some sort of complex potential well model could be applied to the case of low energy neutron scattering.
Feshbach, Porter, and Weisskopf attributed the necessity of using an imaginary term in their potential to the fact that when a neutron interacts with a nucleus, the compound system may form a well-defined excited state whose lifetime will be much greater than the time it normally takes for a neutron to scatter from a potential well. Neutrons which are absorbed into one of these "compound nucleus" states have effectively disappeared from the incident beam; and the presence of an imaginary term in the scattering potential will roughly describe this disappearance since, within the range of the potential, the neutron wave function will be the product of a decaying exponential and an oscillatory wave function.

For convenience in the calculations, Feshbach, Porter, and Weisskopf used a complex square well of form

\[ V(r) = U(r) \left( 1 + i \mathcal{F} \right) \]

where

\[ U(r) = -V_0 \quad \text{for} \quad r \leq R, \]
\[ U(r) = 0 \quad \text{for} \quad r > R, \]

\( \mathcal{F} \) is a constant, and \( R \) is the nuclear radius, \( R = r_0 A^{1/3} \). In spite of the crude form of the potential, they were able to reproduce qualitatively many of the features of Barshall's data by adjustment of the parameters.

Two values of the potential well depth were tried: \( V_0 = 19 \) MeV, and \( V_0 = 42 \) MeV. The smaller value predicted maxima in the s-wave neutron total cross section at the atomic mass
numbers $A = 38$ and $170$, and maxima in the $p$-wave neutron total cross section at $A = 11$ and $90$. The larger value predicted $s$-wave maxima at $A = 11$, $55$, and $150$ and $p$-wave maxima at $A = 27$, $90$, and $216$. For neutron cross sections for $A > 60$, either well predicted the $s$-wave maxima which is known experimentally to be at $A = 150$ and the $p$-wave maxima known to be at $A = 90$. Marshall's data indicated that there was an $s$-wave maximum between $A = 40$-$60$, and therefore the higher value of $V_0$ seemed to give the better fit. Later work by Côté, Bollinger, and Le Blanc (1958) and Marshak and Newson (Marshak and Newson, 1957, and Marshak, thesis, 1956) confirmed the existence of the maximum and indicated that its peak occurred at $A = 50$ and therefore a $V_0$ of about $50$ MeV gave the best fit.

Two values of $\gamma$ were tried: $\gamma = 0.03$, and $\gamma = 0.05$. The effect of increasing $\gamma$ is to flatten the peaks of the resonances and raise the wings; it was found that the lesser value of $\gamma$ gave the best fit to the data.

Although later forms of the optical model are more realistic than the square well, potential depths of about $40$-$50$ MeV still give the best fits to low energy neutron scattering data.

Two of the experimental quantities which can be calculated rather easily from the optical model are the strength functions and the phase shifts for neutrons of a given angular momentum, $\tilde{h}$, bombarding a target of given $A$. The strength function is defined as
\[ Q_L = \frac{\langle \Gamma_L^R \rangle}{\langle D(\theta) \rangle} \]  

(2)

where \( \langle \Gamma_L^R \rangle \) is the energy averaged reduced width of the compound nuclear resonances, and \( \langle D(\theta) \rangle \) is the average spacing between the resonances. These will be discussed in detail in Chapter II.

For the past several years, this laboratory has been carrying out a program to measure strength functions and phase shift ratios by extracting them from average neutron total cross sections measured at neutron energies below the inelastic scattering threshold. These parameters have now been measured for many nuclei throughout the periodic table; they have been compared to available calculations from various forms of the optical model, and with the parameters obtained by different experimenters using other methods of measurement.

Neutron total cross sections averaged over several hundred keV have also been compared to calculations made from the optical model. Since the cross sections can be measured directly, their values are considerably more reliable than those of the strength functions and phase shifts, which must be obtained from a complicated analysis of the data. For this reason, the cross sections form a fruitful test of the optical model for neutron scattering below 1 MeV.

The most extensive survey of strength functions, phase shift ratios, and average neutron total cross sections was done by Divadeenam, Bilpuch, and Newson (Divadeenam, thesis, 1967) who conducted a review of earlier measurements made by Seth et
al. (1964), and Tabony (thesis, 1966); added many new ones in the p-wave giant resonance region; and made extensive comparisons with calculations made from the available forms of the optical model. They also compared the values which had been obtained by Seth et al., Tabony and themselves with the values obtained by other experimenters using other methods of measurement; they discussed at length the merits of the various methods of measurement and analysis.

The purpose of the present thesis is to verify some of the earlier values of the s- and p-wave strength functions, phase shift ratios, and average cross sections obtained in the 3s and 3p giant resonance region and to report new measurements made in the rare earth (4s giant resonance) region.

The calculations mentioned above have been extended, and the new data are compared with them.

It will be noticed that three of the samples used were alkali halide compounds. The term "average neutron total cross section" as applied to them means the cross section averaged over all of the isotopes present in either of the component elements; the spread in mass number is comparable to that in the natural elements of even charge.
CHAPTER II

THEORY

A. Strength Functions and the Optical Potential

In order to clarify the discussion that follows, we will present here an approximate derivation of the connection between the average natural widths of the compound nuclear states \( \langle \Gamma_n \rangle \), the average level spacing \( \langle D \rangle \), and the nucleon-nucleus potential. The treatment is taken from Blatt and Weisskopf (1952).

Suppose that the compound nucleus states are widely separated in energy so that \( \langle \Gamma_n \rangle \ll \langle D \rangle \) and suppose that they are all evenly spaced. The level energy of the \( n \)th state, \( E_n \), can be expressed as

\[
E_n = E_0 + n\langle D \rangle
\]

where \( E_n \) is the energy of one of the states. We can construct the total wave function of the compound system by expanding it in terms of the wave functions of the compound nuclear states. If their spatial dependence is denoted by \( \Psi \), then we can write
\[ \psi = \sum_{n=1}^{N} \alpha_n \phi_n \exp\left(-\frac{iE_n t}{\hbar}\right) = \left[\exp\left(-\frac{iE_0 t}{\hbar}\right)\sum_{n=1}^{N} \alpha_n \phi_n \exp\left(-\frac{iE_n t}{\hbar}\right)\right] \]

It is evident that

\[ |\psi(t + \frac{2\pi \hbar}{\langle D \rangle})|^2 = |\psi(t)|^2 \] (5)

so that \( \psi \) describes the same nucleon configuration at time \( t = t + \frac{2\pi \hbar}{\langle D \rangle} \) as it does at time \( t \). Hence the period of motion \( P \) is

\[ P = \frac{2\pi \hbar}{\langle D \rangle} \] (6)

The compound system was created by a bombarding particle interacting with a target nucleus, and at first the projectile and target formed a state wherein the projectile interacted with the average potential created by all of the nucleons in the target. This is called the single particle state. Every \( P \) seconds the compound system will return to the single particle configuration and the projectile nucleon (or one identical to it) will have enough energy to escape. Since the nuclear potential changes rapidly at the nuclear boundary, there is a distinct possibility that the projectile will be reflected and recreate the compound nucleus. Therefore the probability, \( T \), that the particle can penetrate the nuclear barrier and escape is less than unity even though it has positive energy. The lifetime of the nuclear state can now be expressed as
\[ \mathcal{L} = \frac{P}{T} = \frac{2\pi k}{\langle D \rangle T} \]  

(7)

The lifetime of a state is also related to the natural resonance width \( \langle \Pi_n \rangle \) through the Heisenberg uncertainty principle:

\[ \mathcal{L} = \frac{k}{\langle \Pi_n \rangle} \]  

(8)

The ratio \( \langle \Pi_n \rangle / \langle D \rangle \) can now be related to the transmission probability \( T \) by

\[ \frac{\langle \Pi_n \rangle}{\langle D \rangle} = \frac{2\pi T}{T} \cdot \]  

(9)

In order to remove the effects of the centrifugal barrier and the kinetic energy of the neutron, the reduced width is defined by

\[ \Pi_n^l = \frac{\Pi_n(l)}{\left[ V(l) / E_{\text{nv}} / l \text{ev} \right]^{1/2}} \]  

(10)

where \( V(l) \) is the probability that the neutron can penetrate the centrifugal barrier of a hard sphere, and \( E_{\text{nv}} \) is the neutron energy. Note that the units are corrected by expressing \( E_{\text{ev}}/l \text{ev} \) as a unitless ratio. This is the neutron reduced width used by most experimentalists. The strength function is defined by

\[ S_2 = \frac{\langle \Pi_n^l \rangle}{\langle D(l) \rangle} \]  

(11)
so that

$$S_{\ell} = \frac{2\pi}{T} \left( \frac{E_n (eV) / 1eV}{1} \right)^{1/2} V(t)$$  \hspace{1cm} (12)$$

where $\ell$ refers to neutrons of given angular momentum $\ell$. It should be pointed out that $S_{\ell} = 10^{-4}$ for the black nucleus model when $E_n$ is close to zero, and that this value is independent of the atomic mass of the nucleus. Since the transmission probability (also called the transmission coefficient) is related to the shape and depth of the potential well, the measurement of the strength function provides a tool for determining some of the properties of the optical potential.

When the strength function is measured by observing the resonances themselves, one can simply add up all of the reduced widths and divide by the range of energy over which they were observed. In principle one must correct for the effect of the resonances which could not be observed at his energy resolution but they are so narrow that this correction is usually negligible. A full discussion of this problem is given by Farrell, Bllpuch, and Newson (1966).

B. Forms of the Optical Model Potentials

Since a particle inside the nuclear surface is subject to strong, short-range, attractive forces from the nucleons
Immediately surrounding it, and therefore to a null resultant, the nuclear potential is expected to be nearly constant in the nuclear interior. It is also expected that since the nuclear surface is diffuse, the strength of the potential should fall off more gradually there than does a square well. All potentials fitting these criteria are characterized by a nuclear radius and a parameter related to the diffuseness of the surface.

The interaction of a projectile nucleon with a target nucleus causes rearrangement of the nucleons and hence the promotion of some of them into higher energy states. At low energies in the nuclear interior, all of these states are filled and the operation of the Pauli principle prevents an interaction from taking place. Hence, at low energies, interactions between nucleons in the target and the projectile are strongest at the surface, and the absorbing potential has a dip there. As the energy of the projectile is increased, more of the absorption takes place in the nuclear interior, and the surface peaking becomes less pronounced.

It has been found that variations in the forms of the potential are not important so long as the above criteria are followed. For convenience in comparison of optical model parameters, most writers use the form

\[ V(r) = U f(r) + i W g(r) \]  \( (13) \)

where \( U \) and \( W \) are depths of the real and absorbing potentials,
and $f$ and $g$ are form factors whose maximum values are unity. It is often assumed that $f(r)$ has the Woods-Saxon form:

$$f(r) = \frac{1}{1 + \exp \left[ \frac{(r-Rw)/aw}{2} \right]}$$

(14)

and that $g(r)$ is proportional to the derivative of the Woods-Saxon form:

$$g(r) = \frac{4 \exp \left[ \frac{(r-Rw)/aw}{2} \right]}{\left\{ 1 + \exp \left[ \frac{(r-Rw)/aw}{2} \right] \right\}^2}$$

(15)

The Gaussian form:

$$g(r) = \exp \left\{ -\left[ \frac{(r-Rw)/aw}{2} \right]^2 \right\}$$

(16)

is sometimes used. It will be noticed that both Eqs. (15) and (16) give the required surface peaking for $g$. Many other forms are also used however. The most popular ones are mentioned in Hodgeson (1963).

C. The Effect of Spin-orbit Coupling on the Optical Model

The observation that scattered particles with spin are usually polarized led to the introduction of a term proportional to the dot product of the projectile orbital angular momentum $\vec{l}$ and the projectile spin $\vec{S}$ into the optical model potential. For bound particles this is the spin
dependence that gave successfully the sequence of bound states of the shell model (Mayor and Jenson, 1955). The proper form of the radial dependence for this potential was derived by Fernbach, Heckrotte and Lepore (1955) and Brown (1957) by interpreting the optical model potential as a sum of nucleon-nucleon interactions. Their results indicated that the spin-orbit part should have a radial dependence of the Thomas form:

\[ V_{so} = \lambda \left( \frac{\hbar}{m_n c} \right)^2 \frac{d V(r)}{d r} \hat{\ell} \cdot \vec{\sigma} \]  \( (17) \)

where \( \lambda \) is a multiplicative constant, \( \hat{\ell} \) is the orbital angular momentum vector and \( \vec{\sigma} \) is the Pauli spin operator for the projectile. If one wishes to add on absorptive term to the spin-orbit potential, this can be conveniently expressed as

\[ V_{so} = (\mu_s + i \omega_s) h_{so} \hat{\ell} \cdot \vec{\sigma} \]  \( (18) \)

where

\[ h_{so} = - \frac{1}{r} \frac{d f(r)}{d r} \]  \( (19) \)

\( f(r) \) is the form factor for the real part of the optical model potential. Like the absorbing potential, this has a surface-peaked form.
D. Non-Local Potentials

Studies of nuclear matter by many investigators (Bethe and Goldstone, 1956; Feshbach, 1958; Sugie, 1959; and Namiki, 1960) have shown that the optical potential when derived from the summation of the nucleon-nucleon potentials is non-local. The term means that the potential acting on a particle at \( \mathbf{r} \) depends upon the wave function throughout all space, and thus takes into account the finite size of the particle and its dispersion by nuclear matter. Since the size of the wave packet of the bombarding particle depends upon its energy, calculations using energy independent non-local potentials reproduce the results given by calculations using energy dependent local potentials. Because of the short range of nuclear forces, it is expected that the range of non-locality is rather small. Studies by Wyatt, Wells, and Green (1960) showed that the agreement between experiment and theory obtained by the use of the potential predicted by Brueckner theory with a range of non-locality \( a \sim 0.8 \text{fm} \) was comparable to that which had been obtained previously by the use of local potentials.

Perey and Buck (1962) investigated a non-local potential for the case of neutron scattering, and found that the results which they obtained were comparable to those which had been obtained previously by the use of energy dependent local potentials. They analyzed the neutron differential cross sections of lead measured at 7 and 14 MeV, and used the
resulting potential parameters to calculate total and reaction cross sections for elements ranging from aluminum to lead at energies ranging from 0.4 to 24 MeV. They also calculated the strength functions and the s-wave scattering radius $R_1$ from the same potential, and derived an approximate relation between the non-local potential and the equivalent local potential.

E. Nuclear Deformations

Feshbach, Porter, and Weisskopf (1954) pointed out that the departure of the measured s-wave neutron strength functions at $A \approx 150$ from the values which they had calculated from their spherical square optical potential might be caused by the known deformation of the nuclei in that region (Bohr and Mottleson, 1953; Mottleson and Nilsson, 1955). This hypothesis has since been born out by the calculations of several investigators (Margolis and Troubetzkoy, 1957; Chase, Wilet and Edmunds, 1958; Buck and Perey, 1962; T. Tamura, 1965). The s-wave strength function curve is definitely split by the deformation in the 4s giant resonance region. Similar but not so pronounced effects are indicated in the 3s giant resonance region, where a shoulder appears on the measured s-wave strength function curve at $A = 60-80$, and in the 3p giant resonance region, where a broadening of the measured p-wave strength function is evident. These latter effects may be understood by recalling that the excited states of medium
weight nuclei consist of surface vibrations which give the excited nucleus an average deformation.

The effect of nuclear deformation is to couple the low lying vibrational and rotational states of the target nucleus to the ground state so that they can be directly excited by the projectile. For spherical potentials, resonances will occur at different values of $A$ for neutrons of given angular momentum. For non-spherical potentials, $\vec{l}$ is not conserved so that a mixing of the resonances will occur. The splitting of the s-wave strength function at $A \sim 150$ is caused by the fact that the s- and d-wave giant resonances, which fall very close together, are strongly mixed by the non-sphericity of the nuclei in that region. The $2^+$ (first excited) states of the target nucleus can therefore be directly excited by s-wave neutrons.

The optical model has been extended to this case by accounting for the excitation in one or more of the coupled channels explicitly, and treating the rest of the inelastic channels through the use of a reduced absorbing potential (Tamura, 1965).

This extension can be done by expanding the nuclear radius, $R$, in terms of the spherical harmonics, $Y_{\lambda\mu}$. For the vibrational case,

$$R = R_0 \left( 1 + \sum_{\lambda\mu} \gamma_{\lambda\mu} Y_{\lambda\mu}(\theta, \phi) \right)$$  \hspace{1cm} (20)

and for the rotational case,
\[ R = R_0 \left( 1 + \sum_{\gamma \mu} \beta_{\gamma \mu} Y_{\gamma \mu}(\theta, \phi) \right). \]  

(21)

In the above expressions $\theta$ and $\phi$ are the azimuthal and polar angles respectively, and $R_0$ is the spherical nuclear radius (as in $R_0 = r_0 A^{1/3}$). The $d_{\gamma \mu}$ used in the expansion for the vibrational case, are creation and destruction operators; while the $\beta_{\gamma \mu}$, used in the rotational case, are numbers (Tamura, 1965). Relation (20) or (21) is substituted into the optical potential which, after certain approximations and expansions is separated into

\[ V(r, \theta, \phi) = V_{\text{opt}} + V_{\text{coup}}. \]  

(22)

where $V_{\text{opt}}$ is a spherically symmetric optical potential and $V_{\text{coup}}$ is the coupling potential between channels of different $\tilde{J}$ and $I$ ($I$ is the spin of the target nucleus).

The substitution of $V(r, \theta, \phi)$ into the Shroedinger equation gives rise to a set of coupled differential equations; their solutions lead to the wave functions which one wishes to calculate.

In principle, the method can be extended to any number of excited states; but because of the rapid increase in the complexity of the computations with the number of included states, one chooses the states most strongly coupled to the elastic channel. Most authors, including those mentioned above, choose the $2^+$ state in even-even nuclei. Tamura (1965) has extended the coupled channel formalism to include odd
nuclei and cases where the energy of the excited state of the target is greater than the particle bombarding energy.

The relation of $\delta_{\lambda \mu}$ and $R_{\lambda \mu}$ to the quadrupole deformation parameter, $\beta_2$, is discussed by Tamura (1965), and Stelson and Grodzins (1966). A nuclear deformation would imply the existence of nuclear electric multipole moments, and the quadrupole deformation parameter can be measured independently by the observation of electric quadrupole transitions.

A summary of the quadrupole transition probabilities between the $0^+$ (ground) and $2^+$ (1st excited) states, together with the quadrupole deformation parameters, has been published by Stelson and Grodzins (1966) for a number of even nuclei throughout the mass region.

In Chapter V we will compare the results of our measurements with calculations which assume different types of local and non-local potentials and also with results of some coupled channel calculations.
CHAPTER III
EXPERIMENTAL APPARATUS AND PROCEDURE

A. Apparatus

The apparatus used to make neutron total cross section measurements at Duke University is described in detail elsewhere (Nichols, Bilpuch and Newson, 1959; Bowman, Bilpuch and Newson, 1962). The source of neutrons is the $^7$Li(p,n) reaction using protons supplied by the 3 MeV Van de Graaff accelerator. The proton beam energy is measured by passing the HH+ beam through the cylindrical electrostatic analyzer. The proton beam is incident on a lithium target which has been evaporated in the machine's vacuum system onto a thin tantalum backing.

The neutron collimator consists of a set of male and female polyethylene cones (A, B, C and D in Fig. 1) which define two conical apertures, one on each side of the beam line, which focus on the target (T of Fig. 1). The azimuthal angle between the apertures and a line passing through the center of the collimator is 20°. The male cone, B, may be moved so as to adjust the acceptance angle from 0° to 2°. Cones A and D are not adjustable, but different pairs may be
Figure 1. Schematic Diagram Of The Neutron Collimator. The polyethylene cones A, B, C, and D define the apertures leading into the counter matrices. The apertures form an angle of 20° with the line marked 'Beam direction for 20° collimator', and focus on the cross-hatched area marked 'T'. For proper beam alignment, the proton beam must intersect the lithium target at or very near the center of the cross-hatched area. Cone B may be moved back and forth so as to vary the acceptance angle for the neutrons produced at T from 0° (aperture completely closed) to 2° (aperture fully open). Cones A and D interlock in a stair-step pattern and are cut out so as to form 1/8" x 3/4" apertures, as shown in front view and sectional views. The hole along the 160° beam line is closed in Cone A for measurements at 20°.
used depending upon the acceptance angle desired.

The design of the neutron collimator is such that the sensitive arc of the polar angle of the conical apertures is 120° on each side. When it was originally built, the samples used were either 2" x 4" slabs, or semicircular rings having a 3" inner diameter and 4" outer diameter. They were hung against the face of the collimator over the apertures. This arrangement gives good counting rates, while inscattered and extraneous backgrounds (which are discussed below) are negligible at the forward angles.

Since many of the latest measurements are performed on rare elements or separated isotopes, the samples are necessarily much smaller, and some modifications were necessary. As was discussed by Tabony (1966) and Divadeenam (1967), the practice had been to put the sample near the target so that it would shadow the collimator opening. This arrangement, if carefully used, also gives good counting rates and low backgrounds, but it is susceptible to large systematic errors if any misalignment of the sample occurs or if the acceptance angle is very large.

These large errors come about in the following way: For neutron transmission measurements made with the collimator set at 20°, the main sources of background are the neutrons emitted by the target at some angle other than 20° and scattered into the collimator by the sample (inscattered neutrons) or by the tantalum backing (extraneous neutrons) (Bowman et al., 1962).
As is discussed fully in Divadeenam's thesis, the effect of the inscattered background can be rather large if the sample is placed close to the target and if the collimator acceptance angle is much larger than $1/2^\circ$. Under these conditions, the detectors can 'see' a large volume of the sample from which neutrons could be scattered into the collimator apertures. Furthermore, if the target was misaligned so that the apertures did not focus on it, the detectors would not 'see' neutrons coming directly from the target but would still 'see' those which had been scattered by the sample into the collimator.

When Tabony made the transmission measurements reported in his thesis, the effects outlined above were not fully appreciated. He used an acceptance angle of $2^\circ$ in order to get high neutron beam intensities, and the result was that the inscattered background lowered the apparent cross sections by as much as 10%. This fact had been discovered by the time Divadeenam began measuring the transmissions of the molybdenum isotopes, so he remeasured the transmissions of most of Tabony's samples and was able to correct the remaining ones for the inscattered background.

In the course of this work, Divadeenam made extensive measurements of the inscattered background effect and found that it would be negligible if the collimator acceptance angle was less than $1/2^\circ$ and if the target was very carefully centered at the focal point of the collimator apertures.
In order to avoid any difficulty with in-scattered neutrons in making the measurements reported in this thesis, it was decided that the sample position should be moved from just in front of the target to just in front of the collimator. In this position, the amount of the sample volume which can be 'seen' by the detectors is greatly reduced (if the acceptance angle is not greater than 1/2°) so that neutrons scattered by more than a very small angle will not be counted. Furthermore, neutrons produced at angles other than 20° with respect to the proton beam line and scattered into the collimator by parts of the target chamber, cannot be scattered around the sample; their effect will be that of a very few off-energy neutrons in the beam.

In order to change the sample position, it was necessary that the angular opening of the collimator be reduced enough so that a 1" x 1/2" diameter sample would fully shadow it when placed against the face of the collimator. This was done by constructing a special set of cones to be used at the front of the collimator (cones A and D of Fig. 1). They were designed to interlock with each other in a stair step pattern, and were cut out in such a way as to leave a 3/4" x 1/8" aperture in each side.

These cones were only about 5 1/4" in altitude so that at the higher neutron energies, one would suspect that some neutrons might penetrate all the way through the solid area of the cones and travel up the space between cones B and C of
Fig. 1 and into the counter matrices. Measurements of this 'leakage' were made by measuring the transmission of cones A and D for neutron energies ranging from 30 keV to 650 keV in 10 keV steps, and by comparing transmission measurements made on large and small samples of silver. The large samples were 2" x 4" slabs and completely shadowed the semicircular apertures defined by cones B and C. Any neutrons which 'leaked' through the solid area of cones A and D would have had to have traveled through the large sample also and therefore the 'leakage' had no effect on the transmission measurements of the large sample. The small sample was a 1" square slab and both samples were about 1/4" thick.

Both the measurement of the transmission of the cones and the comparison of the measured transmissions of large and small samples of silver indicated that the 'leakage' was negligible at energies below 370 keV, but rose to about 3\% of the sample-out count at 650 keV. For the measurements on small samples of cerium, neodymium, samarium, gadolinium, dysprosium, holmium, and mercury, this background was subtracted during the reduction of data to cross sections. For the other small sample measurements, which were made sometime later, rubber sheets about 1/4" thick were cut so that they could be placed between the faces of the inner cones (C and B of Fig. 1); and the cones were closed upon them, leaving 3/4" x 1/4" apertures into the counter banks. Comparison of transmission measurements made on large and small samples of silver
Indicated that no 'leakage' occurred with this arrangement.

B. Samples

Samples of metallic cerium, gadolinium, terbium, and holmium had been obtained from Michigan Chemical Company previous to this experiment. They were in the form of 1" x 3" cylinders, and their purity was listed as better than 99%. From these cylinders 1" x 3/4" slabs were cut and were immediately submerged in mineral oil. Samples of neodymium, samarium, gadolinium and dysprosium were obtained from Michigan Chemical Company in the form of machined 1" x 3/4" slabs packed in an inert atmosphere. They were submerged in mineral oil immediately after unpacking. The scandium sample was obtained from A. D. Mackay Company of New York City in the form of a 1" x 3/4" slab. Cerium, neodymium, and samarium are active metals, darkening spontaneously in air if unprotected. The samples of these metals were cleaned in dehydrated ether and canned immediately under an atmosphere of helium in 1 mil silver foil cans sealed with solder. Since gadolinium, dysprosium, holmium, terbium, and scandium are corrosion-resistant metals, these samples had only to be cleaned before use.

The samples of potassium chloride, rubidium bromide, cesium iodide and selenium were obtained from A. D. Mackay Company in powdered form. They were baked out in a laboratory oven for several days at a temperature of 250°C, pressed into
1" x 1" cakes, and canned in 1 mil silver cans sealed with solder. The arsenic sample was obtained from A. D. Mackay bottled under an inert atmosphere, and was pressed and canned as above without baking.

Large samples of copper, zinc, vanadium, and cobalt had been obtained previous to this experiment for use in high resolution total neutron cross section measurements. The vanadium sample was cut into 1" x 3/4" pieces which were clamped together in order to make a sample of adequate thickness. All of the samples were cleaned and weighed before use.

The mercury samples were bottled in 1 1/2" diameter stainless steel cylindrical flasks having 10 mil thick ends and screw tops. The thin sample was 1/4" thick and the thick sample 1/2" thick. Table 1 summarizes the data on the natural samples.

The samples of samarium and neodymium isotopes were obtained from Oak Ridge National Laboratory in the form of powdered tri-valent oxides. They were baked for three days at a temperature of 250°C and canned in 1" diameter stainless steel cans having 10 mil stainless steel end caps in the case of the samarium isotopes and 3 mil silver end caps in the case of the neodymium isotopes. The cans were then sealed with a fine fillet of epoxy resin painted very gently around the joints. The temperature at which the samples were baked was insufficient since these oxides are known to take up water and
Table 1. Data on the Natural Samples

<table>
<thead>
<tr>
<th>Element</th>
<th>A</th>
<th>Dimensions</th>
<th>( n_y (\text{bn}^{-1}) )</th>
<th>( n_z (\text{bn}^{-1}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>KCl</td>
<td>37.3</td>
<td>1&quot;x1&quot;</td>
<td>0.0442</td>
<td>0.0496</td>
</tr>
<tr>
<td>Sc</td>
<td>45.0</td>
<td>1&quot;x 3/4&quot;</td>
<td>0.0390</td>
<td>-</td>
</tr>
<tr>
<td>V</td>
<td>51.0</td>
<td>1&quot;x 3/4&quot;</td>
<td>0.0394</td>
<td>-</td>
</tr>
<tr>
<td>Mn</td>
<td>54.9</td>
<td>1&quot; dia.</td>
<td>0.0264</td>
<td>0.0548</td>
</tr>
<tr>
<td>Co</td>
<td>58.9</td>
<td>2&quot;x4&quot;</td>
<td>0.0332</td>
<td>-</td>
</tr>
<tr>
<td>Cu</td>
<td>63.5</td>
<td>2&quot;x4&quot;</td>
<td>0.05035</td>
<td>-</td>
</tr>
<tr>
<td>Zn</td>
<td>65.4</td>
<td>2&quot;x 4&quot;</td>
<td>0.04412</td>
<td>-</td>
</tr>
<tr>
<td>RbBr</td>
<td>82.7</td>
<td>1&quot;x1&quot;</td>
<td>0.0342</td>
<td>0.0459</td>
</tr>
<tr>
<td>As</td>
<td>74.9</td>
<td>1&quot;x1&quot;</td>
<td>0.0522</td>
<td>-</td>
</tr>
<tr>
<td>Se</td>
<td>79.0</td>
<td>1&quot;x1&quot;</td>
<td>0.0431</td>
<td>0.0534</td>
</tr>
<tr>
<td>CsI</td>
<td>129.4</td>
<td>1&quot; dia.</td>
<td>0.0286</td>
<td>0.0382</td>
</tr>
<tr>
<td>Ce</td>
<td>140.1</td>
<td>1&quot; dia.</td>
<td>0.0245</td>
<td>0.0386</td>
</tr>
<tr>
<td>Nd</td>
<td>144.3</td>
<td>1&quot;x 3/4&quot;</td>
<td>0.0247</td>
<td>-</td>
</tr>
<tr>
<td>Sm</td>
<td>150.4</td>
<td>1&quot;x 3/4&quot;</td>
<td>0.0247</td>
<td>-</td>
</tr>
<tr>
<td>Gd</td>
<td>157.3</td>
<td>1&quot;x 3/4&quot;</td>
<td>0.0255</td>
<td>0.0512</td>
</tr>
<tr>
<td>Tb</td>
<td>158.9</td>
<td>1&quot;x 3/4&quot;</td>
<td>0.0263</td>
<td>-</td>
</tr>
<tr>
<td>Dy</td>
<td>162.5</td>
<td>1&quot;x 3/4&quot;</td>
<td>0.0267</td>
<td>-</td>
</tr>
<tr>
<td>Ho</td>
<td>164.9</td>
<td>1&quot;x 3/4&quot;</td>
<td>0.0272</td>
<td>-</td>
</tr>
<tr>
<td>Hg</td>
<td>200.6</td>
<td>3/2&quot; dia.</td>
<td>0.0278</td>
<td>0.0552</td>
</tr>
</tbody>
</table>
carbon dioxide from the air at any temperature less than 1000°C. We lack the equipment and experience necessary to bake samples at this temperature, and therefore we returned them to Oak Ridge where they baked out and weighed. These weights were communicated to us, and we were able to correct most of the data for the presence of water and carbon dioxide.

This was done by first calculating the corrected cross sections assuming that all of the contaminant was carbon dioxide, and then calculating the cross section assuming that all of the contaminant was water. The true cross section was taken as being the average of the two corrected cross sections, and the uncertainty introduced by the ignorance of the relative amounts of water and carbon dioxide was taken as being half the difference. Those cross section curves whose uncertainties were greater then ±0.37 barn at 50 keV were discarded. The data on the samples of the rare earth isotopes are presented in table 2.

Where transmission measurements were made on canned samples, appropriate compensators were used in taking the sample-out count except in the case of the thick samples of the neodymium isotopes. Here the effect of the silver end-pieces was calculated from the known cross section of silver.
Table 2. Data on the Samples of the Rare Earth Isotopes

<table>
<thead>
<tr>
<th>Isotope</th>
<th>$n_1$(bn$^{-1}$)</th>
<th>$n_2$(bn$^{-1}$)</th>
<th>$n_3$(bn$^{-1}$)</th>
<th>Per Cent Wt Loss</th>
<th>Uncertainty Q 50 Kev</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{142}$Nd</td>
<td>.0116</td>
<td>.0224</td>
<td>.0333</td>
<td>-</td>
<td>± 0.61</td>
</tr>
<tr>
<td>$^{144}$Nd</td>
<td>.0137</td>
<td>.0256</td>
<td>.0385</td>
<td>.51</td>
<td>± 0.36</td>
</tr>
<tr>
<td>$^{146}$Nd</td>
<td>.0136</td>
<td>.0249</td>
<td>.0376</td>
<td>.29</td>
<td>± 0.51</td>
</tr>
<tr>
<td>$^{147}$Nd</td>
<td>.0154</td>
<td>-</td>
<td>-</td>
<td>4.28</td>
<td>± 3.02</td>
</tr>
<tr>
<td>$^{147}$Sm</td>
<td>.0176</td>
<td>-</td>
<td>-</td>
<td>2.480</td>
<td>± 1.32</td>
</tr>
<tr>
<td>$^{148}$Sm</td>
<td>.0143</td>
<td>-</td>
<td>-</td>
<td>1.080</td>
<td>± 0.20</td>
</tr>
<tr>
<td>$^{149}$Sm</td>
<td>.0132</td>
<td>-</td>
<td>-</td>
<td>.17</td>
<td>± 0.37</td>
</tr>
<tr>
<td>$^{152}$Sm</td>
<td>.0133</td>
<td>-</td>
<td>-</td>
<td>.30</td>
<td>± 2.90</td>
</tr>
<tr>
<td>$^{154}$Sm</td>
<td>.01471</td>
<td>-</td>
<td>-</td>
<td>2.27</td>
<td>± 0.30</td>
</tr>
</tbody>
</table>
C. Neutron Energy Considerations

A second energy group of neutrons from the reaction $^7\text{Li}(p,n)^7\text{Be}^*$, which has a threshold at $E_n = 2.378$ MeV, places an upper limit on the energy range for monoenergetic neutrons from the $^7\text{Li}(p,n)$ reaction. When the proton energy $E_p \geq 2.4$ MeV, the intensity of this second group at $\Theta = 20^\circ$ becomes too large to be neglected. At this proton energy, the energy of the neutrons from the reaction $^7\text{Li}(p,n)^7\text{Be}$ is about 650 keV which is therefore the upper limit of the neutron energy for monoenergetic neutrons at $20^\circ$. If $\Theta = 160^\circ$, the upper limit for monoenergetic neutrons occurs at the back angle threshold for neutrons from the $^7\text{Li} p n^7\text{Be}^*$ reaction. This threshold is approximately $E_n = 2.45$ MeV, at which the neutrons from the $^7\text{Li}(p,n)^7\text{Be}$ reaction have an energy of about 230 keV. The energetics and intensities of the $^7\text{Li}(p,n)^7\text{Be}$ and $^7\text{Li}(p,n)^7\text{Be}^*$ are discussed fully by Gibbons and Newson (1960).

It should be mentioned that at $\Theta = 160^\circ$ the intensity of the neutrons emitted at $\Theta < 90^\circ$ and scattered into the collimator by the tantalum backing (extraneous neutrons) is large enough to affect the measurement. The necessary corrections to the data are discussed fully by Bowman et al. (1962). Below 10 keV or so, the intensity of the extraneous neutrons is large enough so that the data is unreliable, even if corrected.
D. Lithium Targets

Targets used for the measurements made with the collimator set at 20° were about 20 keV thick, which gave rise to neutron energy spread of about ±15 keV at a neutron energy of 120 keV, and a spread of about ±12 keV at an energy of 250 keV. Targets used for the measurements made with the collimator set at 160° were about 50 keV thick, which gave rise to a neutron energy spread of about ±5 keV when the neutron energy was 5 keV and a spread of about ±10 keV when the neutron energy was 50 keV. Target thicknesses were somewhat greater in this set of measurements than in those made by Divadeenam or Tabony because restricting the size of the collimator apertures cut the counting rates for a given lithium target to about one tenth of the value associated with their geometry.

E. Procedure

The procedure was to measure the sample transmissions with the neutron collimator set at the 20° for neutron energies ranging from 30 keV to 650 keV in 5 keV steps, rotate the collimator and measure the transmissions from 5 keV to 200 keV. The energy ranges of the forward and back angle measurements thus overlapped from 120 keV to 200 keV, allowing for normalization of the data. Since the neutron counting rate is
much higher at the forward angle than at the back angle, and since extraneous backgrounds and in-scattering are lower, the forward angle data were accepted as correct and the back angle data were normalized to them in case of disagreement. Forward angle data between 30 keV and 120 keV were taken because if necessary they can be approximately corrected for the lower energy group and used.

The collimator acceptance angle was set at $1/2^\circ$ for all data-taking runs. Because of the low counting rates and the need for good statistics, several transmission measurements were made at each energy and averaged. Each measurement took less than two minutes so that slow changes in the collimator detector efficiency were canceled out, but the total sample-out count was always 50,000 counts; hence, the statistical fluctuations in the transmissions were fairly small.

Considerable difficulty was experienced with the back angle measurements because of the lower counting rates and higher backgrounds (Nichols, Bilpuch, and Newson, 1959), because the distance between the vacuum pumps and the target chamber aggravated any problems encountered with the vacuum system. Part way through the run made with the collimator set at 160°, it was discovered that the water tank below the collimator assembly was partly empty. Prior to filling it, backgrounds were measured by closing the collimator completely, counting for a specified number of counts on a McKibben counter placed at 0°, opening the cones, and counting again. This
procedure was repeated from 5 keV to 200 keV in 20 keV steps, and this measurement was used to correct all data taken before the discovery. Filling the water tank caused the background to drop to about 1/8 of its former value.

F. Results of the Cross Section Measurements

Figures 2 and 3 show the results of the total neutron cross section measurements discussed above. The lines are fits to the data made according to the method described in Chapter IV.
Figure 2. Average Neutron Total Cross Sections For The Natural Samples. The lines through the data points are theoretical fits obtained by the method described in Chapter IV. Cross section curves for elements lighter than arsenic show considerable fluctuation due to strong individual levels. In Table 4 s-wave strength functions obtained from these curves are compared with those found from analyzed resonances.
Figure 3. Average Neutron Total Cross Sections For Some Of The Neodymium And Samarium Isotopes. These samples were in the form of oxides, and because the large resonance in the neutron total cross section of oxygen at 437 keV made the extraction of meaningful cross sections from the sample transmissions impossible in that energy region, data between the neutron energies of 350 and 550 keV were discarded. The lines through the data points are again theoretical fits obtained by the method discussed in Chapter IV.
CHAPTER IV
ANALYSIS OF THE DATA

A. Introduction

Our method for determining s-, p-, and d-wave strength functions, the s-wave scattering length, and the p-wave phase shift ratio from average neutron total cross sections was developed some years ago by Seth, Tabony, Bilpuch, and Newson (1964). A detailed discussion is given in Tabony's thesis (1966), and by Tabony and Seth (1968).

The method depends upon the assumption that multi-level interference has a negligible effect on the average total neutron cross section (Feshback, Porter, and Weisskopf, 1954). One may then develop a formula describing the average total neutron cross section, $\langle \sigma_T \rangle$, by averaging over many resonances described by the single-level Breit-Wigner formula including resonance-potential interference. In this way, it can be shown that
\[ \langle \sigma_T \rangle = 4\pi \lambda^2 \sum_{\ell} (2\ell+1) \left[ \sin^2 S'_{\ell} + \frac{\pi}{2} \frac{\langle \Gamma_{\ell} (l) \rangle}{\langle D(l) \rangle} \cos 2S'_{\ell} \right] \] (23)

where \( \langle \Gamma_{\ell} (l) \rangle \) is the average natural width of the resonances, \( \langle D(l) \rangle \) is the average level spacing, \( S'_{\ell} \) is the optical model phase shift for neutrons of angular momentum, \( \ell \), and the other symbols have their usual meanings. The experimentalists' reduced width is defined as

\[ \Gamma'_{\ell} = \frac{\Gamma_{\ell} (l)}{V(l)} \left( \frac{E(eV)}{1eV} \right)^{\frac{1}{2}} \] (24)

where \( V(l) \) is the probability that a neutron of angular momentum, \( \ell \), will penetrate the centrifugal barrier and interact with the surface of an impenetrable sphere. The experimentalists' reduced width is defined in this way to remove the dependence upon the energy and upon the penetrability of the centrifugal barrier. The strength function is defined by

\[ S_{\ell} = \frac{\langle \Gamma_{\ell} \rangle}{\langle D(l) \rangle} \] (25)

and hence, one obtains

\[ \langle \sigma_T \rangle = 4\pi \lambda^2 \sum_{\ell} (2\ell+1) \left[ \sin^2 S'_{\ell} + \frac{\pi}{2} \frac{V(l)}{V} \left( \frac{E(eV)}{1eV} \right)^{\frac{1}{2}} S_{\ell} \cos 2S'_{\ell} \right] \] (26)

This equation describes the average total cross section provided that the samples are thin enough so that self-
absorption is negligible. Tabony (1966) shows that this is so for samples of moderate thickness \( n < 0.1 \text{ bn} \) except for the s-wave resonant cross section for \( E < 100 \text{ keV} \). In this case, it may be shown that the total cross section for s-wave neutrons is

\[
\langle \sigma_T \rangle = -\frac{1}{n} \log_e T_P \left[ 1 - \frac{\langle A'(n\sigma_0, \delta_0', \Delta / \Gamma) \rangle}{\langle \Gamma \rangle} \frac{\langle \Gamma \rangle}{\langle \gamma \rangle} \right]
\]  

(27)

where

\[
T_P = \exp \left( -n \langle \sigma_P \rangle_0 \right);
\]

\( \langle \sigma_P \rangle_0 \) being the shape elastic scattering cross section for s-wave neutrons, and where \( \langle A' \rangle \) is the average area under a resonance having an average total width \( \langle \Gamma \rangle \). \( \langle A' \rangle \) is a function of \( n \), the sample thickness in nuclei/bn,

\[
\sigma_0 = 4\pi^2(2l+1)/(2(2l+1)),
\]

and \( \Delta \) is the amount of Doppler broadening of the resonances by the vibrations of the nucleus in the crystal structure of the sample.

For the purpose of the analysis, \( \langle A' \rangle \), the average area of a resonance obtained by averaging over a group of resonances whose average total width is \( \langle \Gamma \rangle \), is equated to \( A'(\langle \Gamma \rangle) \), the area under a single resonance of width \( \langle \Gamma \rangle \). Tabony's thesis has a good discussion of this point. He shows that the distribution of widths will affect \( \langle A' \rangle \) in such a way that \( \langle A'(\langle \Gamma \rangle) \rangle \neq A'(\langle \Gamma \rangle) \), but that for samples whose thicknesses
are no greater than those reported here, the effect is smaller than the uncertainties in the data and therefore can be neglected. In this method of analysis, we also equate the neutron width, \( \Gamma_\eta \), with the total width, \( \Gamma \). Tabony's thesis also covers this point. During the course of the development of this method, Tabony and his associates verified that for s-wave resonances, the effect of those resonances whose radiative width, \( \Gamma_\gamma \), was comparable to the neutron width, was negligibly small.

As a consequence of Eq. 27, the use of thick samples depresses the apparent s-wave resonant cross section, and therefore measuring the apparent cross sections of samples of differing thicknesses provides a good method for separating the s-wave contribution from those of neutrons of higher angular momenta.

Rather than search on the optical model phase shifts, \( \delta'_l \), which are energy dependent, we search on the ratio \( \delta'_l/\delta_2 \) where \( \delta_2 \) is the phase shift for a neutron scattering off of an impenetrable sphere. The energy dependence is contained in \( \delta'_l \), which is calculated for each energy, and \( \delta_2 \) is taken to be proportional to \( \delta_2 \). The constant of proportionality is the quantity which is, in effect, determined by the fitting procedure.

It is the general custom to express \( \delta'_l/\delta_0 \) by means of the scattering length for \( l = 0 \) neutrons. Recall that in the low-energy limit, the s-wave neutron total cross section is
(Messiah, 1966)

\[ \sigma_{R=0} = 4 \pi R'^2 \]  \hspace{1cm} (29)

\( R' \) is the scattering length and is defined by

\[ R' = -\lim_{k \to 0} \frac{\delta'_0}{k} \]  \hspace{1cm} (30)

where \( \delta'_0 \) is the phase shift for \( l = 0 \) neutrons and \( k \) is the magnitude of the propagation vector of the neutron wave function. If \( R \) denotes the radius of an impenetrable sphere, then

\[ R' = -\lim_{k \to 0} \frac{\delta'_0}{\delta_0} \cdot \frac{\delta_0}{k} = \frac{\delta'_0}{\delta_0} \cdot R. \]  \hspace{1cm} (31)

Since our neutron energy ranged up to only 650 keV, contributions to the total cross section by neutrons having \( l > 3 \) are negligible. In addition, \( \delta'_2/\delta_2 \) was always taken as unity because the contribution by the \( d \)-wave potential scattering at these energies is very small. Hence we are left with the five parameters on which the search was conducted: \( S_0, R', S_1, \delta'_1/\delta_1 \) and \( S_2 \).
B. Fitting Procedure

The analysis computer program, ORION, uses the method of Gaussian linear regressions to find a fit having the least $\chi^2$, where

$$\chi^2 = \frac{1}{N} \sum_{n=1}^{N} \left( \frac{\langle \sigma_T \rangle^{exp} - \langle \sigma_T \rangle^{theor}}{\Delta \langle \sigma_T \rangle^{exp}} \right)^2$$  \hspace{1cm} (32)

$N$ is the number of data points, $\langle \sigma_T \rangle^{theor}$ is the calculated average neutron total cross section at neutron energy $E_n$, $\langle \sigma_T \rangle^{exp}$ is the measured average neutron total cross section at neutron energy $E_n$, and $\Delta \langle \sigma_T \rangle^{exp}$ is the experimental uncertainty of the measured average neutron total cross section at neutron energy $E_n$. A general discussion of this method is given by Melkanoff, Sawada, and Raynal (1966). The values of $A_i$ used to calculate the low energy s-wave cross section are taken from the tables of Seth and Tabony (1965). They are read from a magnetic disk pack at the beginning of execution, and the value of $A_i$ for a particular set of $n_{\text{ref}}$, $\delta_i$, and $\Delta \rho$ is calculated from the tabulated values by quadratic interpolation.

Because of the similarity between the shape of the p-wave potential scattering cross section and that of the d-wave resonant cross section, it was not possible for ORION to find a fit if all five parameters were allowed to vary. Fortunately, $\delta_i'/\delta_i$ is not expected to be different from unity except in the regions of the mass table which lie near the peaks in the p-
wave strength function. Except for rubidium bromide, the measurements reported in this thesis fall outside of these regions and $\delta' / \delta_1$ was taken as unity. For rubidium bromide ($A=83$), we fixed $\delta' / \delta_1$, equal to 1.1 which is the value reported by Divadeenam (1967) for bromine ($A=80$) and yttrium ($A=89$).

While fixing $\delta' / \delta_1$ enabled ORION to find a fit, the fit was not always very sensitive to the parameters. By stepping $S_1$, the parameter to which $X$ was least sensitive, it was found that the program would find acceptable fits for a wide variety of values of $S_0$, $R'$, and $S_2$. By plotting $X^2$ versus $S_1$, it was possible to pick the value of $S_1$ giving the best fit. Acceptable fits were considered to be those for which the value of $X^2$ was less than twice the value of $X^2_{\text{min}}$, the minimum value of $X^2$ obtained in the fitting procedure. Figure 4 illustrates the procedure for the case of natural samarium. Figure 4c shows the plot of $S_1$ vs $X^2$. $X^2_{\text{min}}$ is equal to 20, for which $S_1 = 0.0$. The value of $S_1$ for which $X^2 = 40$ is 1.75 so that $0.0 \leq S_1 \leq 1.75$. Hence, $S_1$ was taken as being $0.88 \pm 0.88$.

With $S_1$ and $\delta' / \delta_1$ both fixed, it was possible to step each of the remaining parameters in turn, and plot $X^2$ versus each of them (Figures 4a, 4b, 4d). The values for which $X^2$ was minimized could then be picked from the graph, and the sensitivity of $X^2$ to the parameters determined. As above, the limits of the parameters were taken as those values for which $X^2$ was twice the minimum $X^2$ or less.
Figure 4. Illustration Of Fitting Procedure For Natural Samarium. 4c shows the plot of $X^2$ vs $S_1$ calculated while letting $S_0$, $R^1$, and $S_2$ vary. 4a, 4b, and 4d show plots of $X^2$ vs $S_0$, $R^1$, and $S_2$ respectively, calculated while fixing $S_1$ equal to 0.88. Note that the $X^2$ scale is multiplied by $N$, the number of data points. For this case $N$ was equal to 60 so that $X^2_{min} = 0.33$. 
Any variation in the specified value of $S_i$ will change the values of $S_0$, $R^i$, and $S_2$ which yield the fit with the least value of $\chi^2$, and this effect must be taken into account when one calculates the final uncertainty in the parameters $S_0$, $R^i$, and $S_2$.

In order to calculate this effect, $S_i$ was fixed at the lower limit, and $S_0$ and $R^i$ were stepped so that a curve of $S_0$ versus $\chi^2$ and $R^i$ versus $\chi^2$ could be plotted. The value of $S_i$ was then fixed at the upper limit, and the procedure repeated. The procedure for natural samarium is illustrated in Fig. 5. Fig. 5a shows the two $\chi^2$ versus $S_0$ curves which will be denoted by

$$\chi^2 = f(S_0) \bigg| \begin{array}{c} S_i = L.L. \end{array}$$ \hspace{1cm} (33)$$

for the curve for which $S$ is equal to its lower limit, and

$$\chi^2 = f(S_0) \bigg| \begin{array}{c} S_i = U.L. \end{array}$$ \hspace{1cm} (34)$$

for the curve for which $S$ is equal to its upper limit. The lower limit of $S_0$ was taken to be the least value of $S_0$ for which $f(S_0) |_{S_i = L.L.} \leq 2\chi^2_{\min} |_{S_i = L.L.}$, where $\chi^2_{\min} |_{S_i = L.L.}$ is the minimum value of $\chi^2$ from the $f(S_0) |_{S_i = L.L.}$ curve; in this case $\chi^2_{\min} |_{S_i = L.L.} = 19.5$ and the lower limit of $S_0$ is 2.25. The upper limit of $S_0$ was chosen to be the greatest value for which $f(S_0) |_{S_i = U.L.} \leq 2\chi^2_{\min} |_{S_i = U.L.}$; in this case $\chi^2_{\min} |_{S_i = U.L.} = 37$ and the upper limit of $S_0$ is 5.2. The value
Figure 5. Illustration of Procedure for Calculating the Range of Possible Error in $S_0$ and $R'$ for Natural Samarium. 5a shows $S_0$ vs $X^2$ calculated with $S_i = 0.0$, and $R'$ and $S_2$ floating. 5b shows $S_0$ vs $X^2$ calculated with $S_i = 1.75$, and $R'$ and $S_2$ floating. 5c shows $R'$ vs $X^2$ with $S_i = 0.0$, and $S_0$ and $S_2$ floating. 5d shows $R'$ vs $X$ with $S_i = 1.75$, and $S_0$ and $S_2$ floating. Note that when $S_i$ is fixed at 1.75, the width of the $S_0$ vs $X^2$ and $R'$ vs $X^2$ curves is about twice as great as when $S_i = 0.0$, which was the value of $S_i$ giving the least $X^2$. Note also that the $X^2$ scale is multiplied by $N$, the number of data points. In this case, $N$ is equal to 60.
of $S_0$ chosen by the program when $S_1 = 0.88$, which was chosen to be the optimum value, was 3.4 so that $S_1$ is taken to be

$$S_0 = 3.4 \pm 1.8$$

(35)

The same procedure was followed for $R^1$. No attempt to calculate an error for $S_2$ was made because the value of $S_2$ chosen by the program depends mostly on the cross section data above 500 keV. Even if $S_2 = 5.0(10^{-4})$, the calculated $d$-wave resonant contribution is only about 0.5 barns at 650 keV, so that any small random or systematic variation in the cross section would greatly perturb the values of $S_2$ found by the fitting procedure. Preliminary studies of the effect of the variation of $S_2$ on the fit indicated that the usual error in $S_2$ would be about $\pm 2.0(10^{-4})$.

C. Discussion of the Results

The results of the analysis of the data presented in this thesis are presented in Table 3 together with the results of Divadeenam's measurements.

The sensitivity of the fits to the parameters depended upon how well the total section curve could be decomposed into the shape-elastic and resonant contributions of the $s$-, $p$-, and
d-wave neutrons. Where the resonant contribution for a particular \( l \)-value, \( \langle \tau_\ell \rangle \), dominated, the total cross section curve had a characteristic shape which made it easy for the program to carry out the decomposition and hence, the fit. Cross sections in the 4s giant resonance region show a sharp rise at the lower neutron energies as the energy is decreased. This rise is caused entirely by \( \langle \tau_0 \rangle \) and hence the s-wave contribution can be separated rather easily. Cross sections in the 3p giant resonance regions show a characteristic drop at the lower neutron energies for decreasing energy because \( \langle \tau_0 \rangle \) is very low so that the contribution due to \( \langle \tau_1 \rangle \) is very apparent and can be isolated. A more complete discussion of these considerations is given in Tabony's thesis.

Where the cross section is measured for elements in the mass regions where \( \langle \tau_0 \rangle \) and \( \langle \tau_1 \rangle \) are likely to be of comparable and moderate size (e.g. \( A \sim 70 \), or \( A \sim 200 \)) the cross section's shape is approximately that of a descending straight line. This makes it impossible to distinguish with much accuracy between a combination of contributions due to \( \langle \tau_0 \rangle \) and \( \langle \tau_1 \rangle \) and \( \langle \tau_0 \rangle \) and \( \langle \tau_1 \rangle \). Within wide limits, a variation in one parameter can be compensated for by variations in the others. As discussed in section A, measurements made on samples of differing thicknesses can sometimes resolve the difficulty if the thickest sample is thick enough to quench the s-wave resonant contribution through self-absorption and thus allow the p-wave contribution to be isolated.
The cross sections for the rare earths were the easiest to analyze because the p-wave strength function is near a minimum in this mass region, and therefore the fits to the data are more sensitive to variations in $S_0$, $R'$, and $S_1$. In addition, these cross sections exhibit little fluctuation, so that $X^{*}_{m,n}$ will be very low, and the sensitivity of $X^{2}_{m,n}$ to any of the parameters will be fairly large.

The cross sections of potassium chloride, scandium, vanadium, cobalt, zinc, and copper proved more difficult because $S_0$ and $S_1$ are of comparable magnitude and fairly low. In addition, the cross sections of these samples exhibited a considerable amount of fluctuation so that $X^{*}_{m,n}$ is always large and is not very sensitive to any of the parameters.

The results for scandium, vanadium, manganese, and cobalt published by Seth, Tabony, Bilpuch, and Newson (1964) were based on the analysis of cross section data taken by Marshak and Newson (1957) over the energy range of 3 to 100 keV. Our data for these elements were combined with those of Marshak and Newson, and our results are obtained from the analysis of the combined data. The only case of significant disagreement occurs for cobalt. In this case, the energy range covered by Marshak and Newson would have included about sixty resonances, whereas ours included about six times as many. When the number of resonances is less than one hundred or so, it is possible that one or two large levels will make a very large contribution to the sum of the reduced widths, and the
contribution by the other resonances will be minor; this will affect the measurement of the strength function. Average level densities for manganese and scandium are about the same as for cobalt, and the fact that our strength functions for these elements agree with those of Marshak and Newson is probably a matter of chance.

The values of $S_o$, $R^i$, and $S_i$ for copper and zinc are not significantly different from those reported earlier.

The cross sections of rubidium bromide, arsenic, and selenium were also difficult to analyze because, as noted above, moderately large contributions by $S_o$ and $S_i$ to the cross sections made the decomposition into the separate contributions difficult. Our results for arsenic disagree with those reported by Divadeenam (1967), but his results are based on the analysis of data taken by the Wisconsin group (M.S. Zucker, 1956), which extended only down to 100 keV, so that the s-wave contribution was not so apparent. In addition, since our fit was not very sensitive to the parameters, the disagreement may not be very significant. The results for rubidium bromide disagree with values which would be expected in this mass region on the basis of the values for neighboring nuclei and of various calculations, but the differences are not significant because of the insensitivity of the fit. The results for selenium do not differ significantly from those reported earlier.
As mentioned in Chapter III, the samples of the rare earth isotopes had become contaminated with water and carbon dioxide, so that it was necessary to correct the data. All data between 350 keV and 550 keV was discarded because the large resonance in the total neutron cross section of oxygen at 437 keV made any meaningful corrections impossible in that energy region. The resulting gap in the data made the fits insensitive to variations in $S_1$. Since $S_1$ is not large in this mass region, nor expected to vary rapidly with $A$, it was chosen to be the value which had been found for the natural element. The uncertainty in the other parameters caused by the uncertainty in $S_1$ was assumed to be the same as that of the natural element, and was folded into the total variation in the usual way.

The effect on the parameters caused by the uncertainty in the relative proportions of water and carbon dioxide in the contaminant was calculated by analyzing a set of data which had been corrected under the assumption that the contaminant was all water, and analyzing a second set of data corrected under the assumption that all of the contaminant was carbon dioxide. The uncertainty in the parameters was taken as half the difference between the parameters obtained from the two sets of data. The parameters themselves were obtained by analyzing a third set of data which was the average of the two sets.

As was expected, $R^1$ was the parameter showing the greatest sensitivity to this variation in the cross section.
Surprisingly, this effect is smaller than some of the other effects contributing to the total uncertainty in the parameters.
CHAPTER V

COMPARISON OF OUR RESULTS WITH THOSE OF OTHER EXPERIMENTERS

Besides the method of measuring strength functions by extracting them from average total cross section measurements, which is outlined above, one may also measure them by measuring and summing the reduced widths of the individual resonances. This method was first applied at Brookhaven National Laboratory, but at that time not enough levels were available to make up a good statistical sample. The method is applicable in the s-wave strength function case if the order of 100 resonances are analysed for any nuclide. Even if this condition is met however, fluctuations caused by intermediate structure can still generate an erroneous value of the s-wave strength function (in the sense that it cannot be compared to an optical model calculation) if the energy interval over which the total cross section measurement was made is 100 keV or less. Tables 4 and 5 show the average cross section results compared to the measurements made from the analysis of individual resonances by various experimenters.

The experimenters at Saclay (Julien and collaborators) have applied the Brookhaven method to obtain p-wave strength functions. Because of their different interference patterns, the separation of p-wave resonances from s-wave resonances is quite practical between $A=40$ and $A=64$. The $2J+1$ sum rule for

\[(64)\]
### Table 4

Comparison of S-wave Strength Functions from Average Cross Sections with Those from the Analysis of Individual Resonances

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Reference Codes:
1. Morgenstern (1963)
2. Rohr and Friedland (1967)
3. Morgenstern (1965)
4. Julien (1965)
5. Garg (1964)
6. Julien (1962)
7. Garg (1964)
8. Garg (1965)
9. LePoittevin (1965)
10. Ribon (1965)
11. Rainwater (1965)
12. Desjardins (1960)
13. Julien (1964)
14. Rahn et al. (1969)

* Separated Isotopes are denoted by an integer atomic mass.
Table 5
Comparison of P-wave Strength Functions from Average Cross Sections with Those Obtained from the Analysis of Individual Resonances

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Note: Sr* indicates the strength parameter; Ref numbers correspond to specific references.
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Table 5 (Continued)
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Reference codes:
1 Bowman (1962)  
2 Farrell (1966)  
3 Le Pointtevin (1965)  
4 Garg (1965)  
5 Ribon (1965)  
6 Morgenstern et al. (1969)  
7 Reported in Table 3  
8 Uttley (1964)  
9 Uttley (1966)

Footnotes to Table 5:
* It is not clear whether or not the strength functions are obtained from the assigned p-wave resonances or from all the non s-wave resonances. However, only assigned resonances have been used in the case of the Duke results. When the unassigned resonances with maximum possible reduced widths are included, no appreciable change is observed in the estimated strength function.

** The strength functions reported by Uttley (references 8 and 9) were obtained by the use of the Harwell method. This is discussed in the text.

*** Separated isotopes are denoted by an integer atomic mass.
the dependence of resonance spacing upon the total spin of the compound nucleus predicts that the p-wave resonances should be relatively numerous, and as is shown in Table 5 both the number of the p-wave resonances and the energy range over which they are measured are favorable for determination of strength functions. The number of s-wave resonances per nuclide is about one third of the number of p-wave resonances.

The experimenters at Harwell (Uttley and collaborators) have used a method in which the s-wave strength function is obtained by using the Brookhaven method for cross section data below 100 keV. To obtain the p-wave strength functions, the s-wave results are extrapolated into the higher energy region and subtracted from the total cross section. The residue is analyzed by using an average cross section method for p-wave strength functions and presumably for d-wave strength functions and p-wave potential scattering. This method is probably as good as the Duke method if the nuclide lies in a mass region far removed from any shell closure so that intermediate structure would be negligible, but in cases such as Sc, Mn, V, and Co, where the s-wave strength function extracted from data ranging up to 100 keV or so is quite different from that extracted from higher energy data (Newson, 1965), the method would give poor results. As noted above in connection with Co, the s-wave strength function extracted from data taken up to as high as 100 keV may still differ from that extracted from higher energy data if intermediate structure is
present. A good discussion of the problems associated with calculating the strength functions from the widths of resolved resonances is given by Newson (1965) and by Divadeenam (1967).
CHAPTER VI

POTENTIAL SCATTERING AND AVERAGED CROSS SECTIONS

A. Introduction

Even over the restricted neutron energy range of these experiments, there are a bewildering number of variations of the optical model, and in order to make even a tentative choice among them, one must proceed with caution. We will first consider the cross sections averaged over an energy range greater than 1/4 MeV and then their largest components, the s-wave potential scattering cross sections. We will thus for the time being minimize the uncertainties introduced by attempting to decompose the cross section into three or more components.

B. \( \langle \sigma_T \rangle \) as a Function of the Atomic Mass Number

Figures 6 and 7 present average neutron total cross sections together with calculations made by Perey and Buck (1962) from their non-local potential and by Divadeenam (Thesis 1967) from the optical model parameters of Moldauer (1963),
Figure 6. $\langle \sigma_T \rangle_{150\text{keV}}$ Versus A. The cross sections were averaged over the energy range of 100 to 650 keV using the trapezoidal rule.
Figure 7. $<\sigma T>_{1470\text{keV}}$ Versus A. The cross sections were averaged over the interval of 300 keV to 650 keV using the trapezoidal rule.
\[ E_n = \frac{300 + 650}{2} \approx 470 \text{ keV} \]

**Perey's Parameters**
- \( W = 8 \text{ MeV} \)
- \( W = 11 \text{ MeV} \)

**Local Optical Model**

\[ \langle \sigma_T \rangle \text{ (barns)} \]

**Mass Number**
- Duke
- Wisconsin
- ANL
## Table 6

Optical Model Parameters

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<th>r</th>
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Footnotes to Table 6:

1. The form of the optical potential used by the workers mentioned in the text is given by Eqns. 14-16 of Chapter II.

2. See Perey and Buck (1962) for the form of this non-local potential.

3. Holdauer used a Gaussian form of the surface-absorption potential (see Eqn. 16) with \( R = r_A A^{1/3} + C \).
Rosen (1965), and Perey and Buck's local potential equivalent to their non-local potential (Perey and Buck, 1962). These parameters are presented in Table 6.

The method by which Perey and Buck arrived at the parameters of their non-local potential and the local potential equivalent to their non-local potential is discussed in Chapter II.

Rosen et al. (1965) used a local optical potential whose parameters were obtained by fitting proton polarization data taken at 10.5 and 14.5 MeV for each of a large number of nuclei. The resulting parameters were averaged to give a set for each energy, thus giving the energy dependence of the real potential. To obtain an average set for neutrons, the geometrical constants and spin orbit strength were fixed at the values obtained for the protons, and the real and imaginary potentials for neutrons were determined by fitting available neutron elastic scattering data at 14.5 MeV. They assumed that the energy dependence which they had found for the proton case was valid for the neutron case.

Moldauer's calculation was done with a spherical, local optical model potential which he modified by introducing a 'fringe' parameter, \( C \), into the surface absorption term \( g(r) \). The form of this absorption term is

\[
g(r) = \exp \left( - \frac{(r - R - C)^2}{b} \right)
\]  

(36)
which is the same as the Gaussian absorption term of Chapter 11 (Eqn. 16) except that \( R + C \) replaces \( R \). This is equivalent to specifying a different nuclear radius for the absorptive potential than for the real potential.

Moldauer varied these parameters to obtain the best fit to the s-wave strength functions, elastic differential cross sections and total cross sections measured below 1 MeV for elements consisting mainly of even-even isotopes for the mass region \( A = 40 - 150 \).

It should be emphasized that Perey and Buck, and Rosen derived their parameters from high energy nucleon scattering data and not from the data to which we are comparing their calculations. The calculation using Moldauer's parameters on the other hand might be expected to agree somewhat better with the experimental points since Moldauer obtained his parameters by fitting similar data.

Since all of these calculations were done with spherical wells, and the 4s giant resonance region is known to be one of highly deformed nuclei, none of the curves fit the entire mass region, and they all fail for \( A > 140 \). In addition, the spherical model calculations predict a more pronounced shoulder near \( A=60 \) than is indicated by the experimental points. The presence of the shoulder causes those calculations which agree with the experimental points near \( A=100 \) to be much too high near \( A=60 \), and those calculations which agree with the data near \( A=60 \) to be too low near \( A=100 \).
Some of the calculated curves do not fit well in either region. Varying Moldauer's parameters improves the fit at $A=100$ at the expense of poorer agreement near $A=60$. Thus the spherical models do not fit very well even in regions of low distortion in the nuclear ground state.

It was first pointed out by Buck and Perey (1962) that the highly excited compound nucleus which resulted from neutron penetration have collective vibrational modes; consequently they are not spherical on the average. Buck and Perey made a calculation of $R'$, $S_0$ and $S_1$ for even-even nuclei over the mass region of $A = 40$ to $A = 240$ using the local optical potential which they had found equivalent to their non-local potential used in the series of calculations discussed in Chapter II. The quadrupole deformation parameters which they used were taken from the measurements of the intrinsic nuclear quadrupole moments available at the time. The values of $S_0$, $R'$, and $S_1$ which they obtained were used by the author of this thesis to calculate the average cross section at 470 keV. The $p$-wave potential scattering contribution was assumed to be the same as that of a hard sphere, and the $d$-wave contribution, which would be less than 0.5 barn at 470 keV, was neglected. The variation of the strength functions with the neutron energy was also neglected but as pointed out by Tabony this variation is not expected to exceed 5% per 100 keV. The strength functions of Buck and Perey were calculated for a neutron energy of 40 keV. It should be noted that the assumption that the strength
functions are independent of energy is basic to our method of analysis. The results of this calculation are presented in Fig. 8a.

Figures 8b and 8c present the results of the calculations done by the author of this thesis using the coupled channel code, JUPITER I of Tamura (1965). In order to minimize computer time, elements were selected at intervals of approximately ten mass units from $A = 48$ to $A = 238$, and the calculations were performed for those even isotopes whose abundance was greater than 5%. A 'best' line was then drawn through the points. The quadrupole deformation parameters, $\beta_2$, were taken from the recent compilation of Stelson and Grodzins (1966). They calculated the $\beta_2$ values from the observed reduced electric quadrupole transition probabilities, $B(E2)$, between the ground state ($0^+$) and the first excited state ($2^+$). The definition of $\beta_2$ and its relation to the dynamic or static distortion of the nuclear shape is given in their article also.

This method of finding $\beta_2$ is unable to give its sign, that is to distinguish oblate ($\beta_2$ negative) from prolate ($\beta_2$ positive). The calculations depicted by Fig. 8b were made using both positive and negative values of $\beta_2$. As will appear later, the difference is negligible compared to the effects of the uncertainties in the absolute values of $\beta_2$.

Only $0^+$(ground) - $2^+$(first excited) state coupling was considered. Nuclei between mass numbers 48 and 138, and 186 and 220 were considered vibrational; nuclei between mass
Figure 3. $\langle \sigma T \rangle_{470 \text{ keV}}$ Versus $A$. The cross sections were averaged over the interval of 300 keV to 650 keV using the trapezoidal rule.
numbers 140 and 186, and above 230 were considered rotational.

Restricting the type of coupling to either purely vibrational or purely rotational is an over simplification of the situation. Recent measurements of the low lying excited states of even nuclei by coulomb excitation utilizing the 'reorientation effect' (which is discussed by Breit, Glucksberg; and Russell; 1956) have revealed that some nuclei previously thought to be pure cases of one or the other types of coupling may be subject to competing deformed and vibrational properties (Simpson, Eccleshall, Yates, and Freeman; 1967).

The samarium isotopes, for example, are in a transitional region (not yet explained) extending from the singly magic (i.e., nearly spherical) \(^{144}\text{Sm}\)(N=82) to the deformed \(^{152}\text{Sm}\) and \(^{154}\text{Sm}\). The same is true of the isotopes of neodymium. The nucleus \(^{130}\text{Ba}\) may also be in a transitional region (Simpson et al., 1967), and the nucleus \(^{114}\text{Cd}\), which was long thought to be one of the best examples of a harmonic vibrational nucleus, has turned out to exhibit a surprisingly large rotational quadrupole moment (de Boer, Stokstad, Symons, and Winther, 1965; Stelson et al., 1965). According to Kumar and Baranger (1968) the region around \(A = 180 - 196\) is also a transitional region. At the low end, one finds that there are large deviations from a pure rotational model for the isotopes of tungsten, while at the higher end there are large deviations from the vibrational model as evidenced by the fact that some
of the platinum isotopes exhibit large static quadrupole moments.

The measurements of the quadrupole transition strengths upon which the calculation of the deformation parameters listed in Stelson and Grodzins is based are subject to errors ranging from a few percent to a factor of two. The error in $\beta_2$ caused by the error in $B(E2)$ was calculated from the definition of $\beta_2 = f(B(E2))$ given in Stelson and Grodzins. The average error in $\beta_2$ turned out to be about $\pm 30\%$ and most errors were within $10\%$ of this average value. In order to find out what the effect of the possible variation of $\beta_2$ on the average total cross section would be, we repeated the calculations twice, once with the $\beta_2$ values fixed at their maximum values, and once with them fixed at their minimum values. These calculations are shown in Fig. 8c as a solid line for the calculation in which the values of $\beta_2$ were fixed at their maxima, and a dashed line for the calculation in which the values of $\beta_2$ were fixed at their minima. It is seen that the calculated cross section is very sensitive to the $\beta_2$ values and the effect of their uncertainties is much greater than that of the change in the sign of $\beta_2$.

It is evident from Fig. 8 that if we used the deformation, $\beta_2$, as a free parameter within the uncertainties indicated in Stelson and Grodzins, a very good fit could be obtained, but this would have the effect of introducing more free parameters into a model that already has too many. In
spite of a greater point scatter in Fig. 8 than is perhaps necessary, it would appear that nothing would be gained in physical understanding by increasing the accuracy of the cross section measurements until sufficiently accurate distortion parameters are available to insert into the optical model.

The comparison of the average cross sections with the various optical model calculations indicates that the coupled channel calculations which include both rotational and vibrational effects are the most promising approach in spite of the uncertainties already mentioned. Consequently we will place particular emphasis on them in what follows. This choice in turn focuses attention on the calculations of Buck and Perey which are the only ones for which we have both the calculated average cross sections and the calculated values of $S_0$, $R'$, and $S_1$.

C. Shape Elastic Scattering

As is discussed below, the s-wave potential scattering cross section averages about half of the total cross section and is the largest single contribution. For this reason, our methods for breaking up the cross section into a number of components should be most accurate for $R'$, the s-wave potential scattering length. In Fig. 9a we have compared the values of $R'$ with the three collective models available for comparison.
Figure 9. S- And P-wave Strength Functions And $M'$. The curve labeled "Fiedeldey and Frahn" was calculated by H. Divadeenar from a potential privately communicated to him by Fiedeldey and Frahn, 1966.
While the three models do not agree with each other very well, they all predict an extremely rapid increase in $R'$ in the neighborhood of mass number 50, a more gradual increase between 180 and 190 and a rapid increase in the neighborhood of 143. The latter is the most striking; separated isotope measurements were made on Nd and Sm to look for this practically discontinuous jump in $R'$, and the measured results leave little doubt that this effect is present. The more gradual increase above 180 is also evident. At low masses there is a decided discrepancy between the results of averaged cross sections and those based on analyses for individual resonances, both from work done in this laboratory on the separated even isotopes of chromium and iron and the Saclay results on manganese and cobalt. Judging from our own experience, $R'$ is very difficult to determine from individual resonances in this neighborhood because of the presence of both resonance-resonance and resonance-potential interference in this region of wide s-wave resonances. For this reason, we have eliminated from Fig. 9a the individual resonance determinations for $R'$ below mass number 80. $R'$ now rises steeply between average mass number 37 (KCl) and 60 as the optical model predicts.

About the best that can be said quantitatively for the three collective models is that they bracket the data. That of Chase et al. is on the whole the best fit even though no account is taken for vibrational deformation for the lighter masses. The Buck and Perey calculations are consistently low
nearly everywhere and indicate the need for revision of the parameters. It should be pointed out that in the case of the calculation of Buck and Perey, the values of $C_2$ which they used may have been subject to errors which were great enough to cause at least some of the disagreement between their calculation and the experimental points. This speculation is reinforced by the observation that in Fig. 8a, calculations of the total cross sections based on the parameters of Buck and Perey do not agree with the experimental data near $A=150$ and that furthermore, the shape of the calculated average cross section curve in that region resembles that of the calculated curve in Fig. 8c in which the values of used in the calculations with JUPITER I were fixed at their lowest permissible values. It has been suggested by Divadeenam that the parameter $r_0$ in the equation for the nuclear radius ($R = r_0 A^{1/3}$) might be changed to get a better fit for our data without affecting the agreement of their model with the higher energy data, from which their parameters were originally derived. One can at least say that the parameters used by Buck and Perey would have to be revised appreciably before a good fit to $R^1$ could be obtained, but there is little point in revising the Buck and Perey parameters to fit our data if the fit with the higher data is spoiled. It would appear that an attempt to reconcile the Buck and Perey calculations with those of Chase et al. would be well worthwhile.
In order to emphasize the fact that the decomposition of the average cross section into its components cannot always be done with much precision, we present in Fig. 9d the experimental average cross section taken at 380 keV together with the residue after subtracting the s-wave contribution, which is the largest component except possibly at the peak of the 3p giant resonance. The residue, which includes s-, p-, and d-wave resonant contributions, as well as p- and d-wave potential contributions averages only about 3 barns and rises above 4 barns only at the peak of the 3p giant resonance. The sharp rise in the average cross section near A=140 is thus seen to be entirely caused by the sharp rise in $R'$; the double peak of the s-wave resonant cross section is barely discernable.

D. P-Wave Strength Functions

In Fig. 9b the Buck and Perey collective calculation is compared to the p-wave strength functions. Up to about mass number 180 the agreement is very good. The fact that between the mass numbers 120 and 180 the experimental points are systematically low may indicate that the deformations have been overestimated since the spherical model calculations performed by Fiedeldey and Frahm are much lower than the experimental points. However, throughout this region the p-wave contribution to the total cross section is not very large and
the experimental uncertainty in the p-wave strength functions may be great enough to account for the discrepancy between experiment and Buck and Perey. The p-wave contribution in this region is difficult to isolate because, as is pointed out in Chapter IV, the cross sections in this region are not very sensitive to the p-wave resonant contribution.

Above about 180 there is some evidence for a sharp minimum just below 200 and a steep rise above that figure. While this may be fortuitous, it is a possible sign that the expected rise in the strength function due to the p-wave giant resonance, which according to the Buck and Perey calculation should peak near A=220, is actually present. Unfortunately, it is not possible to measure points closer to the maximum using our methods since the nuclei between mass numbers 209 and 230 are highly radioactive. The agreement between the individual resonance analysis of the Saclay group, the Harwell results, and ours is very good but it may be fortuitous for the reasons explained above.

Buck and Perey's introduction of vibrational deformation into the optical model appears to have accounted very satisfactorily for the much higher p-wave strength functions in the neighborhood / of A=120 then would be expected on the basis of the Fiedeldey and Frahn spherical model or of the square well which was current at the time that Weston et al. / made the premature suggestion that there was a minimum in the neighborhood of mass number 100. The unfortunate controversy
caused by their suggestion has tended to obscure the fact that the p-wave strength function beyond $A = 100$ was significantly higher than was expected.

E. S-wave Strength Functions

Figure 9c presents the results of various collective model calculations of the s-wave strength functions together with experimental results from the analysis of individual resonances as well as from the analysis of average cross sections. All of the collective calculations qualitatively reproduce the major trends of the experimental points, but none seem to agree very well quantitatively. Buck and Perey speculate that the disagreement between their calculated s-wave strength functions and the experimental values near $A=120$ may indicate that the s- and p-wave neutrons see different optical potentials in some mass regions. The disagreement between the calculated s-wave strength functions near $A=120$ with the experimental data is great enough so that it is probably real rather than just a result of errors in the analysis or measurements.

Figure 10 presents the experimental s- and p-wave strength functions and $R'$ compared with the results of the calculations of Block and Feshbach (1963) and of Moldauer(1963). These are of interest chiefly because both
Figure 10. S- and P-wave Strength Functions and $R'$. The solid line is the calculation of Moldauer (1963), the dot-dash line is the calculation of Block and Feshbach (1963).
calculations were done at least partly to explain the low values of $S_0$ near $A=120$. The calculation of Moldauer is discussed above. It is seen that his calculation of $S_0$ agrees fairly well with the experimental data but it must be remembered that he obtained his optical model parameters by fitting the experimental $S_0$ values as well as other data. His calculation of $S_2$, which he did not fit to the experimental values, is much poorer, especially above $A=100$.

It will be noticed that only the strength functions obtained from average cross section data are compared with Moldauer's calculated curve. As is discussed below, the strength functions obtained from the average cross section data are averaged over a larger energy range than those obtained from individual resonances and are therefore more suitable for comparison to an optical model prediction such as that of Moldauer.

Block and Feshbach speculated that the dip in $S_0$ near $A=120$ and the fluctuations of the measured values of $S_0$ away from a smooth function of $A$ might be due to the effects of the structure of the target nucleus. They treated the nucleon-nucleus potential as being composed of that of a single particle state, related to the shell-model potential, coupled to a one particle-two hole (doorway) state through which the compound configuration had to proceed if it were to form a compound nucleus. Their calculation reproduces the low values of $S_0$ at $A=120$ but seems high near the $3s$ giant resonance
peak at $A \sim 50$. The predicted flat region at $A = 65-85$ is also reproduced, although the scatter of the experimental points makes the shoulder of doubtful significance.

It should be emphasized that the values of $S_0$ obtained from the analysis of individual resonances (denoted by the crosses in Fig. 10d) are higher than those obtained from average cross sections in the region near $A = 50$. Block and Feshbach's calculations agree with the individual resonance data better than they do with the average cross section data, and this is probably because the values from the average cross section data are averaged over an energy range greater than 1/2 MeV and the doorway state effects are averaged out, at least to some extent. For the same reason, the values of $S_0$ obtained from average cross sections fluctuate less than do those from individual resonances.

It should be noted that other evidence for the existence of doorway states has been found. The doorway states are very difficult to observe experimentally because the intermediate structure which they cause in the cross sections is not easily distinguished from the fluctuations in the average cross section caused by the statistical variations in the widths of the compound nucleus resonances. One of the best examples of the evidence for a doorway state appears in the total neutron cross sections of the lead isotopes, which is discussed by Farrell, Kyk, Bilpuch, and Newson (1965).
Divadeenam has discussed much of this strength data; repetition has been avoided where the new data has not altered the earlier data.

F. Conclusion

The comparison of the average cross sections with the various optical model calculations indicates that the coupled channel calculation may be capable of giving the best results, but that a more detailed knowledge of the properties of the low lying states of the target nuclei is needed. Considerable work has been done along these lines since the compilation of Stelson and Grodzins was published, and a new summary of all coupling schemes and parameters would be very useful. Comparisons of the measured average neutron total cross sections with the results of the coupled channel calculations might then help point out where further work on the collective coupling parameters might need to be done.

The comparison of the strength functions with the various optical model calculations revealed little that was new. The results for the nuclei in the rare earth region add to the already convincing evidence that the $4s$ giant resonance is split and also give more details about the variation of $R'$ near $A \sim 150$. The fact that the $s$-wave strength functions measured near $A=50$ varied strongly depending upon the energy interval over which they are averaged has been discussed in
some detail by Newson (1965) with special reference to manganese and also by Divadceenam (1967). This variation may prove to be caused by doorway states.
APPENDIX
APPENDIX
ORION

The formulas which are programmed in ORION are Eqn. 26 of Chapter IV:

\[
\langle \sigma_T \rangle = 4\pi \sum_{l=0}^{2} \left( 2l+1 \right) \left[ \sin^2 \delta' + \frac{l(l+1)}{2} V(0) (E_{\text{cutoff}})^{1/2} \right] \cos 2\delta',
\]

(A1)

and Eqn. 27 of Chapter IV:

\[
\langle \sigma_T \rangle = -\frac{1}{n} \log_e T_P \left[ 1 - \frac{\langle A(n\sigma_0, \delta'_{L, I}, \Gamma/A) \rangle}{\langle n \rangle} \right] \frac{\langle \Gamma \rangle}{\langle D \rangle}
\]

(A2)

where

\[
T_P = \exp \left( -n \langle \sigma_T \rangle_0 \right).
\]

(A3)

Equation A1 is used for energies great enough so that self-absorption of the neutrons in the sample due to the s-wave resonant cross section is negligible. If the self-absorption is not negligible, then Eqn. A2 is used to calculate the s-wave contribution, and the sum in Eqn. A1 is taken only over \( l = 1 \) and 2. The result of Eqn. A1 is then added to the result of Eqn. A2 to give the cross section. In practice, the energy TESTEN
above which Eqn. A1 was used for the s-wave contribution was set high enough so that no discernable discontinuities in the calculated cross section curve occurred at En= TESTEN.

As is discussed in Chapter IV, the table of the area functions A′ is read from a magnetic disk pack at the beginning of execution of ORION. The actual value of A′ for a particular set of values of nσ₀, ξ', and Λ/π is obtained by quadratic interpolation between the tabulated values. A note of caution is in order here. For nσ₀ > 50, ξ' > 0.4, and Λ/π < 2.0, the variation of A′ with these parameters is so rapid that the interpolation procedure gives poor results. For the data analyzed in this thesis, this case does not arise because the samples are thin enough so that nσ₀ (which is inversely proportional to the neutron energy) cannot be greater than 50 at the same time that ξ' (which is proportional to the square root of the neutron energy) is greater than 0.4. If one were analyzing data from very thick samples (n > 2 bn⁻¹) he might have to change the interpolation procedure.

As mentioned in the text, ORION uses a Gaussian linear regression scheme to search for the values of the strength functions and phase shifts giving the best agreement between the experimental data and the formulas A1 and A2. The method is simply sketched out. Let the goodness of fit parameter be defined by
\[
\chi^2 = \frac{1}{N} \sum_{i=1}^{N} \left( \frac{\sigma_i^e - \sigma_i^T}{\sigma_i} \right)^2
\]

where the \( \sigma_i^e \) are the experimentally measured cross sections and \( \sigma_i^T \) the cross sections calculated from Eqns. A1 and A2.

Let \( P_e \) be one of the parameters on which we are going to search (e.g., \( P\_e = S_0, R', S_1, \sigma'_1/S_1 \), or \( S_2 \)). The condition that \( \chi^2 \) be a minimum implies that

\[
\frac{\partial \chi^2}{\partial P_e} = -\frac{1}{N} \sum_{i=1}^{N} \frac{1}{(\sigma_i)} 2(\sigma_i^e - \sigma_i^T) \frac{\partial \sigma_i^T}{\partial P_e} = 0.
\]

Suppose that we make a series of guesses as to the set of \( P \) that give the best fit to the data and suppose that the \( k \)th guess is close enough to the \((k-1)\)st guess to allow us to write

\[
(\sigma^T_i)_{k} = (\sigma^T_i)_{k-1} + \sum_{l=1}^{M} \frac{\partial \sigma^T_i}{\partial P_{e}} \delta P_{e}
\]

where

\[
\delta P_{e} = (P_{e})_{k} - (P_{e})_{k-1}.
\]

Substituting this relation into Eqn. A4 leads to

\[
\sum_{i=1}^{N} (\sigma_i^e - \sigma_i^T) \frac{\partial \sigma_i^T}{\partial P_{e}} = \sum_{m=1}^{M} \sum_{i=1}^{N} \frac{\partial \sigma_i^T}{\partial P_{m}} \cdot \frac{\partial \sigma_i^T}{\partial P_{e}} \delta P_{m}
\]

(A7)
This forms a set of linear equations which may be solved for the $\theta^\prime_2$.

The use of the method involves finding the derivatives of the calculated cross sections with respect to the $P_\ell$. For Eqn. A1, this is straightforward. For Eqn. A2, things are not so simple because we have to find the expression for $\partial A'/\partial \theta_0'$. Since the dependence of $A'$ on $\theta_0'$ is not simple (see Seth and Tabony, 1966), we calculated $\partial A'/\partial \theta_0'$ from the interpolation procedure. The use of quadratic interpolation involves the fitting of a quadratic equation

$$a \theta_0'^2 + b \theta_0' + c = A'$$

(A8)

to the three points: $A'(1), \theta_0'(1); A'(2), \theta_0'(2); A(3), \theta_0'(3)$. When the constants $a$, $b$, and $c$ have been determined, one can calculate $\partial A'/\partial \theta_0'$ from

$$\frac{\partial A'}{\partial \theta_0'} = a \theta_0' + b \ .$$

(A9)

That this method proved adequate is evidenced by the fact that the iterative procedure converged most of the time.

For a more complete discussion, see Melkanoff, Sawada, and Raynall (1966).
LIST OF REFERENCES
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H. H. Barshall, Phys. Rev. 38, 431(1952)


J. M. Blatt and V. F. Weisskopf, Theoretical Nuclear Physics, (John Wiley and Sons, New York, 1952)

B. Block and H. Feshbach, Ann. Phys. 23, 47(1963)


D. M. Chase, L. Hilet, and A. R. Edmunds, Phys. Rev. 110, 1080(1958)


S. Fernbach, W. Heckrotte, and J. V. Lepore, Phys. Rev. 97, 1059(1959)


H. Feshbach, C. E. Porter, and V. F. Weisskopf, Phys. Rev. 96, 448(1954)

H. Fiedeldey and W. E. Frahn, Private communication to M. Divadeenan, 1966


O. Haxel, J. H. D. Jenson, and H. E. Suess, Phys. Rev. 75, 1766L(1949)


J. Julien, Nuclear Structure Study with Neutrons, Proceedings of an International Conference, Antwerp, July 19-23(1965)


B. Margolis and E. S. Troubetzkoy, Phys. Rev. 106, 105(1957)


M. G. Mayer, Phys. Rev. 75, 1069L(1949)


P. A. Moldauer, Nucl. Phys. 47, 65(1963)


M. Namiki, Prog. Theor. Phys. (Japan) 23, 629(1960)


D. J. Schwartz and R. B. Hughes, Neutron Cross Sections, BNIL 325 (1958)


P. H. Stelson and L. Grodzins, Nuclear Data 1, 21(1966)

A. Sugie, Prog. Theor. Phys. (Japan) 21, 681(1959)


T. Tamura, Rev. of Mod. Phys. 37, 679(1965)


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