Low-Energy Proton-Deuteron Elastic Scattering and the "$A_y$ Puzzle"

by

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ABSTRACT

MICHAEL H. WOOD: Low-Energy Proton-Deuteron Elastic Scattering and the “A_y Puzzle”
(Under the Direction of E. J. Ludwig)

Angular distributions of cross sections and complete sets of analyzing powers have been measured for d-p scattering at a center-of-mass energy of 667 ± 1 keV. This set of high-precision data was compared to the most recent variational calculations with the nucleon-nucleon potential AV18 alone and with AV18 plus the UR three-nucleon potential. The calculations have the best agreement with the cross-section data while the comparison with the tensor analyzing powers showed good agreement. However, a comparison between the vector analyzing powers revealed differences of approximately 40% in the maxima of the angular distributions. These VAP discrepancies have been observed at $E_{c.m.} \geq 2.0$ MeV as well as in the n-d scattering case. After the n-d experiments, the problem has been labelled the “A_y Puzzle”. A $\chi^2$/datum analysis was performed with the lowest values achieved by the inclusion of a three-nucleon force in the theoretical calculations. This fact provides more credence for the existence of three-nucleon forces. However, the $\chi^2$/datum for the VAPs remained approximately 100. To investigate the root of the problem in addition to other differences between the theory and the data, a single-energy phase shift analysis (PSA) was performed. The best fits to the VAP data displayed an $\approx 20\%$ increase in the the mixing parameter $\epsilon_3^2$ and an increase in the splitting between $^4P_J$ phase shifts from the variational calculations with the AV18+UR potentials.
ACKNOWLEDGMENTS

Gone are the days when Rutherford and his students sat in a dark room counting the flashes from alpha particles hitting a phosphorescent screen. Today’s research in nuclear physics requires large particle accelerators and many man-hours. This project could not have been completed entirely by myself, and all those involved deserve my greatest thanks.

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Education has always been important to me. I did not attend graduate school because of the poor career opportunities of a bachelor’s degree in physics.
Instead, my graduate degree extends my knowledge of a subject that has inspired me since high school. Therefore, I should thank a few people who have aided me in my education before coming to UNC. Drs. Hall Crannell and Daniel Sober at CUA gave me my first research job and provided me with a good foundation in experimental physics. Special thanks go to Fr. Jon Woodward, S.J. for his inspiration. I consider him the greatest mathematician that I have ever known.

Every Tarheel fan knows that Carolina is the best. Luckily, my in-laws Joseph, Judy, and Joe DiCostanzo, who are huge UNC fans, steered me toward this Southern part of heaven. I thank them for pointing out the merits of this university and for providing loving care for my son Joe. Without them, I would not have been able to devote as much time to my studies. I also appreciate the many, many times that Mary DiCostanzo has made me feel at home in North Carolina.

Life would not be the same without my family. They have always shown genuine interest in my work and have wished the best for me. For this, I want to thank my brothers and sisters Kevin, Amy, Chris, Beth, Tim, Meg, and Jo. I am eternally grateful to my parents Kathleen and Herbert for all they have done for me. Not only did they pay for twelve years of Catholic education, but they never pushed me to be something I am not. I will always be appreciative of the freedom they gave me to fulfill my dreams, even if it was something as esoteric as being a nuclear physicist.

So many thanks go to my wife Dee and my son Joe. Without a doubt, Joe is the best son a father could ask for. He can make me forget about the troubles of the day with a single smile. Finally, Dee deserves this degree as much as I do. With the long hours, random schedules, and numerous stresses associated with graduate school, she has always supported me. She has made this experience worthwhile. Thank you, Dee.

Michael H. Wood

June 2000
To Dee

Love You Always
CONTENTS

LIST OF TABLES ................................................................. xvii
LIST OF FIGURES ............................................................. xix
Chapter

I. Introduction ................................................................. 1
   1.1 Current Theory ....................................................... 4
      1.1.1 Conventional Potentials ................................. 6
      1.1.2 Current Theoretical Methods .......................... 11
   1.2 Previous Work ...................................................... 14
   1.3 The “A_y Puzzle” ................................................... 17
   1.4 Is the Problem in NN forces? ................................... 18
   1.5 Or Is the Problem in 3N forces? .............................. 20

II. Analyzing Power Formalism ........................................ 23
   2.1 What is Polarization? ........................................... 23
   2.2 Extraction of A_zz .................................................. 26
      2.2.1 Pol-Unpol Method for A_zz ............................. 27
      2.2.2 Pol-Pol Method for A_zz ................................. 27
   2.3 Extraction of A_xz and A_yy at β = 45° ...................... 28
      2.3.1 Pol-Unpol Method .......................................... 28
      2.3.2 Pol-Pol Method ............................................. 29
   2.4 Extraction of A_y and A_yy at β = 90° ....................... 30
      2.4.1 Pol-Unpol Method .......................................... 30
      2.4.2 Pol-Pol Method ............................................. 31
2.5 Proton beams and $A_y$ ........................................ 31
  2.5.1 Pol-Unpol Method .................................. 32
  2.5.2 Pol-Pol Method .................................... 32
2.6 Spherical Tensor Analyzing Powers ....................... 34

III. Experimental Procedure .................................. 35
  3.1 Cross-section Measurements ............................. 37
    3.1.1 Relative Angular Distribution .................... 37
    3.1.2 Absolute Normalization ............................ 41
  3.2 Tensor Analyzing Powers ............................... 44
    3.2.1 TAPs Measurements ................................ 44
  3.3 Vector Analyzing Powers ............................... 47
    3.3.1 Coincidence Measurements ....................... 48

IV. Data Analysis ............................................ 57
  4.1 Analyzing Powers .................................... 58
    4.1.1 Tensor Analyzing Powers ......................... 60
    4.1.2 Vector Analyzing Powers ......................... 61
  4.2 Relative Cross-Section ................................ 63
    4.2.1 Monitor Detectors ................................ 64
    4.2.2 Reconstructing Contaminant Peaks ............... 66
  4.3 Absolute $\sigma(\theta)$ Normalization .................. 72

V. Phase-Shift Analysis ..................................... 79
  5.1 Theoretical Calculations of Phase Shifts .............. 79
  5.2 Previous Phase-Shift Analyses ........................ 83
  5.3 Phase Shifts from Experimental Data .................. 84

VI. Discussion ................................................ 89
  6.1 Comparison to Theory ................................ 89
    6.1.1 The Absolute Cross-Section ....................... 89
    6.1.2 The Tensor Analyzing Powers ..................... 92
    6.1.3 The Vector Analyzing Powers ..................... 94
# LIST OF TABLES

1.1 Comparison between measurements of static deuteron properties and calculations with AV18 .................................................. 10
1.2 Comparison between measurements of $^3$He and $^3$H binding energies and calculations with both AV18 and AV18+UR .......................... 10
1.3 Previous measurements for d-p scattering observables ................................................................. 14
3.1 Reactions and resonances employed for 52°-beamline magnet calibration ......................................................... 37
3.2 Detector summary for relative cross-section measurements .......................................................... 40
3.3 Angle offsets for the 52° chamber ................................................................. 40
3.4 Detector summary for TAPs measurements ................................................................. 45
3.5 Mylar foil thickness used at various detector angles ........................................................ 46
3.6 Detector parameters for $^2$H($\vec{p}$, $p$)$^2$H coincidence experiment .................................................. 50
3.7 Detector parameters for $^1$H($\vec{d}$, $d$)$^1$H coincidence experiment .................................................. 51
4.1 Average tensor polarization for TAPs experiments ........................................................ 61
4.2 Average vector polarization for $^2$H($\vec{p}$, $p$)$^2$H scattering experiment ........................................ 63
4.3 Angles where contaminant peaks were problematic ........................................................ 67
4.4 Coefficients for the exponential-Gaussian fits ........................................................ 70
4.5 Ratio of measured $^1$H($p$, $p$)$^1$H yields to measured d-p scattering yields ........................................ 74
4.6 Ratio of $^1$H($p$, $p$)$^1$H cross-sections to d-p scattering cross-sections calculated from current theories ........................................ 76
4.7 Summary of uncertainties in absolute cross-section measurement .................................................. 78
5.1 Angular momentum and the mixing parameters ........................................................ 82
5.2 The $\chi^2_N$ for each observable after phase-shift analysis ........................................................ 85
5.3 Phase shifts and mixing parameters from final phase-shift analysis ........................................ 87
5.4 Values $\chi^2_N$ from final phase-shift analysis ........................................................ 88
6.1 The $\chi^2_N$ for each observable ........................................................ 96
B.1 Energies of polarized deuteron beams for $p_{zz}$ measurements ........................................ 117
C.1 Tensor analyzing powers $T_{20}$ and $T_{21}$ for $^1\text{H}(\vec{d},d)^1\text{H}$ scattering
at $E_{c.m.} = 667$ keV .................................................. 126

C.2 Tensor analyzing powers $A_{y\nu}$ and $T_{22}$ for $^1\text{H}(\vec{d},d)^1\text{H}$ scattering
at $E_{c.m.} = 667$ keV .................................................. 127

C.3 Vector analyzing power for $^1\text{H}(\vec{d},d)^1\text{H}$ scattering at $E_{c.m.} = 667$ keV . 128

C.4 Vector analyzing power for $^2\text{H}(\vec{p},p)^2\text{H}$ scattering at $E_{c.m.} = 667$ keV . 128

C.5 Absolute cross section for $^1\text{H}(d,d)^1\text{H}$ scattering at $E_{c.m.} = 667$ keV . 129

C.6 Vector analyzing powers for calibrated $^{12}\text{C} (\vec{d},p)^{13}\text{C}$ polarimeter . . . 130
# LIST OF FIGURES

1.1 Previous absolute cross-section measurements at $E_{c.m.} = 667$ keV.  
1.2 Ratio of experimental to theoretical cross-section at $E_{c.m.} = 667$ keV.  
1.3 Drawing of the Jacobi coordinate system.  
1.4 Comparison between theoretical calculations and d-p scattering data at $E_{c.m.} = 432$ keV.  
1.5 Difference in $A_y$ between experiment and theory versus $E_{c.m.}$.  
1.6 Difference in $iT_{11}$ between experiment and theory versus $E_{c.m.}$.  
2.1 Definition of the Madison Convention.  
3.1 Kinematics for a 2 MeV deuteron beam on a hydogenated carbon target.  
3.2 Experimental setup for relative cross-section measurement.  
3.3 Detector electronics used for singles mode.  
3.4 Sample spectrum for relative cross-section measurement.  
3.5 Sample spectrum for $^{197}$Au$(d,d)^{197}$Au scattering at $\theta_{lab} = 140.0^0$.  
3.6 Experimental setup for TAPs measurements.  
3.7 Electronics setup for $^{12}$C$(\bar{d},d)^{12}$C veto.  
3.8 Experimental setup for VAPs measurements.  
3.9 Electronics setup for coincidence mode.  
3.10 Coincidence spectrum of four-detector configuration.  
3.11 Laboratory angle to center-of-mass angle conversion for $^1$H$(d,d)^1$H and $^1$H$(d,p)^2$H scattering.  
3.12 Electronics setup for coincidence mode with two detectors.  
4.1 Ratio of monitor detector yields.  
4.2 Relative cross-section for $^2$H$(p,p)^2$H scattering at $E_{c.m.} = 667$ keV.  
4.3 Ratio of $^2$H$(p,p)^2$H scattering cross-section measurements to the theoretical calculations.  
4.4 Spectrum of $^1$H$(d,d)^1$H and $^1$H$(d,p)^2$H scattering at $\theta_{lab} = 10^0$.  
4.5 Spectrum of $^1$H$(d,p)^2$H scattering at $\theta_{lab} = 60^0$.  

xix
Chapter 1

Introduction

The research detailed in this dissertation was motivated by the fact that a solvable expression for the strong nuclear force has not been derived from first principles. I should quickly make the disclaimer in this second sentence that this thesis will not be providing one. Quantum Chromodynamics or QCD can describe the interactions of quarks and gluons, but the present status of the theory is that it cannot describe in detail the interaction between two nucleons. What this thesis will discuss is an experimental study of the three-nucleon system via deuteron-proton scattering as a means to illuminate the properties of the strong nuclear force. The region of interest is low interaction energies ($E_{c.m.} = 667 \pm 1$ keV to be exact). In this low-energy regime, the interaction of the nucleons is understood by the exchange of virtual mesons. Moreover, the range provided by the interaction is not short enough for quark degrees of freedom to play any sizable role. It is surprising that after almost a hundred years of intense searching by theorists and experimentalists, the exact form of the strong nuclear force remains elusive.

The first question to ask is why concentrate on three nucleons instead of a simpler system such as two nucleons. The response is that the three-body system is a much richer system to explore at this time. Two-body observables are well described by the existing potential models. For proton-proton scatter-
ing ([Bar82]) in this energy region, the vector analyzing power $A_y$ has an order of magnitude smaller value than that of deuteron-proton scattering. Systematic errors in the vector analyzing power measurements would have a smaller effect in the d-p scattering experiments than the p-p scattering experiments. Furthermore, an unanswered question is whether three-body forces are needed when the number of nucleons in the system increases. It is possible that the nucleon-nucleon (NN) interaction is all that is required to describe a many nucleon system. Over the past twenty-five years, there have been numerous potential models ([Lac80, Mac87, Sto94, Wir95]) developed based on the current NN database. There have been various theoretical methods ([Ber86, Kie93, Pud95]) developed recently utilizing the conventional potential models. The models and methods will be discussed in Section 1.1. This thesis will provide a stringent test for the theoretical calculations in their attempt to describe the nuclear dynamics.

As it was stated in the first paragraph, these experiments were conducted at a center-of-mass energy of 667 keV. This low energy was chosen to simplify the scattering process in the number of partial waves involved and to check previous measurements made at the same energy. The scattering is below the deuteron-breakup threshold so there is only one exit channel (elastic scattering) to consider (disregarding the very low probability of radiative capture and Bremsstrahlung). With only elastic scattering possible, complicated detection systems such as $\Delta E - E$ or spectrometers were not necessary. A very simple experimental setup was employed instead. Also, the absolute cross-section for d-p scattering was measured in the past by two different experiments, one by Kocher and Clegg [Koc69] and the other by Huttel et al. [Hut83a]. Figure 1.1, overlays the results of the two measurements. At this resolution of the graph, there is no apparent disagreement. However, when the data are divided by a current calculation by Kievsky et al. [Kie94] as shown in Figure 1.2, a discrepancy arises in the forward angle data points. The data of Kocher and Clegg are a flat distribution with values about 1% greater than the theory. On the other hand, the measure-
Figure 1.1: Previous absolute cross-section data at $E_{c.m.} = 667$ keV. The triangles are the data of Kocher and Clegg [Koc69] and the squares were measured by Huttel et al. [Hut83a]. The errors are smaller than the symbols.

The measurement of Huttel et al. is not a flat distribution but has a positive slope. As the theoretical predictions have become more accurate in recent years, a disagreement of 3-4% is very important for understanding the scattering process. The S-waves have the largest strengths in d-p scattering at this low energy [Kie96] and provide the principle contribution to the cross-section. A measurement of the cross-section with high precision will clear up this disagreement.

To understand the different partial waves involved in the d-p system, a measurement of the cross-section is not sufficient. By using polarized proton and deuteron beams of various polarization states, a phase-shift analysis of the experiments can be performed, and the different strengths of the phases can be obtained. The phase-shift analysis will modify the specific phases in order to reproduce the data and will clarify which partial waves of the current potential
models or theoretical methods are incorrect. Therefore, the cross-section as well as a complete set of analyzing powers (see Chapter 2) were measured.

1.1 Current Theory

Even though a complete theoretical description for the nucleon-nucleon force has not been obtained, many properties have been found through experiments. A list of several of these properties are found in [Wal95] and are as follows:

1. The NN interaction is essentially the one-pion-exchange potential (OPEP).
This potential has the form [Yuk35]

\[ V^\pi(r) = \frac{g^2_{\pi}}{4\pi c^2} \left( \frac{m_\pi}{M} \right)^2 \frac{1}{3} \frac{1}{\vec{r}_1 \cdot \vec{r}_2} \left\{ \vec{\sigma}_1 \cdot \vec{\sigma}_2 + S_{12} \left[ 1 + \frac{3}{m_\pi r} + \frac{3}{(m_\pi r)^2} \right] \right\} e^{-m_\pi r} \]  

(1.1)

where \( r \) is the internucleon distance, \( g_{\pi} \) is the \( \pi N \) coupling constant, \( m_\pi \) is the pion mass, \( M \) is the nucleon mass, and \( \sigma_i \) (\( \tau_i \)) are spin (isospin) operators. The quantity \( S_{12} \) is the tensor operator and takes the form of

\[ S_{ij} = 3(\sigma_i \cdot \hat{r})(\sigma_j \cdot \hat{r}) - \vec{\sigma}_i \cdot \vec{\sigma}_j. \]  

(1.2)

The interaction between the nucleons is seen as the exchange of a virtual pion. The OPEP contains the following nuclear properties:

- At very large internucleon distances, the force between the nucleons decreases rapidly so that at a distance of approximately 10 fm the nuclear force is negligible.

- The potential is not completely a central potential. There is a non-central or tensor force in the NN potential. The deuteron has a non-zero quadrupole moment in its ground state which can only be explained by the addition of a non-central component.

- There is a strong spin-dependence in the NN force. The Pauli operators for intrinsic spin and isospin operators for each nucleon are coupled in the OPEP.

2. Experimentally, it has been observed that a polarized beam has a certain probability of scattering in a preferred direction which can be described by spin-dependent forces. This observation has been attributed to an \( \vec{L} \cdot \vec{S} \) interaction between the nucleons.

3. At short distances, the NN potential becomes repulsive. It acts as if there is a core at \( \approx 0.5 \) fm pushing away other nucleons.
4. The NN potential is approximately charge symmetric and charge independent. This statement means that there is no difference between the proton and a neutron besides the electric charge. Charge symmetry says that nn scattering and pp scattering without the Coulomb interaction are the same. Charge independence implies that nn, pp and np scattering are all equivalent, again without the Coulomb force. From nn and pp scattering-lengths measurements, charge symmetry is upheld to a large extent. However, the np scattering length is about 30% larger than the other two, leading to a breaking of charge independence. As an indirect argument for charge-independence breaking (CIB), the neutron and proton have slightly different masses due to their different quark contributions. In addition, the quark structure of the pions contributes to a mass difference between the charged and the neutral pions. The difference in nucleon as well as pion structure may lead to a difference between np and pp/nn scattering.

1.1.1 Conventional Potentials

When solving the three-nucleon (3N) system, one constructs a non-relativistic Hamiltonian of the form

\[ H = \sum_i T_i + \sum_{i<j} V_{ij}(NN) + \sum_{i<j<k} W_{ijk}(3N), \]  

(1.3)

where the three particles are summed over the index \( i \) and \( T_i \) is the kinetic energy of particle \( i \). What is unknown are the NN and 3N potentials \([V_{ij}(NN)\] and \(W_{ijk}(3N)\)). In an attempt to model the NN force, phenomenological potentials have been created to reproduce the aforementioned properties. Over the past three decades, there have been many models produced such as the Paris potential [Lac80, Pig94], the charge-dependent Bonn (CD Bonn) potential [Mac87, Jia92], the Nijmegen potential [Sto94], and the Argonne V18 (AV18) potential [Wir95] to name a few. Potentials such as AV18 have been derived in configuration space and others such as CD Bonn in momentum space. All of
these potentials include the one-pion-exchange (OPE) explicitly. The long-range part is obtained by the exchange of a virtual pion and quark degrees of freedom are ignored. Most of the models differ in the description of the short-range region and in the presence of non-localities. The potentials have differing descriptions of the core. A hard core with strong repulsion or a softer core are the traditional approaches. In order to improve the accuracy in the description of the force, the most recent models include charge-symmetry breaking (CSB) and charge-independence breaking (CIB) terms. Of course, in addition to the strong force, the NN interaction includes electromagnetic effects.

Although the NN potentials come in different forms, they are treated in a similar manner. In order to reproduce properties of the NN system, the potentials are parameterized and fit to the current NN database. There are approximately 4300 data points from NN experiments below $E_N = 500$ MeV. The database, in addition to elastic scattering observables, includes experimental values for the binding energy, the magnetic moment, and quadrupole moment of the deuteron as well as pp, np, and nn scattering lengths. Not all of the experimental data are included in the fits by each group that has formulated a potential. Some of the data are excluded if the values lie a certain number of standard deviations (for example 3 in the Nijmegen fits [Ber90a]) away from the fits. To illustrate the form of these potentials, the AV18 will be described in detail.

The AV18 potential can be broken down into three parts: the electromagnetic (EM) interaction, the OPE interaction, and a short-range phenomenological part. The decomposition of the potential is represented as

$$V(NN) = V^{EM}(NN) + V^\pi(NN) + V^R(NN).$$

(1.4)

The potential has been generalized for the three possible NN scenarios: pp, np, or nn. First, the EM term will be discussed for each situation. The three EM
potentials have the following forms

\[ V^{EM}(pp) = V_{C1}(pp) + V_{C2}(pp) + V_{DF}(pp) + V_{VP}(pp) + V_{MM}(pp), \]

\[ V^{EM}(np) = V_{C1}(np) + V_{MM}(np), \]

\[ V^{EM}(nn) = V_{MM}(nn), \]

(1.5)

where the C1 and C2 terms are the one- and two-photon exchanges, DF is the Darwin-Foldy term, VP represents the vacuum polarization, and MM is the magnetic moment term. Each term has its own operator form which is described in [Wir95]. For the OPE, \( V^\pi(NN) \) contains a \( \pi N \) coupling constant and a pion-exchange potential which contains spin-dependent terms and a tensor operator (see equation 1.1). Inside the pion-exchange term are short-range exponential cutoffs with parameters to fit the shape of the nuclear potential. It is interesting to note that the \( V^\pi(pp) \) and \( V^\pi(nn) \) terms only depend on a \( \pi^0 \) exchange whereas the \( V^\pi(np) \) term can describe the exchange of a neutral and charged pion. The form of \( V^\pi(NN) \) reflects that fact. For the exact form of the OPE potential, the reader is again referred to [Wir95]. The last part of the NN potential is the short-range and intermediate-range term, and it contains the remaining properties not found in the OPE term. This part has been parameterized as a sum of central, \( L^2 \), tensor, spin-orbit, and quadratic spin-orbit operators and has the form

\[ V^R(NN) = v^c(r) + v^P(r)L^2 + v^t(r)S_{12} + v^L(r)\vec{L} \cdot \vec{S} + v^{L2}(r)(\vec{L} \cdot \vec{S})^2, \]

(1.6)

where the \( v^i \) functions contain the parameters for the fit to the database. Again, more detail is found in [Wir95].

This formalism for the AV18 potential is utilized to show the physical meaning behind each term in the potential. Since the \( V^\pi(NN) \) and \( V^R(NN) \) have similar operator structure, the two terms can be combined into a concise operator basis of the form

\[ V_{ij} = \sum_{p=1,18} v^p(r)O^p_{ij}, \]

(1.7)
where $O_{ij}^p$ is a given operator and $v^p(r)$ contains the parameters for the fit. The first fourteen operators are

$$O_{ij}^{p=1,14} = 1, \bar{\tau}_i \cdot \bar{\tau}_j, \bar{\sigma}_i \cdot \bar{\sigma}_j, (\bar{\tau}_i \cdot \bar{\tau}_j)(\bar{\sigma}_i \cdot \bar{\sigma}_j), S_{ij}, S_{ij}(\bar{\tau}_i \cdot \bar{\tau}_j), L \cdot \bar{S},$$

$$\bar{L} \cdot \bar{S}(\bar{\tau}_i \cdot \bar{\tau}_j), L^2, L^2(\bar{\tau}_i \cdot \bar{\tau}_j), L^2(\bar{\sigma}_i \cdot \bar{\sigma}_j), L^2(\bar{\sigma}_i \cdot \bar{\sigma}_j)(\bar{\tau}_i \cdot \bar{\tau}_j),$$

$$(\bar{L} \cdot S)^2, (\bar{L} \cdot \bar{S})^2(\bar{\tau}_i \cdot \bar{\tau}_j),$$

(1.8)

where $\bar{\sigma}$ is a Pauli spin matrix, $\bar{\tau}$ is a Pauli isospin matrix, and $S_{ij}$ is the tensor operator. The remaining four operators provide CIB and are as follows

$$O_{ij}^{p=15,18} = T_{ij}, (\bar{\sigma}_i \cdot \bar{\sigma}_j)T_{ij}, S_{ij}T_{ij}, (\tau_{zi} + \tau_{zj}),$$

(1.9)

where $T_{ij}$ is a tensor operator for isospin and takes the form

$$T_{ij} = 3\tau_{zi}\tau_{zj} - \bar{\tau}_i \cdot \bar{\tau}_j.$$  

(1.10)

The previous dissection of the AV18 is not intended to bore the reader but to elucidate how complicated the expression of a NN potential has become. So that all of the effects of the strong nuclear force may be included, more and more terms have to be added to the expansion of operators. Currently, the fit for the AV18 potential contains approximately 40 free parameters. For the case of the AV18, the potential has not been fit to the data directly but to the phases shifts produced by the Nijmegen phase-shift analysis [Sto93] of the NN database from 0-350 MeV. After all of the parameters are fit, the conventional potentials reproduce the NN system remarkably well. Table 1.1 lists a few properties of the deuteron and the calculations with AV18 of those quantities. The fit achieved by the parameterization gives a $\chi^2$/datum $\approx 1$.

When the conventional NN potentials were applied to the $A = 3$ system, a problem arose. The NN potentials alone did not reproduce the experimentally-determined $^3$He or $^3$H binding energies. Table 1.2 shows the measured binding energies and those calculated with the NN model potentials. Since the NN potentials reproduced the deuteron properties, the conclusion is that there are
Table 1.1: Static properties of the deuteron and AV18 calculations.

<table>
<thead>
<tr>
<th></th>
<th>Experiment</th>
<th>AV18</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>BE</td>
<td>2.224575(9) [vdL82]</td>
<td>2.224575</td>
<td>MeV</td>
</tr>
<tr>
<td>$r_d$</td>
<td>1.953(3) [Kla86]</td>
<td>1.967</td>
<td>fm</td>
</tr>
<tr>
<td>$\mu_d$</td>
<td>0.857406(1) [Lin65]</td>
<td>0.847</td>
<td>$\mu_0$</td>
</tr>
<tr>
<td>$Q_d$</td>
<td>0.2859(3) [Eri83]</td>
<td>0.270</td>
<td>fm$^2$</td>
</tr>
</tbody>
</table>

Table 1.2: Binding energy calculations with AV18 and AV18+UR.

<table>
<thead>
<tr>
<th>Isotope</th>
<th>Experiment [Aud93]</th>
<th>AV18</th>
<th>AV18+3NF</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^3$He</td>
<td>7.72</td>
<td>6.91</td>
<td>7.73</td>
<td>MeV</td>
</tr>
<tr>
<td>$^3$H</td>
<td>8.48</td>
<td>7.61</td>
<td>8.49</td>
<td>MeV</td>
</tr>
</tbody>
</table>

three-nucleon forces (3NF) present. This reasoning is the first indirect evidence of the existence of a 3NF. To alleviate this binding energy problem, phenomenological 3N potentials were created in the spirit of the NN potentials. The several 3N potentials which have stood the test of time are the Tuscon-Melbourne (TM) [Coo79], the Brazil (BR) [Coe83], and the Urbana (UR) [Pud95]. All have a similar format in that each contains a two-pion exchange potential with intermediate- and short-range phenomenological terms. The potentials use cutoff functions to restrict the nucleons from being too close to each other. The various models differ in the strength and form of these cutoffs. The 3N potentials are adjusted to reproduce the correct trinucleon binding energies. Referring back to Table 1.2, the calculations reproduce the binding energies once a 3N force has been included. However, it is not surprising that the binding energies are predicted so well since the potentials are fit for that purpose. Thus, another motivation for the d-p scattering experiments in this thesis is to verify whether the NN and 3N potentials can reproduce scattering observables.
Although the NN and 3N potentials have been downplayed for their phenomenological nature, the models are very helpful as inputs into the theoretical calculations of the 3N system. Even with a potential in hand, the solutions are not trivial. Techniques need to be developed to solve a system of three bodies.

The first step in this calculation is to transform from a coordinate system where the position of each nucleon is its distance from the center of mass to a set of more powerful Jacobi coordinates

\[
x_i = r_j - r_k
\]
\[
y_i = \frac{1}{\sqrt{3}}(r_j + r_k - 2r_i).\]  

Figure 1.3 displays the Jacobi coordinates graphically. Next, the total 3N wavefunction is rewritten as a sum of three separate nucleon amplitudes

\[
\Psi = \psi(x_i, y_i) + \psi(x_j, y_j) + \psi(x_k, y_k).\]  

Each amplitude corresponds to a total angular momentum and isospin state. Thus, the amplitude can be decomposed into separate radial and angular mo-
momentum functions as

\[ \psi(\vec{x}_i, \vec{y}_i) = \sum_{\alpha=1}^{N_c} \Phi(\vec{x}_i, \vec{y}_i) \left\{ \left[ (Y_{Ls}(\hat{x}_i) s^{ik})_1 \otimes s^j \right] S \otimes Y_L(\hat{y}_i) \right\} _{J J_z} \left[ \hat{t}^{jk \mu} \right] _{TT_z} (1.13) \]

The quantity in the \{ \} _{J J_z} carries the angular momentum coupling. The isospin coupling is contained in \[ TT_z \]. With the spin and isospin dependence factored out, the problem is reduced to solving for \( \Phi(\vec{x}_i, \vec{y}_i) \), the spatial function of the nucleon.

To solve the problem further, there are two methods which are most effective. One is the Faddeev method. This calculation involves reducing the Schrödinger equation with a wave-function that encompasses all three nucleons to three Faddeev equations (FE) with the single-nucleon amplitudes. By this reduction, three simpler wave equations are solved instead of a single complicated one. Furthermore, due to symmetry considerations, in the case of three-equal particles, only one of the FE has to be solved. In any case, each FE is a 6-dimensional integro-differential equation and the usual method of solving the equation involves transforming it to an infinite set of two-dimensional integro-differential coupled equations. Using numerical techniques, these equations are truncated after a partial-wave decomposition of the NN potential. The disadvantage of these techniques is that it is difficult to deal with potentials of infinite range, like the Coulomb potential, safely. A solution for the d-p system is extremely difficult and has not been found in a mathematically-rigorous way. On the same note, the neutron-deuteron system can be solved rigorously, since for all intents and purposes, the nuclear potential is of finite range and the Coulomb force does not enter. The Faddeev method has been largely used in both configuration and momentum space [Ber86].

Another approach is to employ a variational procedure to solving the 3N Schrödinger equation. This method has been worked out in detail in [Kie93]. For the scattering states, the complex Kohn variational principle is utilized [Kie97a]. The wavefunctions are separated into two parts, a short-range part and an asymp-
totic part. The asymptotic wavefunction is proportional to an incoming and outgoing wavefunction

$$\psi_A(x_i, y_i) \rightarrow \psi_A^{\text{In}}(x_i, y_i) + \sum_{L'S' SS'} U_{L'SL'S}' \psi_A^{\text{Out}}(x_i, y_i),$$

where $U_{L'SL'S}'$ is the collision matrix. This matrix contains all of the phase-shift information and will be discussed in more detail in Chapter 5. The variational method is used to solve for the collision matrix which is needed to calculate the scattering observables. The solution of the scattering wavefunctions is a principal feature of the variational method. The calculations allow for comparisons with both the double-charged (p-d) system as well as the single-charged (n-d) system. Both methods (variational and Faddeev) show excellent agreement with each other in calculating n-d scattering observables [Kie98a]. However, only the variational approach can produce rigorous calculations for p-d scattering.

One last method which deserves some notice is that of chiral perturbation theory (CPT) [Wei79]. It is an analytical approach to solving for few-nucleon observables by employing field-theory techniques. In CPT, a Lagrangian of the system is formed where the strength of its terms are set by two scales, the pion-decay constant $f_\pi \approx 93$ MeV and the large-mass QCD scale $\Lambda \approx 1$ GeV. The terms of the Lagrangian are organized by inverse powers of $f_\pi$ and $\Lambda$ in order to truncate the expansion of the Lagrangian and determine which terms can be neglected. By setting these arbitrary scales, low-energy observables can be calculated while ignoring the quark-gluon interactions present inside the nucleons. Calculations of observables from CPT have not reached the accuracy of the Faddeev or variational methods, but some useful information has come out of CPT studies. One example is the organization of $2\pi$-exchange 3NF with CPT [Fri99]. Here, the TM, UR, and Texas 3NF were compared term by term with a Lagrangian constructed with CPT. Friar et al. found that one term in TM did not fit into their counting scheme and was not first-order in the Lagrangian. All of the terms in the UR and Texas potentials were of leading order. Thus, from
Table 1.3: Previous measurements for d-p scattering observables.

<table>
<thead>
<tr>
<th>$E_{c.m.}$ (MeV)</th>
<th>Observables</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.67 - 6.7</td>
<td>$\sigma(\theta)$</td>
<td>[Koc69]</td>
</tr>
<tr>
<td>1.7 - 4.3</td>
<td>$A_y$, $iT_{11}$, $T_{20}$, $T_{21}$, $T_{22}$</td>
<td>[Whi79]</td>
</tr>
<tr>
<td>1.0</td>
<td>$iT_{11}$, $T_{20}$, $T_{21}$, $T_{22}$</td>
<td>[Whi79]</td>
</tr>
<tr>
<td>0.27 - 0.67</td>
<td>$\sigma(\theta)$</td>
<td>[Hut83a]</td>
</tr>
<tr>
<td>0.53 - 0.67</td>
<td>$A_y$</td>
<td>[Hut83a]</td>
</tr>
<tr>
<td>2.0</td>
<td>$\sigma(\theta)$, $A_y$, $iT_{11}$, $T_{20}$, $T_{21}$, $T_{22}$</td>
<td>[Knu93]</td>
</tr>
<tr>
<td>1.3 - 2.7</td>
<td>$\sigma(\theta)$, $A_y$</td>
<td>[Shi95]</td>
</tr>
<tr>
<td>1.7 - 2.0</td>
<td>$iT_{11}$, $T_{20}$, $T_{21}$, $T_{22}$</td>
<td>[Shi95]</td>
</tr>
<tr>
<td>0.432</td>
<td>$\sigma(\theta)$, $A_y$, $iT_{11}$, $T_{20}$, $T_{21}$, $T_{22}$</td>
<td>[Bru00, Bru98, Kar99]</td>
</tr>
<tr>
<td>0.163, 0.211</td>
<td>$\sigma(\theta)$</td>
<td>[Bla95]</td>
</tr>
</tbody>
</table>

their examination, they showed that the UR, Texas, and TM (minus one term) potentials provide similar descriptions of the 3NF. The next big step will be to actually calculate binding energies and other observables from CPT.

### 1.2 Previous Work

The investigation of d-p scattering below the deuteron breakup threshold has not been restricted to theoretical investigations. Table 1.3 lists previous elastic scattering measurements at energies near and below the deuteron breakup threshold. The disagreement of the cross-section measurements at $E_{c.m.} = 667$ keV by Kocher and Clegg [Koc69] and Huttel et al. [Hut83a] was discussed earlier in this chapter. Previous cross-section measurements [She47, Bro52] before 1969 will not be discussed since their uncertainties are 2 to 3 times that measured by Kocher and Clegg. Along with the cross-section data, Huttel et al. measured $A_y$ angular distributions from $E_{c.m.} = 530$ to $667$ keV with uncertainties on the order of $5 \times 10^{-4}$. With this data and the vector and tensor analyzing powers of White
et al. [Whi79], Huttel et al. [Hut83b] conducted an energy-dependent phase-shift analysis over the energy range of $E_{c.m.} = 530$ keV to 2.0 MeV. Their analysis found that the VAP data were influenced by the $^4P_J$ phase shifts and that the splitting between them was less than $5^\circ$. The fits to the cross-section data did not show any sensitivity to a splitting of the phase shifts. With the TAP data, they had difficulty in fitting the backward angles over the entire energy range.

Experiments of d-p scattering have been one of the interests of our group in recent years. Cross-section measurements at $E_{c.m.} = 163$ keV and 211 keV [Bla99] were obtained and were used in the first precise determination of the doublet and quartet S-wave scattering lengths. This work also confirmed the existence of a singularity in the $^2S_{1\over 2}$ effective-range function. Angular distributions of $\sigma(\theta)$, $A_y$, $iT_{11}$, $T_{20}$, $T_{21}$, and $T_{22}$ [Kie97b, Bru98] were measured at $E_{c.m.} = 432$ keV. Figure 1.4 shows comparisons between the calculations and measurements of a set of analyzing powers measured by our group at this energy. The calculations of the tensor analyzing powers (TAP), $T_{30}$ and $T_{21}$, display good agreement with the data with only small discrepancies. Although it is not shown, the cross-section is reproduced very well by the theory. The noticeable discrepancy is in the vector analyzing powers (VAP), $A_y$ and $iT_{11}$. Here the disagreement is on the order of 40% with the theory giving much smaller values. The problem with the vector analyzing powers was first seen in $A_y$ measurements for n-d scattering [Tor91] and has been labelled the “$A_y$ Puzzle”. More about the problem will be elaborated on in Section 1.3. Improved descriptions of $A_y$ and $iT_{11}$ were possible when the $^4P_{1\over 2}$ phase shift was reduced by 1.5% and the $\epsilon_{1\over 2}$ parameter was increased by 15% in the variational calculations. Changes such as these were not found when the UR 3N potential was included in the calculations.

There are two previous data sets at $E_{c.m.} = 2$ MeV [Knu93, Shi95] which are important for this discussion. Comparisons between the 2 MeV measurements and the theoretical predictions show similar agreement in the TAP distributions and similar disagreement in the VAP distributions. Knutson et al. [Knu93] per-
formed a single-energy phase shift-analysis on their data set. The results of this analysis were compared to phase shifts calculated approximately using the Faddeev equations including NN interactions only. They found the largest discrepancies with the $^2S_{1/2}$ phase shift and the $S$-$D$ mixing parameter, $\eta_{S,D}$. There were small differences between the analysis and the theoretical calculations for splitting of the $P$-wave phase shifts ($< 0.5^\circ$). However, they claimed the splitting was necessary to achieve good fits to the data. The single-energy phase-shift analysis of Kievsky et al. [Kie96] included the data of Knutson et al. and Shimizu et al. [Shi95], and the results were compared to phase shifts obtained from the variational calculations. This second analysis confirmed the findings of Knutson.

Figure 1.4: Comparison between variational calculations with AV18 and UR potentials and selected analyzing power data for d-p scattering at $E_{c.m.} = 432$ keV
et al. In particular, the discrepancies involving the $^2S_1^>$ phase shift and $\eta_{1^+}$ were reduced by the inclusion of a 3N potential. In addition, the fits to $A_y$ and $iT_{11}$ were improved by changes of 3.4% and 12% in the $^4P_1^>$ phase shift and $\epsilon_{1^-}$ parameter, respectively.

1.3 The “$A_y$ Puzzle”

As I mentioned in the discussion of the variational calculations, the largest discrepancy between theory and experiment is with the VAPs. At $E_{c.m.} = 432$ keV, the difference is as large as 40%. The discrepancy is not restricted to the variational calculations. Similar differences are present between nucleon-deuteron scattering data and Faddeev calculations with current potential models [Wit94]. From $E_{c.m.} = 3.3$ to 5.7 MeV, $A_y$ measurements for n-d scattering disagree with Faddeev calculations by approximately 25% [Tor91]. As stated in section 1.1.2, the variational calculations and Faddeev calculations agree very well for n-d scattering observables [Kie98a] leading to the conclusion that the discrepancy is not from the chosen method but from the potentials. Since the problem exists in both the p-d and n-d systems with approximately the same percent discrepancy, the solution does not lie in CSB or CIB. The neutron and proton apparently act alike. It also does not appear to be an electromagnetic problem. The respective EM terms are inserted in both systems. In addition, the effect of the magnetic moments was even investigated by Stoks [Sto98] and found not to provide the solution. Figures 1.5 and 1.6 show that the discrepancy does have an energy dependence. These graphs are plots of the difference between experimental data and theoretical calculations normalized to the data. They show a decreasing discrepancy as the energy increases. The disagreement mostly disappears at approximately $E_{c.m.} = 30$ MeV [Glo96].
Figure 1.5: Difference in $A_y$ between experiment and the variational calculations of Kievsky et al. [Kie93] with respect to $E_{\text{c.m.}}$. The data correspond to the maximum of the angular distribution. The circle is a previous measurement by our group [Bru98]. The squares and the triangle are the data of Shimizu et al. [Shi95] and Huttel et al. [Hut83a].

1.4 Is the Problem in NN forces?

There are speculations that the source of the “$A_y$ Puzzle” may be in the NN interactions. Witala and Glöckle [Wit91] showed that $A_y$ for n-d and p-d scattering at $E_{\text{c.m.}} \geq 6.7$ MeV was influenced by the strength of the $^3P_J$ phase shifts in the NN interactions. They found that the calculated $A_y$ changed dramatically over the Faddeev calculations with the Bonn B potential when $^3P_0$, $^3P_1$, and $^3P_2$ were left out of the calculations individually. Hüber et al. [Hüb95] found that the $^3P_J$ phase shifts in the NN system can influence the $^4P_J$ phase shifts in the 3N system. They discovered that $^3P_0$ affects $^4P_{\frac{3}{2}}$, changes in $^3P_1$ influences $^4P_{\frac{1}{2}}$ and $\epsilon_{\frac{3}{2}}$- mixing parameter, and $^3P_2$ influences $^4P_{\frac{5}{2}}$. Multiplicative
Figure 1.6: Difference in $iT_{11}$ between experiment and the variational calculations of Kievsky et al. [Kie93] with respect to $E_{c.m.}$. The data correspond to the maximum of the angular distribution. The circles are previous measurements of our group [Kar99]. The squares and triangles are the data of Shimizu et al. [Shi95] and of Gruebler et al. [Grü83], respectively.

Factors of 0.976 for $^3P_0$, 0.912 for $^3P_1$, and 1.16 for $^3P_2$ were able to change the $^4P_J$ phase shifts and the $\epsilon_{\frac{1}{2}-}$, and $\epsilon_{\frac{3}{2}-}$ parameters sufficiently to match $A_y$ data to calculations for n-d scattering at $E_{c.m.} = 2$ MeV to 5.7 MeV [Tor98]. The NN scattering data were still reproduced by calculations with the modified $^3P_J$ phase shifts. In fact, Bittner and Kretschmer [Bit79b] found that many different choices for the $^3P_J$ reproduced the NN scattering data equally well. They showed that $3^3P_1$ could be less than or greater than $2|3^3P_0| + |3^3P_2|$, and the calculations described equally-well the $A_y$ angular distribution for p-p scattering at $E_p = 6.14$ MeV. Tornow et al. [Tor98] found $^3P_0$, $^3P_1$, and $^3P_2$ to be uncertain by 20%, 25%, and 135%, respectively, corresponding to a 2% change in the
phase shifts for p-p scattering at $E_{c.m.} = 2.0$ MeV. The uncertainties were even larger for n-p scattering phase shifts (on the order of 100%, 240%, and 300%, respectively). Measurements of $A_y$ and $iT_{11}$ for d-p scattering at $E_{c.m.} = 667$ keV could show the energy dependence of the $^3P_J$ phase shifts at an energy where NN scattering data do not exist.

1.5 Or Is the Problem in 3N forces?

The alternative solution to the "$A_y$ Puzzle" comes from the existence of a 3NF. There is already a need to include a phenomenological 3N potential beyond the phenomenological NN potentials to arrive at the correct $^3$He and $^3$H binding energies. The development of 3NF has been very recent (within the last ten years). The study of the 3NF has much room to grow as was the case of the NN forces some thirty years ago. Hüber and Friar [Hüb98] suggested that the solution to the "$A_y$ Puzzle" has to be a 3NF since their changes to the NN $^3P_J$ phase shifts improved the 3N $A_y$ description but altered the predictions of NN observables unsatisfactorily. One new 3NF term have been proposed in the form of a phenomenological $\vec{L} \cdot \vec{S}$ force by Kievsky [Kie99]. The force constructed by Kievsky improves the description of the VAP which will be discussed later in Section 6.2. The goal is to gain a more physical understanding of the 3NF instead of adjusting parameters to fix the binding energies.

The VAP problem is the strongest evidence that something is missing in our understanding of NN and 3N interactions. The current theoretical methods are capable of calculating the partial waves present in the system, how these wave-functions are shifted from their initial state, and how much the partial waves mix with one another. With the complete set of analyzing powers and cross-section data in this dissertation, a phase-shift analysis is also conducted to elucidate the degree of shifting from the initial state as well as the strength of the mixing. By comparing the two calculations (one from experimental data and the other
from theory), the deficient wavefunctions are isolated. From this knowledge, new physics can be developed to fix the deficiency and in the process, learn more about basic nuclear dynamics.
Chapter 2

Analyzing Power Formalism

The strong nuclear force has the curious feature of being a spin-dependent force. The $\vec{L} \cdot \vec{S}$ force is a cornerstone of the nuclear Shell Model. The tensor portion has the form $S_{ij} = 3(\sigma_i \cdot \hat{r})(\sigma_j \cdot \hat{r}) - \vec{\sigma} \cdot \vec{\sigma}$. For nuclear scattering and reactions, the spin dependence must be taken into consideration. Since both nucleons and nuclei carry some angular momentum $\vec{L}$ and intrinsic spin $\vec{S}$, a polarized beam of particles will scatter from a nuclear target with an angular dependence on azimuthal angle $\phi$. A detailed description of working with polarization is given in [Ohl73].

2.1 What is Polarization?

A beam is a grouping of a given number of particles generally moving in the same direction. The beam is unpolarized if there are equal populations of particles in each spin substate $m_S$. Since protons have $\vec{S} = \frac{1}{2}$ which gives substates of $m_S = +\frac{1}{2}$ (spin up) and $m_S = -\frac{1}{2}$ (spin down), an unpolarized proton beam has half of its constituents spin up and half spin down. The beam is polarized when a majority of the constituents sits in one substate.

For a beam of spin-$\frac{1}{2}$ particles such as protons, only a vector polarization is defined. Since most polarized-ion sources like the ABPIS at TUNL are cylindrically symmetric, the polarization is given with respect to the z-axis or the
direction of the momentum of the incident beam. The vector polarization is denoted as \( p_z \) and is defined as

\[
P_z = N_+ - N_-, \tag{2.1}
\]

where \( N_+ \) and \( N_- \) are the fractional populations of particles in the \( m_S = +\frac{1}{2} \) and \( m_S = -\frac{1}{2} \) substates, respectively.

Deuterons are spin-1 particles and therefore exist in three substates \( m_S = +1, -1, \) and 0. With the +1 and -1 substates, a deuteron beam can be vector polarized similar to the proton beam and will have the same expression for \( p_z \) as in equation 2.1. With the inclusion of the third substate \( (0) \), a deuteron beam can have a tensor polarization as well. The tensor polarization is labelled as \( p_{zz} \) and is defined as

\[
P_{zz} = N_+ + N_- - 2N_0, \tag{2.2}
\]

where \( N_+ \), \( N_- \), and \( N_0 \) are the fractional populations of particles in the \( m_S = +1, -1, \) and 0 substates, respectively. Since \( N_+ + N_- + N_0 = 1 \) by definition, equation 2.2 is simplified to

\[
P_{zz} = 1 - N_0 - 2N_0 = 1 - 3N_0. \tag{2.3}
\]

With equations 2.1 and 2.3, the ranges of the vector and tensor polarizations are

\[-1 \leq p_z \leq +1, \quad -2 \leq p_{zz} \leq +1. \tag{2.4}\]

At most accelerator facilities, there are invariably numerous magnets to transport the beam. These magnets might also precess the spin axis. It is most useful to define the coordinate system for the spin axis at the target for which we employ the Madison Convention [Mad71], shown in Figure 2.1. The z-axis is defined as the direction of the incident beam momentum \( \hat{k}_{in} \). The y-axis is given as the cross product \( \hat{k}_{in} \times \hat{k}_{out} \) where \( \hat{k}_{out} \) is the direction of the momentum of the scattered particle. Finally, the x-axis is defined to make a right-handed coordinate system.
It is interesting that the y-axis is dependent on the direction of the scattered particle. The spin axis $\vec{S}$ is established by two angles $\beta$ and $\phi$. The angle $\beta$ is the angle between $\vec{S}$ and the z-axis while $\phi$ is the angle between the projection of $\vec{S}$ in the xy-plane and the y-axis. The angle $\phi$ is defined to be positive and increasing.

Assuming that parity is conserved, an equation describing the scattering of reaction process with a polarized beam can be expanded to include $p_z$, $p_{zz}$, $\beta$, and $\phi$. The general equation for spin-$\frac{1}{2}$ particles is given as [Ohl73]

$$I^{(i)}(\theta, \beta, \phi) = I_o(\theta) \left\{ 1 + p_z^{(i)} A_y(\theta) \sin \beta \cos \phi \right\}, \quad (2.5)$$

where $I^{(i)}(\theta, \beta, \phi)$ is the number of counts detected in $\theta$ for a given $(\beta, \phi)$ and beam polarization, and $I_o(\theta)$ is the number counts detected for the same setup with an unpolarized beam. The superscript $i$ denotes the polarization states 2 and 3 which represent the up and down states, respectively. The unpolarized
state or state 1 is denoted by the subscript \( o \) for historical reasons. For deuteron beams and tensor polarization, the general count rate equation is [Ohl73]

\[
I^{(i)}(\theta, \beta, \phi) = L_o(\theta) \left\{ 1 + \frac{3}{2} p_{zz}^{(i)} A_y(\theta) \sin \beta \cos \phi - p_{zz}^{(i)} A_{xx}(\theta) \sin \beta \cos \beta \sin \phi \\
+ \frac{1}{2} p_{zz}^{(i)} A_{yy}(\theta) \sin^2 \beta \cos 2\phi \\
+ \frac{1}{4} p_{zz}^{(i)} A_{zz}(\theta) \left[ 3 \cos^2 \beta - 1 + \sin^2 \beta \cos 2\phi \right] \right\},
\]

(2.6)

where the variables have the same meaning as in equation 2.5. The \( A \)'s are the analyzing powers which show the change in the cross section when polarized beams are employed. Each analyzing power can be isolated by selecting a specific \( \beta \) and detector geometry (which determines \( \phi \)). In equations 2.5 and 2.6, there is a vector analyzing power \( A_y \). Although the notation is the same, these quantities are not equal for proton and deuteron beams. In the discussion of \( A_y \), different expressions will be derived for each beam type.

The analyzing powers can be obtained by two different algebraic methods labelled Pol-Unpol and Pol-Pol. In the Pol-Unpol configurations, ratios of the polarized yields to the unpolarized yields are calculated to determine the analyzing powers. In the Pol-Pol method, the unpolarized yield is divided out when a ratio of the state 2 to state 3 yields are created. The results are the same for both Pol-Unpol and Pol-Pol methods. If all three polarization states are used, the desired analyzing power can be determined in two different manners as a cross check. To explain this point further, an expression for each analyzing power will be reproduced for both the Pol-Unpol and Pol-Pol cases.

### 2.2 Extraction of \( A_{zz} \)

If the spin axis is aligned with the direction of the incident beam in the reaction plane, \( \beta = 0^\circ \) and \( \phi \) is undefined. In this arrangement, equation 2.6
reduces to

\[
I^{(i)}(\theta, 0^\circ, \phi) = I_o(\theta) \left\{ 1 + \frac{1}{2} p^{(i)}_{zz} A_{zz}(\theta) \right\}.
\]  

\[ \text{(2.7)} \]

### 2.2.1 Pol-Unpol Method for \( A_{zz} \)

For a pair of detectors placed symmetrically on the left and right of the beam direction, ratios of counts can be made for each detector as follows

\[
L^{(i)} = \frac{I^{(i)}(\theta, 0^\circ, \phi)}{I_o(\theta)} = 1 + \frac{p^{(i)}_{zz} A_{zz}(\theta)}{2} \quad \text{and}
\]

\[
R^{(i)} = \frac{I^{(i)}(\theta, 0^\circ, \phi)}{I_o(\theta)} = 1 + \frac{p^{(i)}_{zz} A_{zz}(\theta)}{2}.
\]  

\[ \text{(2.8)} \]

From these equations, \( A_{zz} \) can be measured with a single detector or both. The statistics of each detector can be added to give

\[
A_{zz}^{(i)}(\theta) = \frac{L^{(i)} + R^{(i)} - 2}{p^{(i)}_{zz}}.
\]  

\[ \text{(2.9)} \]

Finally, \( A_{zz} \) is the only analyzing power for deuteron beams which can be non-zero at \( \theta = 0^\circ \). Equation 2.8 can be used for the \( 0^\circ \) angle. From equation 2.8, the ratio of counts for both state 2 and state 3 are dependent on the unpolarized counts. This means that \( L^{(2)} \) is correlated with \( L^{(3)} \) when \( A_{zz}^{(2)} \) and \( A_{zz}^{(3)} \) are averaged. The same statement is true for \( R^{(2)} \) and \( R^{(3)} \). To avoid this correlation of measurements, another formalism is obtained which utilizes only state 2 and state 3 polarization states.

### 2.2.2 Pol-Pol Method for \( A_{zz} \)

For the Pol-Unpol measurements, all three substates are needed. However, \( A_{zz} \) can be measured without the unpolarized state (state 1). Utilizing only states 2 and 3 will be called the Pol-Pol method. Writing out equation 2.7 for both states 2 and 3 gives

\[
I^{(2)}(\theta, 0^\circ, \phi) = I_o(\theta) \left\{ 1 + \frac{1}{2} p^{(2)}_{zz} A_{zz}(\theta) \right\} \quad \text{and}
\]

\[
I^{(3)}(\theta, 0^\circ, \phi) = I_o(\theta) \left\{ 1 + \frac{1}{2} p^{(3)}_{zz} A_{zz}(\theta) \right\}.
\]  

\[ \text{(2.10)} \]
The ratio of the right-detector counts is defined as

\[
    r = \frac{I^{(2)}(\theta, 0^\circ, \phi)}{I^{(3)}(\theta, 0^\circ, \phi)} = \frac{2 + p^{(2)}_{zz} A_{zz}(\theta)}{2 + p^{(3)}_{zz} A_{zz}(\theta)}. \tag{2.11}
\]

After some simple algebraic gymnastics, a solution for \( A_{zz} \) is

\[
    A_{zz}(\theta) = \frac{2(r - 1)}{p^{(2)}_{zz} - r p^{(3)}_{zz}}. \tag{2.12}
\]

A similar expression for the left detector can be found with the ratio \( l \).

While it may not be evident to the reader, the Pol-Pol formalism was preferred for the d-p scattering experiments. By reducing the time spent in the unpolarized state, the polarized yields were accumulated faster than if all three polarization states had equal time. Therefore, the measurements were conducted in a shorter period of time. Another advantage to the Pol-Pol method is that the measurements have slightly smaller statistical uncertainties. The error in the Pol-Unpol method is dependent on the errors in the yields of all three states. The Pol-Pol uncertainty is dependent on the uncertainties in the yields of state 2 and state 3. For these reasons, the time the beam was in state 1 was reduced for this research.

2.3 Extraction of \( A_{xz} \) and \( A_{yy} \) at \( \beta = 45^\circ \)

2.3.1 Pol-Unpol Method

If the spin axis is \( 45^\circ \) with the incident beam direction and in the reaction plane, then \( \beta \) is \( 45^\circ \). The placement of the left and right detectors depends on whether the spin axis is \( 45^\circ \) to the left or right of the z-axis. If the spin axis is to the right, \( \phi = 90^\circ \) and \( 270^\circ \) define left and right detectors, respectively. For \( \phi = 90^\circ \) and \( \beta = 45^\circ \), equation 2.6 becomes

\[
    I^{(i)}(\theta, 45^\circ, 90^\circ) = I_o(\theta) \left\{ 1 - \frac{p^{(i)}_{zz} A_{zz}(\theta)}{2} - \frac{p^{(i)}_{zz} A_{yy}(\theta)}{4} \right\}. \tag{2.13}
\]
For $\phi = 270^\circ$ and $\beta = 45^\circ$, we have

$$I^{(i)}(\theta, 45^\circ, 270^\circ) = I_o(\theta) \left\{ 1 + \frac{p_{zz}^{(i)} A_{zz}(\theta)}{2} - \frac{p_{zz}^{(i)} A_{yy}(\theta)}{4} \right\}. \quad (2.14)$$

Again, defining the ratios of polarized yields to unpolarized yields provides the expressions

$$L^{(i)} = I^{(i)}(\theta, 45^\circ, 90^\circ)/I_o(\theta) = 1 - \frac{p_{zz}^{(i)} A_{xx}(\theta)}{2} - \frac{p_{zz}^{(i)} A_{yy}(\theta)}{4}$$

and

$$R^{(i)} = I^{(i)}(\theta, 45^\circ, 270^\circ)/I_o(\theta) = 1 + \frac{p_{zz}^{(i)} A_{xx}(\theta)}{2} - \frac{p_{zz}^{(i)} A_{yy}(\theta)}{4}. \quad (2.15)$$

Subtracting $R^{(i)}$ from $L^{(i)}$ yields

$$A_{xx}^{(i)}(\theta) = \frac{L^{(i)} - R^{(i)}}{p_{zz}^{(i)}}. \quad (2.16)$$

If the spin axis is $45^\circ$ to the left of the incident beam direction, $A_{xx}$ has an opposite sign. If $R^{(i)}$ and $L^{(i)}$ are added, the $A_{xx}$ term disappears, and we are left with

$$A_{yy}^{(i)}(\theta) = \frac{-2(L^{(i)} + R^{(i)}) - 2}{p_{zz}^{(i)}}. \quad (2.17)$$

However, it has been shown that $A_{yy}$ determined in this manner at $\beta = 45^\circ$ is very sensitive to misalignments of the spin axis [Gei98].

### 2.3.2 Pol-Pol Method

Exact solutions to $A_{xx}$ and $A_{yy}$ at $\beta = 45^\circ$ can be determined using only states 2 and 3. By defining the detector ratios $l = I^{(2)}(\theta, 45^\circ, 90^\circ)/I^{(3)}(\theta, 45^\circ, 90^\circ)$ and $r = I^{(2)}(\theta, 45^\circ, 270^\circ)/I^{(3)}(\theta, 45^\circ, 270^\circ)$, equations 2.13 and 2.14 become

$$l = \frac{4 - 2p_{zz}^{(2)} A_{xx}(\theta) - p_{zz}^{(2)} A_{yy}(\theta)}{4 - 2p_{zz}^{(3)} A_{xx}(\theta) - p_{zz}^{(3)} A_{yy}(\theta)}$$

$$r = \frac{4 + 2p_{zz}^{(2)} A_{xx}(\theta) - p_{zz}^{(2)} A_{yy}(\theta)}{4 + 2p_{zz}^{(3)} A_{xx}(\theta) - p_{zz}^{(3)} A_{yy}(\theta)}. \quad (2.18)$$

After much algebraic twisting and turning, the following expressions appear

$$A_{xx}(\theta) = \left[ \frac{r - 1}{p_{zz}^{(2)} - rp_{zz}^{(3)}} \right] - \left[ \frac{l - 1}{p_{zz}^{(2)} - lp_{zz}^{(3)}} \right]$$

and

$$\text{(2.19)}$$
\[ A_{yy} (\theta) = -2 \left[ \frac{r - 1}{p_{zz}^{(2)} - r p_{zz}^{(3)}} + \frac{l - 1}{p_{zz}^{(2)} - l p_{zz}^{(3)}} \right]. \]  

(2.20)

These expressions have the advantage that the correlation due to the unpolarized state is eliminated. For \( A_{yy} \), the sensitivity to misalignments of the spin orientation lead us to an alternative means of determining \( A_{yy} \), i.e. rotating the spin axis so \( \beta = 90^\circ \).

### 2.4 Extraction of \( A_y \) and \( A_{yy} \) at \( \beta = 90^\circ \)

**2.4.1 Pol-Unpol Method**

Another determination of \( A_{yy} \) makes use of \( \beta = 90^\circ \) measurements. The spin axis is perpendicular to the reaction plane and oriented “up” with respect to a left/right pair of detectors. The left detector has \( \phi = 0^\circ \) and the right detector has \( \phi = 180^\circ \). Inserting these values for \( \beta \) and \( \phi \) into equation 2.6 gives

\[
I^{(i)} (\theta, 90^\circ, 0^\circ) = I_o (\theta) \left\{ 1 + \frac{3}{2} p_{zz}^{(i)} A_y (\theta) + \frac{1}{2} p_{zz}^{(i)} A_{yy} (\theta) \right\} \quad \text{and} \\
I^{(i)} (\theta, 90^\circ, 180^\circ) = I_o (\theta) \left\{ 1 - \frac{3}{2} p_{zz}^{(i)} A_y (\theta) + \frac{1}{2} p_{zz}^{(i)} A_{yy} (\theta) \right\}. \quad (2.21)
\]

Following the convention of \( L^{(i)} \) and \( R^{(i)} \), we have the simpler forms of

\[
L^{(i)} = I^{(i)} (\theta, 90^\circ, 0^\circ) / I_o (\theta) = 1 + \frac{3 p_{zz}^{(i)} A_y (\theta)}{2} - \frac{p_{zz}^{(i)} A_{yy} (\theta)}{2} \quad \text{and} \\
R^{(i)} = I^{(i)} (\theta, 90^\circ, 180^\circ) / I_o (\theta) = 1 - \frac{3 p_{zz}^{(i)} A_y (\theta)}{2} - \frac{p_{zz}^{(i)} A_{yy} (\theta)}{2}. \quad (2.22)
\]

When the combination of \( L^{(i)} - R^{(i)} \) is taken, \( A_y \) appears as

\[
A_y^{(i)} (\theta) = \frac{L^{(i)} - R^{(i)}}{3 p_{zz}^{(i)}}. \quad (2.23)
\]

The quantity \( A_{yy} \) is formulated into

\[
A_{yy}^{(i)} (\theta) = \frac{L^{(i)} + R^{(i)} - 2}{p_{zz}^{(i)}} \quad (2.24)
\]

when \( L^{(i)} \) and \( R^{(i)} \) are added.
2.4.2 Pol-Pol Method

The extraction of $A_y$ and $A_{yy}$ is much more difficult with only states 2 and 3 than the other analyzing powers previously discussed. Making the ratios

$$l = \frac{I^{(2)}(\theta, 90^\circ, 0^\circ)}{I^{(3)}(\theta, 90^\circ, 0^\circ)} \quad \text{and} \quad r = \frac{I^{(2)}(\theta, 90^\circ, 180^\circ)}{I^{(3)}(\theta, 90^\circ, 180^\circ)}, \quad (2.25)$$

and $r$ is subtracted from $l$, we achieve a single expression of the form

$$A_y = \frac{\frac{2}{3}(l - r)[1 + \frac{1}{3}p_{zz}^{(3)}A_{yy}(\theta)]}{2p_{zz}^{(2)} - p_{zz}^{(3)}(l + r)}. \quad (2.26)$$

Unfortunately, $A_y$ in the Pol-Pol method is dependent on the tensor polarization $p_{zz}^{(3)}$ and $A_{yy}$. When most polarized ion sources maximize $p_z$, they also produce a small amount of $p_{zz}$. The ABPIS at TUNL provides on average $p_{zz} < 5\%$ when $p_z$ is optimized. Luckily, for $^1\text{H}(d, d)^1\text{H}$ scattering, $A_{yy}$ is on the order of $10^{-2}$, and the product $p_{zz}A_{yy}$ is much less than 1. Thus, the following approximation to equation 2.26 can be made

$$A_y \approx \frac{\frac{2}{3}(l - r)}{2p_{zz}^{(2)} - p_{zz}^{(3)}(l + r)}. \quad (2.27)$$

Similarly, a solution for $A_{yy}$ is found when the $r$ and $l$ equations are added, and $A_{yy}$ is isolated

$$A_{yy} = \frac{2(l + r) + 3p_{zz}^{(3)}A_y(\theta) - 4}{2p_{zz}^{(2)} - p_{zz}^{(3)}(l + r)}. \quad (2.28)$$

Like the $A_y$ case and its dependence on $p_{zz}$, $A_{yy}$ by the Pol-Pol method cannot be separated from its vector counterpart. However, with the approximation that $p_zA_y$ is much less than $A_{yy}$, equation 2.28 becomes

$$A_{yy} \approx \frac{2(l + r - 2)}{2p_{zz}^{(2)} - p_{zz}^{(3)}(l + r)}, \quad (2.29)$$

which is a much more useful formula.

2.5 Proton beams and $A_y$

The experimental setup for $A_y$ with polarized proton beams is the same as in the case of vector-polarized deuteron beams in Section 2.4. The spin axis
is oriented normal to the reaction plane at $\beta = 90^\circ$. A left and right detector pair defines $\phi = 0^\circ$ and $180^\circ$, respectively. With this information, equation 2.5 simplifies to

$$I^{(i)}(\theta, 90^\circ, 0^\circ) = I_o(\theta)\left\{1 + p_z^{(i)}A_y(\theta)\right\}$$

and

$$I^{(i)}(\theta, 90^\circ, 180^\circ) = I_o(\theta)\left\{1 - p_z^{(i)}A_y(\theta)\right\}.$$  \hspace{1cm} (2.30)

### 2.5.1 Pol-Unpol Method

If all three states are used, detector yield ratios are defined as

$$L^{(i)} = \frac{I^{(i)}(\theta, 90^\circ, 0^\circ)}{I_o(\theta)}$$ and

$$R^{(i)} = \frac{I^{(i)}(\theta, 90^\circ, 180^\circ)}{I_o(\theta)},$$  \hspace{1cm} (2.31)

and equation 2.30 is rewritten as

$$L^{(i)} = 1 + p_z^{(i)}A_y(\theta)$$ and

$$R^{(i)} = 1 - p_z^{(i)}A_y(\theta).$$  \hspace{1cm} (2.32)

The analyzing power $A_y$ takes the form

$$A_y^{(i)}(\theta) = \frac{L^{(i)} - 1}{p_z^{(i)}}$$ and

$$A_y^{(i)}(\theta) = -\frac{R^{(i)} - 1}{p_z^{(i)}}.$$  \hspace{1cm} (2.33)

Unlike the $A_y$ from the deuteron beam experiment, only a single detector is necessary. To utilize the statistics of a pair of detectors, a $L^{(i)} - R^{(i)}$ combination from equation 2.32 gives

$$A_y^{(i)}(\theta) = \frac{L^{(i)} - R^{(i)}}{2p_z^{(i)}}.$$  \hspace{1cm} (2.34)

### 2.5.2 Pol-Pol Method

The Pol-Pol formalism is a more favorable setup. For a polarized proton beam, the "unpolarized state" or state 1 is created by turning off the transition
units at the source. However, state 1 retains a measurable amount of polarization even with the transition units off due to the magnetic moment of the proton and residual magnetic fields of the sextupole magnets at the polarized-ion source. The effect is negligible for deuteron beams since the magnetic moment of the deuteron is much smaller than the proton. Thus, it is better to avoid state 1 altogether with proton beams.

When ratios of state 2 and state 3 yields are made, equation 2.30 transforms to

\[
\begin{align*}
  l &= \frac{1 + p_z^{(2)} A_y(\theta)}{1 + p_z^{(3)} A_y(\theta)} \\
  r &= \frac{1 - p_z^{(2)} A_y(\theta)}{1 - p_z^{(3)} A_y(\theta)}.
\end{align*}
\] (2.35)

This equation yields the following results

\[
\begin{align*}
  A_y(\theta) &= \frac{l - 1}{p_z^{(2)} - l p_z^{(3)}} \quad \text{and} \\
  A_y(\theta) &= -\frac{(r - 1)}{p_z^{(2)} - r p_z^{(3)}}.
\end{align*}
\] (2.36)

As in the Pol-Unpol scenario, only one detector is needed.

The vector polarizations can be solved exactly from equation 2.35 and have the following forms

\[
\begin{align*}
  p_z^{(2)} &= \frac{2rl - l - r}{A_y(r - l)} \quad \text{and} \\
  p_z^{(3)} &= \frac{(l + r - 2)}{A_y(r - l)}. \quad (2.37)
\end{align*}
\]

With this equation, \( p_z \) can be measured without state 1. An alternative form of the vector polarization is \( \overline{p}_z = (p_z^{(3)} - p_z^{(2)})/2 \) and \( \delta p_z = (p_z^{(3)} - p_z^{(2)})/2 \). When \( p_z^{(3)} \) and \( p_z^{(2)} \) are substituted into these equations, the full expressions are

\[
\begin{align*}
  \overline{p}_z &= \frac{(r - 1)(l - 1)}{A_y(l - r)} \quad \text{and} \\
  \delta p_z &= \frac{rl - 1}{A_y(r - l)}. \quad (2.38)
\end{align*}
\]
Conversely, the dependence of $A_y$ on $\overline{p}_z$ and $\delta p_z$ is

\[ A_y = \frac{(r - 1)(l - 1)}{\overline{p}_z(l - r)} \quad \text{and} \quad A_y = \frac{r l - 1}{\delta p_z(r - l)} \quad \text{for} \quad r > l. \quad (2.39) \]

For the experiment in this project, $\overline{p}_z$ was employed and not $\delta p_z$. When the magnitudes of $p_z^{(2)}$ and $p_z^{(3)}$ are close in value as was the case in my measurement, $\delta p_z$ is close to zero, and a large number of counts is required to achieve reasonable errors. Therefore, $\overline{p}_z$ was utilized to calculate $A_y$ in this work.

### 2.6 Spherical Tensor Analyzing Powers

The analyzing powers denoted by the $A$'s are in a Cartesian coordinate system. For a phase-shift analysis and comparison to theoretical calculations, a spherical coordinate system is more convenient and traditional. For spin-1 particles, the conversion from Cartesian to spherical analyzing powers is

\[ i T_{11} = \frac{\sqrt{3}}{2} A_y, \quad (2.40) \]

\[ T_{20} = \frac{1}{\sqrt{2}} A_{zz}, \quad (2.41) \]

\[ T_{21} = -\frac{1}{\sqrt{3}} A_{xz}, \quad \text{and} \quad (2.42) \]

\[ T_{22} = -\frac{1}{2\sqrt{3}} (A_{zz} + 2A_{yy}). \quad (2.43) \]
Chapter 3

Experimental Procedure

As stated in the previous chapter, the goal of the research project is to measure angular distributions for $\sigma(\theta)$, $A_y$, $iT_{11}$, $T_{20}$, $T_{21}$, and $T_{22}$ for the d-p elastic scattering. In designing the most efficient experiment for each observable, several different factors had to be taken into account. Such factors include choosing the detectors to maximize resolution, selecting foils to exclude particles from reactions with target contaminants, and determining which reaction to use for beam polarimetry. A critical concern was the small magnitude of the analyzing powers observed in the angular distributions. This necessitated taking a large number of counts at each angle, and careful consideration was needed to determine techniques to maximize count rate. The measurements are arranged in three groups with similar experimental techniques: cross-section, tensor analyzing powers, and vector analyzing powers.

To illustrate how the experiments needed careful planning, Figures 3.1 is a plot of the kinematics of a 2 MeV deuteron beam on a hydrogen target. This figure shows the expected energy of the outgoing particle with respect to the angular position in the laboratory. Since there was $^{12}$C in the target, numerous $^{12}$C$(d, p)^{13}$C* reactions are present. There are certain angle settings where the outgoing protons have the same energy as the protons and deuterons from d-p scattering. Another interesting feature of $^1$H$(d, d)^1$H elastic scattering is that the
Figure 3.1: Kinematics of reaction products for a 2 MeV deuteron beam on a hydrogenated carbon target.

deuteron can only be detected at laboratory angles $\leq 30^\circ$ as seen in the plot.

For each experiment, the beam was accelerated with the 10-MV FN tandem accelerator at TUNL and bent by a dipole magnet into the 62-cm scattering chamber on the 52° beamline. The dipole magnet and a feedback system with FN tandem kept the beam energy to within $\pm 8$ keV [Fis97] as determined by the calibration of the magnet system. The magnet was calibrated just prior to the start of this project. The calibration was obtained by increasing the magnetic field and measuring the yield of the reactions in Table 3.1 over the given resonances. The magnet constant $K$ was extracted by the following expression

$$K = \frac{E m}{f^2 q^2} \left(1 + \frac{E}{2mc^2}\right),$$

(3.1)

where $f$ is the NMR frequency which sets the magnetic field and $E$ is the kinetic energy of a particle of mass $m$ and charge $q$. The new magnet constant was
found to be 13.182 ± 0.005 [Fis97]. This value was used to set the magnet B-field throughout the experiments.

Two different beam sources were used depending on whether the beam was unpolarized or polarized. The Direct Extraction Negative Ion Source (DENIS) produced unpolarized beams of deuterons and protons for the cross-section measurements. For the analyzing power experiments, the Atomic-Beam Polarized Ion Source (ABPIS) [Cle95] provided the polarized deuteron or proton beams. The ABPIS has a Wien filter which rotated the spin axis to give the desired orientation on target. The average beam intensity on target for each experiment was approximately 200 nA. The typical beam spot on a target after an experiment was approximately 2 mm x 3 mm.

More specific details for each experiment are provided below.

### 3.1 Cross-section Measurements

#### 3.1.1 Relative Angular Distribution

Experimentally, the differential cross-section is

\[
\sigma(E, \theta) = \frac{I_0(\theta)}{N_b n_t \epsilon \Delta \Omega},
\]

where \(I_0(\theta)\) is the number of scattered particles, \(N_b\) is the number of incident particles, \(n_t\) is the number of target atoms per cm², \(\epsilon\) is the detector efficiency, and \(\Delta \Omega\) is the solid angle. To accurately determine the cross-section, each one of
the quantities had to be measured precisely. In these experiments, the detectors were silicon surface-barrier detectors and the efficiencies were $100 \pm 0.1\%$ [Zab80] with the uncertainty arising from inelastic scattering from $^{28}$Si. Beam currents were obtained by collecting the charge with a Faraday cup placed at the back of the scattering chamber while the target ladder was grounded. However, knowing the target thickness was a problem since target material scattered out of the targets. The solution to this problem was to measure an angular distribution of the relative cross-section and to normalize this measurement to a known reaction. This process bypassed an actual measurement of the target thickness and solid angle.

As stated in the introduction, the relative cross-section experiment was conducted in the 62-cm scattering chamber on the 52$^\circ$ beamline at TUNL. A beam of deuterons produced by DENIS was accelerated by the FN tandem accelerator to give an energy in the center of the target of 2.00 MeV and directed down the 52$^\circ$ beamline. We decided to measure $^1$H($d, d$)$^1$H because a normalization scheme was available which will be described in the Section 3.1.2. The relative cross-section for the reverse scattering $^2$H($p, p$)$^2$H at $E_p = 1.00$ MeV was measured as a cross-check, but at this time, is not normalized. Both relative cross-sections were measured in the same manner although the $^1$H($d, d$)$^1$H experiment is explained in the next paragraph.

A schematic of the experimental setup is shown in Figure 3.2. The target ladder held three thin hydrogenated carbon foils (refer to Appendix A), one 10 $\mu$g/cm$^2$ carbon foil, and an empty target frame. The empty frame allowed the beam to travel directly to the Faraday cup for tuning purposes. The hydrogenated carbon foils contained approximately $0.5 \times 10^{18}$ H/cm$^2$ and $1.0 \times 10^{18}$ C/cm$^2$ and had to be thin to allow the scattered and recoiled particles to escape with the least amount of energy lost. Thin targets also keep energy loss and divergence of the beam to a minimum. From the stopping powers [And77] of $^1$H and $^{12}$C and their abundances in the target, the energy loss was calculated to
be approximately 5 keV. Therefore, to have the beam energy at the center of the target to be 2.000 MeV, the terminal voltage of the tandem accelerator was set to accelerate the beam to 2.002 MeV. The elastically-scattered deuterons and recoiled protons were counted in two pairs of silicon surface-barrier detectors spaced 10.0° apart and placed symmetrically with respect to the beam direction. The detectors were rotated to give an angular range of $\theta_{lab} = 7.0^\circ$ to $64.0^\circ$ ($\theta_{c.m.} = 21.0^\circ$ to $166.0^\circ$). Table 3.2 shows the nominal thickness and angular acceptance of each detector. Another two pairs of silicon detectors were mounted to the chamber wall and remained stationary throughout the experiment. The front pair was set at $15.0^\circ$ while the rear pair was at $42.0^\circ$. The counts in the latter four detectors were used to normalize the yields in the rotating detectors to account for deviations of the beam position on the target as well as target deterioration. A description of the monitor detectors is also given in Table 3.2.

The carbon foil was employed to calibrate the zero-angle position of rotating detectors in the chamber at the beginning of the experiment. The right pair of detectors measured the yield of elastically scattered deuterons from $^{12}\text{C}(d,d)^{12}\text{C}$ in the angular range of $35.0^\circ$ to $-35.0^\circ$ in $5.0^\circ$ steps while the left pair remained fixed. The same process was repeated for the left pair while the right pair stayed
Table 3.2: Summary of detector thickness and geometry for the relative cross-section measurements.

<table>
<thead>
<tr>
<th>Detector</th>
<th>Nominal thickness (μm)</th>
<th>Slit size HxV (mm)</th>
<th>Solid angle (msr)</th>
</tr>
</thead>
<tbody>
<tr>
<td>R1, L1</td>
<td>100</td>
<td>0.8 × 9.5</td>
<td>0.18</td>
</tr>
<tr>
<td>R2, L2</td>
<td>300</td>
<td>1.6 × 9.5</td>
<td>0.37</td>
</tr>
<tr>
<td>RMON1, LMON1</td>
<td>100</td>
<td>1.6 × 9.5</td>
<td>0.26</td>
</tr>
<tr>
<td>RMON2, LMON2</td>
<td>300</td>
<td>1.6 × 9.5</td>
<td>0.26</td>
</tr>
</tbody>
</table>

Table 3.3: List of the angle offsets for the four chamber detectors in the 52° scattering chamber. The second pair of detectors was 10° behind the first pair.

<table>
<thead>
<tr>
<th>Detector</th>
<th>Angle offset</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>R1</td>
<td>0.125°</td>
<td>0.003°</td>
</tr>
<tr>
<td>L1</td>
<td>0.199°</td>
<td>0.003°</td>
</tr>
<tr>
<td>R2</td>
<td>0.108°</td>
<td>0.002°</td>
</tr>
<tr>
<td>L2</td>
<td>0.142°</td>
<td>0.002°</td>
</tr>
</tbody>
</table>

fixed. The yields were fit with the formula for Rutherford scattering to determine a dependence of yield versus angle. The values of the offsets are summarized in Table 3.3.

The signals from each detector were processed by an Ortec 142 preamplifier at the chamber before being amplified and shaped by the electronics in the control room. A schematic of the electronics setup is displayed in Figure 3.3. The unipolar output of the amplifier was sent directly to a Linear Gate & Stretcher (LGS) where it was shaped for processing as an energy signal. The bipolar signal from the amplifier was sent to a Timing Single-Channel Analyzer (TSCA) to create a TTL gate. At this point, the TTL gate and linear energy signals are square pulses which was what we needed for the type of Analog-to-Digital Converter (ADC) that was utilized. We used Northern NS621 ADCs which are peak-sensing ADCs and have conversion rates of 50 MHz. Six Northern
ADCs were available so the RMON1 (RMON2) and LMON1 (LMON2) detector signals were routed into individual ADCs. The ADCs communicated with a VAX computer by means of a CAMAC crate and a 16-bit microcomputer called a Microprogrammed Branch Driver. The VAX computer stored the digitized signals into spectra with a series of TUNL XSYS routines. A sample spectrum for $^1$H($d,d)^1$H scattering at $\theta_{lab} = 26.0^\circ$ is shown in Figure 3.4. Scaler signals were also sent to the CAMAC scalers and, subsequently, to the computer which were used to calculate the dead-time of the electronics. The scalers registered the signals from a beam-current integrator, a pulser, and a clock.

### 3.1.2 Absolute Normalization

The absolute cross-section was achieved by normalizing the relative $\sigma(\theta)$ for d-p scattering to the well-known $^1$H($p,p)^1$H cross-section. The fact that DENIS can produce proton and deuteron beams simultaneously is the key to making this part of the experiment possible. The scattering chamber was left with the same setup as for the relative measurement, and four new hydrogenated carbon foils and a Au-target were put on the target ladder. The detectors on the movable plates remained stationary at $25.0^\circ$ and $35.0^\circ$. These angles provided three normalization points $^1$H($d,d)^1$H and $^1$H($d,p)^2$H for $25.0^\circ$ and $^1$H($d,p)^2$H
for 35.0°. One target was used as a practice target for tuning the deuteron beam at $E_d = 2.0$ MeV. Once a good beam tune was obtained, the three remaining targets were cycled through the beam. Each target remained in the beam for a very short period of time ($\approx 8 \mu C$) to reduce target deterioration. Only if 4.000 MeV protons and 2.000 MeV deuterons have the same magnetic rigidity will both beams hit approximately the same spot on the target. This was accomplished by adjusting the dipole magnet after the source and raising the FN tandem terminal voltage. After the proton beam was put on target, the three targets were cycled as in the deuteron beam case. This process of switching from deuteron beam to proton beam back to deuteron beam was repeated four times in order to understand the degradation of the hydrogenated carbon targets. The
beam-switching was performed on the order of $\approx 1 - 2$ minutes. From these tests, approximately 1% of the target atoms were depleted for each short run.

The Au target was utilized to determine systematic problems such as inaccurate beam current integration. At incident energies of $E_d = 2.0$ MeV and $E_p = 4.0$ MeV, the $^{197}$Au($d,d$)$^{197}$Au and $^{197}$Au($p,p$)$^{197}$Au cross-sections should follow the well-known Rutherford formula

$$
\sigma(\theta_{c.m.}) = \left(\frac{2Z^2e^2}{4\pi\varepsilon_0}\right)^2 \left(\frac{1}{4E_{c.m.}}\right)^2 \frac{1}{\sin^4 \frac{\theta_{c.m.}}{2}}.
$$

For our tests, the detectors pairs were placed at $140.0^\circ$ and $150.0^\circ$. The target was $171 \mu g/cm^2$ of natural Au evaporated on a $10 \mu g/cm^2$ carbon foil. At the end of each cycle of hydrogenated carbon targets, the Au target was placed in the beam (either deuteron or proton). Figure 3.5 is a typical spectrum of $^{197}$Au($d,d$)$^{197}$Au scattering at $\theta_{lab} = 140.0^\circ$. A similar spectrum was found for $^{197}$Au($p,p$)$^{197}$Au scattering. The ratio $^{197}$Au($d,d$)$^{197}$Au and $^{197}$Au($p,p$)$^{197}$Au scattering measure-
ments was found to be within 0.6% of the calculations.

### 3.2 Tensor Analyzing Powers

The dependence on the azimuthal angle $\phi$ is left out of equation 3.2 because it does not play a part in the cross-section when unpolarized beams are present. However, when experiments are conducted with polarized beams, the equation expands to account for $\phi$. The observable $A_{zz}$ is the only analyzing power not dependent on $\phi$. The development of the analyzing powers is discussed in Section 2. This section will describe how each specific analyzing power was measured.

#### 3.2.1 TAPs Measurements

The three TAPS ($A_{zz}, A_{zx}, A_{yy}$) were measured in a mode similar to that used to obtain the differential cross-sections in Section 3.1. The polarized deuteron beam was produced by the ABPIS, and beam polarization was measured online with a polarimeter based on the $^3\text{He}(\vec{d}, p)^4\text{He}$ reaction. The details of the tensor polarimetry are given in Appendix B. Again, three thin hydrogenated carbon foils and an Au target were attached to the target ladder as well as an empty target ring for tuning purposes. The Au target was employed to test on how accurately we could measure the analyzing powers with a tensor-polarized beam. This time, the thickness of the hydrogen targets was $1.0 \times 10^{18}$ H/cm$^2$ and $2.0 \times 10^{18}$ C/cm$^2$. In the polarized beam experiments, three pairs of silicon-surface barrier detectors were positioned symmetrically with the beam direction. A schematic of the chamber setup is shown in Figure 3.6. The front pair (R1, L1) was separated from the middle pair (R2, L2) by 15.0° which in term was separated from the back pair (R3, L3) by 12.0°. These angular spacings were chosen to find angles where the proton peaks of $^{12}\text{C}(\vec{d}, p)^{13}\text{C}^*$ would not overlap in the energy spectrum with the deuteron or proton peaks from d-p scattering. Descriptions of detector properties are given in Table 3.4. Furthermore, thin mylar foils were
placed on the rear collimators at certain angle settings to block any heavy recoils from the target such as carbon and to separate the elastically-scattered deuteron peak from proton peaks (either from $^1\text{H}(d, p)^2\text{H}$ or $^{12}\text{C}(d, p)^{13}\text{C}$ excited states) in the spectra. A list of angles where TAPs data were taken as well as the thickness of mylar attached to the detectors is found in Table 3.5.

The electronics scheme shown in Figure 3.3 and described in Section 3.1 was used to process the detector signals. One addition to the electronics setup was a $^{12}\text{C}(d, d)^{12}\text{C}$ veto for spectra when the detector was placed at very forward angles ($\theta_{lab} < 20.0^\circ$). Figure 3.7 is a diagram of the carbon-veto electronics.
Another complication to the electronics and data acquisition was the need for spin-state switching. To reduce the effects of slow changes in the beam position and amplifier gain, more electronics were assembled to send signals to the ABPIS to switch the transition units every $\frac{1}{3}$ of a second. The transition units at the source polarize the deuteron beam in one of the three spin states. Thus, every second, the deuteron beam has been polarized in sequence in all three states. More on the spin-state switching electronics can be found in [Gei98]. Three spin-state routing bits were sent from the spin-state switching electronics to the Northern ADCs. These signals synchronize the detector signals for beam in a given spin state to the proper detector spectrum in the computer. In this manner, the computer collected three spin-state spectra and a spectrum of the sum of the three spin states for each detector. Since the magnitude of the TAPs is on the order of $10^{-2}$, approximately $4 \times 10^6$ counts in both $^1\text{H}(\vec{d},d)^1\text{H}$ and $^1\text{H}(\vec{d},p)^2\text{H}$ peaks in the sum spectra were collected in order to provide the desired statistical accuracy.

Similar Au targets to those used in the cross-section experiment were used for the TAP experiments. Here, $^{197}\text{Au}(\vec{d},d)^{197}\text{Au}$ scattering was utilized to check
the systematic uncertainty in the experimental setup when a polarized beam was used. The TAP for $^{197}\text{Au}(d,d)^{197}\text{Au}$ scattering should be $< 10^{-4}$ at this energy [Kam85]. For the $A_{xz}$ measurement, the detector pairs were positioned at $130^\circ$, $145^\circ$, and $157^\circ$. For the $A_{yy}$ experiment, the placement of the detector pairs was $120^\circ$, $135^\circ$, and $147^\circ$. Unfortunately, no measurements of $^{197}\text{Au}(d,d)^{197}\text{Au}$ scattering were conducted for the $A_{zz}$ experiment. For $A_{xz}$ and $A_{yy}$, a value of zero was measured to a level of $10^{-4}$ which was well below the statistical uncertainties of our d-p scattering measurements.

3.3 Vector Analyzing Powers

The VAPs measurements proved to be experimentally very difficult. The magnitude of the VAPs is on the order of $10^{-3}$. The singles method described in the cross-section and TAPs sections of this chapter was not feasible since a higher data rate was required resulting in pile-up of the $^{12}\text{C}(d,d)^{12}\text{C}$ background under the peaks of interest. With such small analyzing powers, the data are very sensitive to errors with background determination which has an analyzing power
on the order of $10^{-2}$ at an incident deuteron energy of 1.300 MeV. Therefore, a new technique was employed. As with the tensor analyzing power measurements, the polarization of the beam was measured online for both the $A_y$ and $iT_{11}$ experiments. For the proton beam, the $^6\text{Li}(\vec{p},^3\text{He})^4\text{He}$ reaction was employed for polarimetry. In the deuteron beam case, a polarimeter based on the $^2\text{H}(d, p)^3\text{H}$ reaction was used. The reader is referred to Appendix B for an explanation of these polarimeters.

### 3.3.1 Coincidence Measurements

With VAPs on the order of $10^{-3}$, the singles method would not give reliable results. Instead, a scheme for measuring coincidences between the elastically-scattered particle and its recoiled partner from the target was employed. The discussion will describe the $\vec{p}$ beam measurement for $A_y$, but the procedure was the same for the $\vec{d}$ beam experiment for $iT_{11}$ with a few substitutions besides the obvious fact of the different beams. Both experiments were conducted in the chamber on the $52^\circ$ beamline. The final energies in the center of the target for the $\vec{p}$ beam and the $\vec{d}$ beam were 1.000 MeV and 2.000 MeV, respectively. For the $A_y$ experiment, four thin deuterated-carbon foils with $1.0 \times 10^{18}$ D/cm$^2$ and $2.0 \times 10^{18}$ C/cm$^2$ were added to the target ladder along with the empty target ring for tuning the beam onto the polarimeter. For the $iT_{11}$ case, four thin hydrogenated-carbon foils of the same thickness were used.

For $^2\text{H}(\vec{p}, p)^2\text{H}$ scattering, a coincidence was obtained between the elastically-scattered proton and the deuteron knocked out of the target. Two pairs of silicon surface-barrier detectors were arranged symmetrically with the beam direction. One pair of detectors was 100 $\mu$m thick, and the other pair had thicknesses of 300 $\mu$m. A drawing of the chamber setup is in Figure 3.8. Here is an example of how the coincidence setup worked. When the proton was detected in a front detector (R1), the back detector (L2) was positioned at the kinematically-determined angle so as to detect the recoiled deuteron. The collimator of L2 was
set to accept the spread in angle and energy of the deuterons due to the angular acceptance of R1. Tables 3.6 and 3.7 lists the different detector parameters for each angle of the $A_y$ and $iT_{11}$ angular distributions, respectively. The R1-L2 pair in effect acted as a detector on the right-hand side of the chamber detecting one event. The L1-R2 pair acted as a left detector giving a left-right geometry with which to measure the VAPs. Thin mylar foils with a thickness of 2 µm were attached to the collimators to block the heavy recoiled particles such as carbon. The foils eliminated any chance of false coincidences between the proton and a heavy recoil.

Figure 3.9 shows an example of the electronics setup for a single pair of detectors (say R1-L2) in the coincidence method. The coincidence signal was generated in the Time-to-Amplitude Converter (TAC). To create a TAC signal, there had to be a START and a STOP signal. The signal from the process with the higher count rate was delayed with a cable delay of 120 ns to create the STOP. In Figure 3.9, the time signal went from the L2 preamplifier and was shaped by a Timing-Filter Amplifier (TFA). The output of the TFA was sent to a Constant Fraction Discriminator (CFD) where a timing signal was generated.
Table 3.6: List of detector parameters for the $^2\text{H}(\bar{p}, p)^2\text{H}$ coincidence experiment.

<table>
<thead>
<tr>
<th>$\theta_{\text{c.m.}}$</th>
<th>Detected particle</th>
<th>$\theta_{\text{lab}}$</th>
<th>Front coll. (mm)</th>
<th>Rear coll. (mm)</th>
<th>Dist. from target (cm)</th>
<th>Snout length (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>61.7°</td>
<td>p</td>
<td>42.1°</td>
<td>6.4 × 12.7</td>
<td>6.4 × 12.7</td>
<td>12.1</td>
<td>5.1</td>
</tr>
<tr>
<td></td>
<td>d</td>
<td>59.1°</td>
<td>6.4 × 9.5</td>
<td>6.4 × 9.5</td>
<td>12.1</td>
<td>5.1</td>
</tr>
<tr>
<td>87.8°</td>
<td>p</td>
<td>61.6°</td>
<td>6.4 × 12.7</td>
<td>6.4 × 12.7</td>
<td>12.1</td>
<td>3.8</td>
</tr>
<tr>
<td></td>
<td>d</td>
<td>46.2°</td>
<td>6.4 × 12.7</td>
<td>6.4 × 12.7</td>
<td>16.5</td>
<td>3.8</td>
</tr>
<tr>
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<td>p</td>
<td>80.2°</td>
<td>6.4 × 12.7</td>
<td>6.4 × 12.7</td>
<td>12.1</td>
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</tr>
<tr>
<td></td>
<td>d</td>
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<td>6.4 dia.</td>
<td>6.4 dia.</td>
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<td>2</td>
</tr>
<tr>
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<td>p</td>
<td>90.0°</td>
<td>6.4 × 12.7</td>
<td>6.4 × 12.7</td>
<td>12.1</td>
<td>5.1</td>
</tr>
<tr>
<td></td>
<td>d</td>
<td>30.0°</td>
<td>6.4 × 12.7</td>
<td>3.2 × 6.4</td>
<td>12.1</td>
<td>5.1</td>
</tr>
<tr>
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<td>p</td>
<td>113.6°</td>
<td>6.4 × 12.7</td>
<td>6.4 × 12.7</td>
<td>12.1</td>
<td>5.1</td>
</tr>
<tr>
<td></td>
<td>d</td>
<td>19.6°</td>
<td>6.4 × 12.7</td>
<td>3.2 × 6.4</td>
<td>15.9</td>
<td>5.1</td>
</tr>
<tr>
<td>52.0°</td>
<td>p</td>
<td>35.2°</td>
<td>6.4 × 12.7</td>
<td>6.4 × 12.7</td>
<td>14.6</td>
<td>5.1</td>
</tr>
<tr>
<td></td>
<td>d</td>
<td>64.0°</td>
<td>6.4 × 9.5</td>
<td>6.4 × 9.5</td>
<td>12.1</td>
<td>5.1</td>
</tr>
<tr>
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<td>6.4 × 12.7</td>
<td>6.4 × 12.7</td>
<td>12.1</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>d</td>
<td>52.2°</td>
<td>6.4 × 12.7</td>
<td>6.4 × 12.7</td>
<td>12.1</td>
<td>5.1</td>
</tr>
</tbody>
</table>

from the risetime of the signals. The CFD can give very fast timing signals (our time resolution ranged from 10 to 40 ns). From the CFD, the signal went to a 755 which generates three identical NIM signals. One signal went to the TAC as the START. The other signal went to another 755 which acted as a coincidence module with a resolving time of 200 ns. A signal was not created from the second 755 unless it received a signal from both the R1 and L2 detectors. If this criterion was met, the second 755 sent two signals out as gates. One gate was sent to the L2 LGS to be timed with the energy signal, and the other gate triggered the ADC to accept the eventual coincidence signal from the TAC. The same path was followed by the R1 signal which, in this case, created the STOP for the TAC. The output from the TAC went to a Northern ADC. A pulser signal was also
Table 3.7: List of detector parameters for the $^1\text{H}(\vec{d}, d)^1\text{H}$ coincidence experiment.

<table>
<thead>
<tr>
<th>$\theta_{c.m.}$</th>
<th>Detected particle</th>
<th>$\theta_{lab}$</th>
<th>Front coll. (mm)</th>
<th>Rear coll. (mm)</th>
<th>Dist. from target (cm)</th>
<th>Snout length (cm)</th>
</tr>
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<tbody>
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<td>26.1°</td>
<td>6.4 × 12.7</td>
<td>3.2 × 9.5</td>
<td>14.5</td>
<td>5.1</td>
</tr>
<tr>
<td></td>
<td>p</td>
<td>46.1°</td>
<td>6.4 × 12.7</td>
<td>6.4 × 12.7</td>
<td>10.7</td>
<td>5.1</td>
</tr>
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<td>d</td>
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<td>6.4 × 12.7</td>
<td>4.8 × 12.7</td>
<td>22.1</td>
<td>3.8</td>
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<td></td>
<td>p</td>
<td>21.0°</td>
<td>6.4 × 12.7</td>
<td>3.2 × 6.4</td>
<td>14.5</td>
<td>3.8</td>
</tr>
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<td>61.1°</td>
<td>d</td>
<td>19.4°</td>
<td>4.8 × 12.7</td>
<td>4.8 × 9.5</td>
<td>23.5</td>
<td>5.1</td>
</tr>
<tr>
<td></td>
<td>p</td>
<td>59.4°</td>
<td>6.4 × 12.7</td>
<td>6.4 × 12.7</td>
<td>12.1</td>
<td>5.1</td>
</tr>
<tr>
<td>50.0°</td>
<td>d</td>
<td>16.2°</td>
<td>4.8 × 12.7</td>
<td>4.8 × 9.5</td>
<td>23.3</td>
<td>5.1</td>
</tr>
<tr>
<td></td>
<td>p</td>
<td>65.0°</td>
<td>none</td>
<td>12.7 dia.</td>
<td>10.2</td>
<td>5.1</td>
</tr>
<tr>
<td>74.1°</td>
<td>d</td>
<td>22.9°</td>
<td>6.4 × 12.7</td>
<td>3.2 × 9.5</td>
<td>15.9</td>
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<tr>
<td></td>
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<td>5.1</td>
</tr>
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<td>6.4 × 12.7</td>
<td>3.2 × 9.5</td>
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<td>3.8</td>
</tr>
<tr>
<td></td>
<td>p</td>
<td>46.1°</td>
<td>none</td>
<td>12.7 dia.</td>
<td>10.7</td>
<td>3.8</td>
</tr>
<tr>
<td>54.8°</td>
<td>d</td>
<td>17.6°</td>
<td>4.8 × 12.7</td>
<td>4.8 × 9.5</td>
<td>23.3</td>
<td>5.1</td>
</tr>
<tr>
<td></td>
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<td>46.1°</td>
<td>6.4 × 12.7</td>
<td>6.4 × 12.7</td>
<td>10.2</td>
<td>5.1</td>
</tr>
<tr>
<td>120.0°</td>
<td>d</td>
<td>30.0°</td>
<td>6.4 × 12.7</td>
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<td>14.5</td>
<td>5.1</td>
</tr>
<tr>
<td></td>
<td>p</td>
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<td>6.4 × 12.7</td>
<td>3.2 × 9.5</td>
<td>14.5</td>
<td>5.1</td>
</tr>
</tbody>
</table>

Put through the electronics in order to measure the deadtime of the system. The coincidence and pulser signals were positioned in the spectrum by adjusting the amount of delay in the STOP signal. The two peaks were separated further in the spectrum by added delay to the pulser signal. The pulser signal was delayed prior to being sent to the preamplifier. A typical time coincidence spectrum is shown in Figure 3.10.

There was an additional feature to the coincidence electronics which is presented in Figure 3.9. The R1 path has a $^{12}\text{C}(\vec{p}, p)^{12}\text{C}$ veto which was employed when one detector pair was at a very forward angle. The signal from the TFA was fanned out by a 744 linear fan-out into two signals. Both signals went to
Figure 3.9: Electronics setup for coincidence measurement using four detectors.

separate CFDs. The unwanted $^{12}$C($\vec{p}, p$)$^{12}$C pulses always had a higher amplitude signals corresponding to d-p scattering peaks (see Figure 3.4). A threshold was set below the $^{12}$C($\vec{p}, p$)$^{12}$C signal in one CFD, and the CFD output was sent to the VETO of the 755. The threshold of the other CFD was set below the $^2$H($\vec{p}, p$)$^2$H or $^2$H($\vec{p}, d$)$^3$H signal, and its output was sent to the input of the 755. This circuit acted as a window to veto out the $^{12}$C($\vec{p}, p$)$^{12}$C signals from the electronics and reduce the dead-time.

There was one last twist to the experimental setup in the coincidence mode.
In both the $^2\text{H}(\vec{p}, p)^2\text{H}$ and $^1\text{H}(\vec{d}, d)^1\text{H}$ kinematics, there is a “magic” angle where the elastically-scattered particle exits at the exact same angle as the corresponding recoil on the opposite side of the beam direction. Figure 3.11 is a plot of the relationship between $\theta_{lab}$ to $\theta_{c.m.}$ for both particles. In the laboratory, the elastically-scattered particle travels to one side (to the right, for example) and the recoil moves to the opposite side (to the left). To achieve a left-right asymmetry, the front and back detectors would be positioned in the same location. The “magic” angle data were taken with a two-detector coincidence setup which is displayed in Figure 3.12. For this angle setting, one pair of detectors (R1 and L1) was placed in the chamber. The signal from the TFA of R1 was fanned out by a 744, two signals went to separate CFDs. A low threshold was set for the deuteron amplitude in one CFD, and a higher threshold was set in the other.
Figure 3.11: Laboratory angle to center-of-mass angle conversion for d-p scattering. The solid and dashed curves represent $^1\text{H}(d, d)^1\text{H}$ and $^1\text{H}(d, p)^2\text{H}$ scattering, respectively.

Figure 3.12: Electronics setup for coincidence measurement using two detectors.
CFD to allow only the proton signal. Therefore, the R1 proton signal and the L1 deuteron signal provided one TAC signal, and the R1 deuteron signal and the L1 proton signal created the other TAC. The coincidence spectra for a “magic” angle were similar to the spectrum shown in Figure 3.10.

It was not essential to store the singles energy spectra; however, they provided for useful redundancy during each experiment. The energy signal for each detector was sent from the preamplifier to the amplifier where it was shaped and increased in magnitude. The unipolar output of the amplifier went to a LGS where it was timed with a gate which will be described shortly. The gate from the G&D generator and energy signal from the LGS went to a Northern ADC. The energy signal for each detector was stored in the computer. The singles events were prescaled to avoid high computer deadtime. Only one out of every ten singles events were stored in the computer. A visual inspection of a singles spectrum verified the angle of the detector, and that an approximately equal number of counts appeared in the peak which caused the STOP in the coincidence as appeared in the coincidence spectrum.

With both coincidence methods, the contaminant reactions were blocked entirely. Without the carbon contamination to hurt the separation of the peaks and background subtraction, we were able to enlarge the angular acceptance of the detectors and increase the count rate for the experiments. We were capable of measuring $iT_{11}$ and $A_y$ angular distributions in approximately five days per analyzing power.
Chapter 4

Data Analysis

Chapter 3 gave a description of the experimental procedures and the raw data collection for \( ^1H(d,d)^1H \) and \( ^2H(p,p)^2H \) elastic scattering. This chapter will explain how the raw data were refined and the observables were extracted.

Except for the forward and backward angles in the cross-section experiment, the analysis was the same for each experiment. The XSYS display and software were utilized to subtract background under peaks and to sum the counts in the peaks. A gate was set around each peak and the pulser, individually, to record the total number of peak counts and background counts. To subtract the background, a wider gate was placed which included the background below and above the peak as well as the peak itself. An exponential fit was constructed with the channels in the background gate to match the shape of the background. Background counts in the peak gate were subtracted from total counts in the peak gate to give the true peak sum. The total sums and true peak sums were printed to an output file along with the scaler values. The peak sum files were inputted into FORTRAN codes written to convert the sums into the desired observable. Chapter 2 and the following sections discuss the algorithms in the codes to calculate each observable and its uncertainty.

The analysis of analyzing power data will be discussed first followed by the cross-section data. This order is reversed from Chapter 3 since the analyzing
power data were less complicated to analyze than the cross-section.

4.1 Analyzing Powers

All of the analyzing powers were measured with the Pol-Pol method (see Chapter 2). Each TAP and VAP equation has some combination of the ratio of detector yields ($l$ or $r$). In Pol-Pol notation, $l$ and $r$ are proportional to the ratio $I^{\parallel 2}/I^{\parallel 3}$ where the angular dependence in the counts $I^{\parallel 2}$ and $I^{\parallel 3}$ has been ignored for simplicity. The yield for state 2 is defined as

$$Y^{\parallel 2} = \frac{I^{\parallel 2}}{BC I^{\parallel 2}} \times dtc^{\parallel 2},$$

(4.1)

where $I^{\parallel 2}$ is the background-subtracted peak counts from the spectrum, $BC I^{\parallel 2}$ is collected charge in state $(2)$, and $dtc^{\parallel 2}$ is the dead-time correction which is defined as

$$dtc^{\parallel 2} = \frac{pulser^{\parallel 2}}{I^{\parallel 2}_{pul}}.$$

(4.2)

The quantities $pulser^{\parallel 2}$ and $I^{\parallel 2}_{pul}$ are the pulser’s counts stored directly in a scaler and the pulser’s counts stored in the spectrum, respectively.

The error in the yield was found by using the propagation of errors [Bev92]. If $f(a,b,c,...)$ is a function of the independently measurable quantities $a,b,c,...$, the error in $f(a,b,c,...)$ is

$$\Delta f(a,b,c,...) = \sqrt{\left(\frac{\partial f}{\partial a}\Delta a\right)^2 + \left(\frac{\partial f}{\partial b}\Delta b\right)^2 + \left(\frac{\partial f}{\partial c}\Delta c\right)^2 + \cdots},$$

(4.3)

where $\Delta a$, $\Delta b$, and $\Delta c$ are the errors in $a$, $b$, and $c$, respectively. Subsequently, the error in $Y^{\parallel 2}$ is

$$\Delta Y^{\parallel 2} = \sqrt{\left(\frac{dtc^{\parallel 2}}{BC I^{\parallel 2}}\Delta I^{\parallel 2}\right)^2 + \left(\frac{I^{\parallel 2}}{BC I^{\parallel 2}}\Delta dtc^{\parallel 2}\right)^2}
= Y^{\parallel 2}\sqrt{\left(\frac{\Delta I^{\parallel 2}}{I^{\parallel 2}}\right)^2 + \left(\frac{\Delta dtc^{\parallel 2}}{dtc^{\parallel 2}}\right)^2}.$$
The errors $\Delta I^{(2)}$ and $\Delta dtc^{(2)}$ are defined as

$$\Delta I^{(2)} = \sqrt{I^{(2)}}$$

and

$$\Delta dtc^{(2)} = dtc^{(2)} \frac{\Delta I^{(2)}_{pul}}{I^{(2)}_{pul}} = dtc^{(2)} \sqrt{\frac{|pulser - I^{(2)}_{pul}|}{I^{(2)}_{pul}}}.$$  \hspace{1cm} (4.6)

Equation 4.6 takes into account the statistical error in the number of spectrum pulser counts missed instead of those that are counted. The effect is to give a small error when more spectrum pulser pulses were counted (low deadtime) and a large error when more pulser counts are missed (high deadtime). It also implies that higher pulser rates are needed for higher deadtimes. Substituting these expressions into equation 4.4 provides the error in $Y^{(2)}$ as

$$\Delta Y^{(2)} = Y^{(2)} \sqrt{\frac{1}{I^{(2)}} + \frac{|pulser - I^{(2)}_{pul}|}{(I^{(2)}_{pul})^2}}.$$  \hspace{1cm} (4.7)

Similar expressions can be found for $Y^{(3)}$ and $\Delta Y^{(3)}$. Applying the same error analysis to for the right detector ratios for the Pol-Pol and Pol-Unpol formalism provides the following expressions

$$\Delta r = |r| \sqrt{\left(\frac{\Delta Y^{(2)}_R}{Y^{(2)}_R}\right)^2 + \left(\frac{\Delta Y^{(3)}_R}{Y^{(3)}_R}\right)^2},$$

$$\Delta R^i = |R^i| \sqrt{\left(\frac{\Delta Y^{(i)}_R}{Y^{(i)}_R}\right)^2 + \left(\frac{\Delta Y^{(1)}_R}{Y^{(1)}_R}\right)^2},$$  \hspace{1cm} (4.8, 4.9)

where $Y^{(i)}_R$ and $\Delta Y^{(i)}_R$ are the right detector yield and its associated error. Similar equations can be made for $l$ and $L^i$. With the rudimentary error analysis finished, we can move on to the errors in the analyzing powers.
4.1.1 Tensor Analyzing Powers

For the TAPs, the measurable quantities were \( r, l, p_{zz}^{(2)}, \) and \( p_{zz}^{(3)} \). From equation 2.12, the error for \( A_{zz} \) was as follows

\[
\Delta A_{zz} = \left[ \left( \frac{\partial A_{zz}}{\partial r} \Delta r \right)^2 + \left( \frac{\partial A_{zz}}{\partial l} \Delta l \right)^2 + \left( \frac{\partial A_{zz}}{\partial p_{zz}^{(2)}} \Delta p_{zz}^{(2)} \right)^2 + \left( \frac{\partial A_{zz}}{\partial p_{zz}^{(3)}} \Delta p_{zz}^{(3)} \right)^2 \right]^{\frac{1}{2}}
\]

\[
= \left( \frac{A_{zz}}{l + r - 2} \right)^2 \left\{ \left( p_{zz}^{(2)} - p_{zz}^{(3)} \right)^2 \left[ (\Delta r)^2 + (\Delta l)^2 \right] + 4 \left[ (l + r - 2) \Delta p_{zz}^{(2)} \right]^2 + \left[ (l + r - 2)(l + r) \Delta p_{zz}^{(3)} \right]^2 \right\}^{\frac{1}{2}}. \tag{4.10}
\]

Since a large number of counts per run was accumulated from the polarimeter reaction and the analyzing power was large (see Section B), the Pol-Unpol equations were used to calculate the beam polarization. The uncertainties in \( p_{zz}^{(i)} \) were calculated from

\[
\Delta p_{zz}^{(i)} = \frac{1}{A_{zz}^{pol}} \sqrt{\left( \Delta R^{(i)} \right)^2 + \left( \Delta L^{(i)} \right)^2}, \tag{4.11}
\]

where \( A_{zz}^{pol} \) is the analyzing power of the polarimeter.

Similarly, \( \Delta A_{xz} \) and \( \Delta A_{yy} \) were found from equations 2.19 and 2.29, respectively, and take on the forms

\[
\Delta A_{xz} = \left\{ \left( p_{zz}^{(2)} - p_{zz}^{(3)} \right)^2 \left[ \frac{(\Delta r)^2}{p_{zz}^{(2)} - rp_{zz}^{(3)}} - \frac{(\Delta l)^2}{p_{zz}^{(2)} - lp_{zz}^{(3)}} \right] \right\}^{\frac{1}{2}}
\]

\[
+ \left[ \frac{r - 1}{p_{zz}^{(2)} - rp_{zz}^{(3)}} - \frac{l - 1}{p_{zz}^{(2)} - lp_{zz}^{(3)}} \right]^2 \Delta p_{zz}^{(2)},
\]

\[
+ \left[ \frac{r(r - 1)}{p_{zz}^{(2)} - rp_{zz}^{(3)}} - \frac{l(l - 1)}{p_{zz}^{(2)} - lp_{zz}^{(3)}} \right]^2 \Delta p_{zz}^{(3)} \}^{\frac{1}{2}}. \tag{4.12}
\]

\[
\Delta A_{yy} = \left( \frac{A_{yy}}{l + r - 2} \right)^2 \left\{ \left( p_{zz}^{(2)} - p_{zz}^{(3)} \right)^2 \left[ (\Delta r)^2 + (\Delta l)^2 \right] + 4 \left[ (l + r - 2) \Delta p_{zz}^{(2)} \right]^2 + \left[ (l + r - 2)(l + r) \Delta p_{zz}^{(3)} \right]^2 \right\}^{\frac{1}{2}}, \tag{4.13}
\]

60
Table 4.1: Average tensor polarization for TAPs experiments.

<table>
<thead>
<tr>
<th>Data</th>
<th>$p_z^{(2)}$</th>
<th>$\Delta p_z^{(2)}$</th>
<th>$p_z^{(3)}$</th>
<th>$\Delta p_z^{(3)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_{zz}$</td>
<td>-0.824</td>
<td>0.025</td>
<td>0.677</td>
<td>0.020</td>
</tr>
<tr>
<td>$A_{xz}$</td>
<td>-0.630</td>
<td>0.019</td>
<td>0.631</td>
<td>0.019</td>
</tr>
<tr>
<td>$A_{yy}$</td>
<td>-0.899</td>
<td>0.032</td>
<td>0.833</td>
<td>0.039</td>
</tr>
</tbody>
</table>

where the error in the beam polarizations were calculated from

$$
\Delta p_z^{(i)} = \frac{1}{|A_{xx}^{pol}|} \sqrt{(\Delta R^{(i)})^2 + (\Delta L^{(i)})^2} \text{ and}
$$

(4.14)

$$
\Delta p_z^{(i)} = \frac{1}{|A_{yy}^{pol}|} \sqrt{(\Delta R^{(i)})^2 + (\Delta L^{(i)})^2}
$$

(4.15)

for the $A_{xz}$ and $A_{yy}$ experiments, respectively. Table 4.1 shows the average tensor polarization for each TAP measurement. The final step in the TAP analyses was to convert from Cartesian to spherical coordinates. Equations 2.41 to 2.43 were employed in these conversions. The associated errors were found to be

$$
\Delta T_{20} = \frac{1}{\sqrt{2}} \Delta A_{zz}, \tag{4.16}
$$

$$
\Delta T_{21} = \frac{1}{\sqrt{3}} \Delta A_{xz}, \text{ and} \tag{4.17}
$$

$$
\Delta T_{22} = \sqrt{\frac{1}{12} (\Delta A_{zz})^2 + \frac{1}{3} (\Delta A_{yy})^2}. \tag{4.18}
$$

The results for $T_{20}$, $T_{21}$, and $T_{22}$ measurements are shown in Chapter 6 along with a comparison to the theoretical calculations.

4.1.2 Vector Analyzing Powers

For the VAP experiments, the variables were $r$, $l$, $p_z^{(2)}$, and $p_z^{(3)}$. To avoid confusion about $A_y$ from $^1H(\bar{d}, d)^1H$ and $A_y$ from $^2H(\bar{p}, p)^2H$, the deuteron beam case will be discussed first in its entirety followed by the proton beam analysis.
For $^1H(d, d)^1H$ scattering, $A_y$ was determined with equation 2.27. The error in the measurement was

$$
\Delta A_y = \frac{3}{4} \left( \frac{A_y}{l-r} \right)^2 \left\{ \left[ \left( p_z^{(2)} - lp_z^{(3)} \right) \Delta r \right]^2 + \left[ \left( p_z^{(2)} - rp_z^{(3)} \right) \Delta l \right]^2 \\
+ \left[ (l-r) \Delta p_z^{(2)} \right]^2 + \left[ \frac{1}{2} \left( p^2 - r^2 \right) \Delta p_z^{(3)} \right]^2 \right\}^{\frac{1}{2}},
$$

(4.19)

where $\Delta r$ and $\Delta l$ are described by equation 4.8. The beam polarization for each run was used to calculate $A_y$ for that run. To give a sense of what polarizations were achieved, the average polarization over the length of the experiment was $p_z^{(2)} = -0.554 \pm 0.017$, and $p_z^{(3)} = 0.346 \pm 0.010$. Finally, $iT_{11}$ was determined by equation 2.40, and the error was produced by

$$
\Delta iT_{11} = \frac{\sqrt{3}}{2} \Delta A_y.
$$

(4.20)

A discussion of the $iT_{11}$ results can be found in Chapter 6.

For the proton beam experiment, $A_y$ was measured with the combination of $p_z^{(2)}$ and $p_z^{(3)}$ as well as $\overline{p}_z$ and $\delta p_z$ (see Section 2.5.2). For the $p_z^{(2)}$ and $p_z^{(3)}$ scenario, the TAC peak sums were inserted into equation 2.36. The uncertainty in $A_y$ was determined by

$$
(\Delta A_y)_r = \left( \frac{A_y}{r-1} \right)^2 \left\{ \left[ \left( p_z^{(2)} - p_z^{(3)} \right) \Delta r \right]^2 + \left[ (r-1) \Delta p_z^{(2)} \right]^2 \\
+ \left[ r(r-1) \Delta p_z^{(3)} \right]^2 \right\}^{\frac{1}{2}},
$$

(4.21)

and

$$
(\Delta A_y)_l = \left( \frac{A_y}{l-1} \right)^2 \left\{ \left[ \left( p_z^{(2)} - p_z^{(3)} \right) \Delta l \right]^2 + \left[ (l-1) \Delta p_z^{(2)} \right]^2 \\
+ \left[ l(l-1) \Delta p_z^{(3)} \right]^2 \right\}^{\frac{1}{2}}.
$$

(4.22)

For the $\overline{p}_z$ and $\delta p_z$ situation, the $A_y$ determination was achieved by equation 2.39. The quantity $\Delta A_y$ was obtained by

$$
\Delta A_y = |\overline{p}_z A_y| \left[ A_y^2 \left( \frac{\Delta r}{(r-1)^2} \right)^2 + A_y^2 \left( \frac{\Delta l}{(l-1)^2} \right)^2 + \left( \frac{\Delta \overline{p}_z}{\overline{p}_z} \right)^2 \right]^{\frac{1}{2}},
$$

(4.23)

62
Table 4.2: Average vector polarization for $^2$H($\vec{p}, p)^2$H scattering experiment. The errors are purely statistical. Section 2.5.2 provides the definitions for $\bar{p}_z$ and $\delta p_z$.

<table>
<thead>
<tr>
<th>Data</th>
<th>Polarization</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_z^{(2)}$</td>
<td>-0.644</td>
<td>0.003</td>
</tr>
<tr>
<td>$p_z^{(3)}$</td>
<td>0.657</td>
<td>0.003</td>
</tr>
<tr>
<td>$\bar{p}_z$</td>
<td>0.650</td>
<td>0.002</td>
</tr>
<tr>
<td>$\delta p_z$</td>
<td>0.007</td>
<td>0.002</td>
</tr>
</tbody>
</table>

\[ \Delta A_y = \frac{1}{|\delta p_z|} \left[ A_y^2 \left( \frac{(l^2 - 1)\Delta r}{(r - l)^2} \right)^2 + A_y^2 \left( \frac{(r^2 - 1)\Delta l}{(r - l)^2} \right)^2 + (A_y \Delta (\delta p_z))^2 \right]^{\frac{1}{2}}. \]  

The average vector polarizations over the length of the experiment are given in Table 4.2. The determinations of $A_y$ by both the $p_z^{(i)}$ method and the $\bar{p}_z$ method were within the errors of each other. The $\delta p_z$ configuration was never used since it requires a greater number of counts to determine $A_y$. The final $A_y$ angular distribution is given in Chapter 6 accompanied by the theoretical predictions.

4.2 Relative Cross-Section

The manner in which the differential cross-section data were collected is described in Section 3.1. Numerous techniques were employed to reduce the raw data. For the most part, the peak counts were summed and analyzing powers calculated using XSYS-supported software. At the extreme forward and backward angles, peak-fitting was employed since the XSYS background subtraction was inadequate. The peak-fitting method will be described in a subsequent section.

The first quantity measured was the relative cross-section. For a single detector, the relative cross-section (as an example choose the right detector) is given by

\[ \sigma_{rel}(\theta) \propto \frac{Y_R}{Y_M} \]  

(4.25)
where $Y_R$ is the right detector yield and is given by equation 4.1. The quantity $Y_M$ is the sum of the four monitor detector yields and takes the form

$$Y_M = Y_{RM1} + Y_{LM1} + Y_{RM2} + Y_{LM2}.$$  \hspace{1cm} (4.26)

To average out the effect of the beam striking different spots on the target, a left-right symmetric pair of detectors with identical solid angles was used. Having two detectors also doubled our statistics. Thus, the relative cross-section becomes

$$\sigma_{rel}(\theta) \propto \frac{Y_R + Y_L}{Y_M}. \hspace{1cm} (4.27)$$

Utilizing equation 4.3, the error in $\sigma_{rel}(\theta)$ is formulated as

$$\Delta \sigma_{rel}(\theta) = \left| \sigma_{rel}(\theta) \right| \left[ \frac{(\Delta Y_R)^2 + (\Delta Y_L)^2}{(Y_R + Y_L)^2} \right] + \left( \frac{(\Delta Y_{RM1})^2 + (\Delta Y_{LM1})^2 + (\Delta Y_{RM2})^2 + (\Delta Y_{LM2})^2}{(Y_M)^2} \right)^{\frac{1}{2}}. \hspace{1cm} (4.28)$$

### 4.2.1 Monitor Detectors

The four monitor detectors were fixed to the wall of the chamber and sat in the reaction plane. For some angle settings, one pair of monitor detectors was blocked when the chamber detectors were within their angular range. Four monitor detectors were necessary so that during the entire experiment, there was always a pair of monitor detectors counting. However, the relative cross-section depends on the sum of all monitor detector counts. To remain consistent, the missing monitor detector yields were reconstructed. Plots were made of the ratio of front detector yields to back detector yields versus run number. Figure 4.1 is a sample of the plots. Since ratios of front-to-back monitor yields are within errors of each other, a weighted average of the data was made. For both the left and right pair of monitor detectors, the average value of their ratio of yields was calculated and was used when any of the monitor detectors was blocked. To find the reconstructed monitor sum $Y_M$, the following expressions were used

$$Y_M = Y_{RM2}(1 + b_R) + Y_{LM2}(1 + b_L), \hspace{1cm} (4.29)$$
Figure 4.1: Ratio of d-p scattering yields of the front monitor detectors to the back monitor detectors. Graph (a) is the ratio of LM1 yields to LM2 yields. Similarly, graph (b) displays ratio of RM1 yields to RM2 yields. The ratios are constant within 0.7%.

\[
Y_M = Y_{RM1} \left( 1 + \frac{1}{b_R} \right) + Y_{LM1} \left( 1 + \frac{1}{b_L} \right),
\]

where equations 4.29 and 4.30 rebuild \( Y_M \) when RM1 and LM1 are blocked and when RM2 and LM2 are blocked, respectively. The coefficients \( b_R \) and \( b_L \) are the reconstruction ratios for the right and left monitor pairs, respectively. The value for \( b_R \) was \( 7.218 \pm 0.036 \), and \( b_L \) was calculated to be \( 7.141 \pm 0.035 \). The errors
in $Y_M$ now become

$$
\Delta Y_M = \left\{ \left[ (1 + b_R) \Delta Y_{RM2} \right]^2 + \left[ (1 + b_L) \Delta Y_{LM2} \right]^2 \\
+ \left( Y_{RM2} \Delta b_r \right)^2 + \left( Y_{LM2} \Delta b_l \right)^2 \right\}^{\frac{1}{2}}, \quad \text{and} \quad (4.31)
$$

$$
\Delta Y_M = \left\{ \left[ \left( 1 + \frac{1}{b_R} \right) \Delta Y_{RM1} \right]^2 + \left[ \left( 1 + \frac{1}{b_L} \right) \Delta Y_{LM1} \right]^2 \\
+ \left( \frac{Y_{RM1}}{b_R^2} \Delta b_R \right)^2 + \left( \frac{Y_{LM1}}{b_L^2} \Delta b_L \right)^2 \right\}^{\frac{1}{2}}, \quad (4.32)
$$

depending on which pair was blocked.

### 4.2.2 Reconstructing Contaminant Peaks

Working with hydrogenated or deuterated carbon targets, one has to deal with the $^{12}C + d$ or $^{12}C + p$ peaks in the spectra. From kinematics, the peak coming from elastic scattering on $^{12}C$ was always at a higher energy than the peaks of interest. The only complication from elastic scattering from $^{12}C$ was background in the spectra taken at the forward most angles and this topic is saved for the next section. In the case of a proton beam, $^1H(p, p)^1H$ was the only other contaminant reaction, and it never merged with a peak of interest. The real troubles were protons resulting from $^{12}C(d, p_n)^{13}C^*$ reactions. There were three proton peaks from $^{12}C(d, p_n)^{13}C^*$ reactions to the first three excited states of $^{13}C$, and their kinematics are displayed in Figure 3.1. Table 4.3 displays the angles where contaminants interfered with the peaks of interest. The yield for the $^1H(d, p)^2H$ peak at these angles can be extracted if an estimate for the yield of the contaminant reaction is made. Such estimates were determined by tracking the ratio of contaminant yield to the $^{12}C(d, d)^{12}C$ yield over the entire angular range. The points were fit to a third-order polynomial, and the ratio was calculated for the angles in question from the fit. The ratio was next multiplied by the $^{12}C(d, d)^{12}C$ yield at the problematic angle to estimate the yield in the
contaminant reaction. Finally, the true yield from the d-p scattering is found by subtracting out the estimated contaminant yield. The error in the estimated contaminant yields was approximately 0.5% due the error in the fits and the statistical uncertainty in the $^{12}\text{C}(d,d)^{12}\text{C}$ yields. Removing the contaminant yields resulted in the reduction in $\sigma_{\text{rel}}(\theta)$ by $\approx 10\%$ due to $^{12}\text{C}(d,p_1)^{13}\text{C}^*$, $\approx 0.6\%$ due to $^{12}\text{C}(d,p_2)^{13}\text{C}^*$, and $\approx 0.9\%$ due to $^{12}\text{C}(d,p_3)^{13}\text{C}^*$ at the angles where it was a problem.

At this point in the discussion, it should be stated that no more corrections were needed for $\sigma_{\text{rel}}(\theta)$ for $^2\text{H}(p,p)^2\text{H}$ scattering. Peak-fitting was not necessary for the very forward angle data and contaminants were not a problem. Figure 4.2 is a plot of the $\sigma_{\text{rel}}(\theta)$ normalized to the theoretical calculation at $\theta_{\text{c.m.}} = 87.3^\circ$. With the normalization from the theoretical calculations including the AV18 and UR potentials, the theoretical curve reproduces the data well. Figure 4.3 shows the ratio of experiment to theory when the experimental points are normalized to the theoretical calculations $\theta_{\text{c.m.}} = 87.3^\circ$. From this plot, the data points are never more than one standard deviation from 1.0. The difficulty with the $^2\text{H}(p,p)^2\text{H}$ measurement is that the absolute cross-section cannot be determined.

Table 4.3: Angles where a contaminant peak has merged with a $^1\text{H}(d,p)^2\text{H}$ scattering peak.

<table>
<thead>
<tr>
<th>$\theta_{\text{lab}}$</th>
<th>Contaminant</th>
</tr>
</thead>
<tbody>
<tr>
<td>$21^\circ$</td>
<td>$^{12}\text{C}(d,p_1)^{13}\text{C}^*$</td>
</tr>
<tr>
<td>$22^\circ$</td>
<td>$^{12}\text{C}(d,p_1)^{13}\text{C}^*$</td>
</tr>
<tr>
<td>$23^\circ$</td>
<td>$^{12}\text{C}(d,p_1)^{13}\text{C}^*$</td>
</tr>
<tr>
<td>$46^\circ$</td>
<td>$^{12}\text{C}(d,p_2)^{13}\text{C}^*$</td>
</tr>
<tr>
<td>$47^\circ$</td>
<td>$^{12}\text{C}(d,p_2)^{13}\text{C}^*$</td>
</tr>
<tr>
<td>$48^\circ$</td>
<td>$^{12}\text{C}(d,p_2)^{13}\text{C}^*$</td>
</tr>
<tr>
<td>$52^\circ$</td>
<td>$^{12}\text{C}(d,p_3)^{13}\text{C}^*$</td>
</tr>
<tr>
<td>$54^\circ$</td>
<td>$^{12}\text{C}(d,p_3)^{13}\text{C}^*$</td>
</tr>
</tbody>
</table>
Figure 4.2: Relative cross-section for $^2\text{H}(p, p)^2\text{H}$ scattering at $E_{c.m.} = 667$ keV. The solid curve is the variational calculation with AV18+UR. The data are normalized to the theoretical calculations with the AV18+UR at $\theta_{c.m.} = 87.3^\circ$.

Figure 4.3: Ratio of $^2\text{H}(p, p)^2\text{H}$ scattering cross-section measurements to the theoretical calculations. The data have been normalized to the theoretical calculations with the AV18+UR at $\theta_{c.m.} = 87.3^\circ$. 
Figure 4.4: Spectrum of $^1\text{H}(d, d)^1\text{H}$ and $^1\text{H}(d, p)^2\text{H}$ scattering at $\theta_{lab} = 10^\circ$.

experimentally with the present setup.

For the determination of the absolute $\sigma(\theta)$, the $^1\text{H}(d, d)^1\text{H}$ scattering data were taken. The last step in the analysis of the $\sigma_{rel}(\theta)$ with the deuteron beam was to properly subtract the background at the very forward and backward angles. At the forward angles, the $^{12}\text{C}(d, d)^{12}\text{C}$ peak has a tail which the other peaks sit on as seen in Figure 4.4. For these cases, the peak was fit with a Gaussian form, and the background was fit with an exponential function. The PAW [Cou95] software was used to make the fits. The Gaussian fit has three parameters, the height $A$, the width $\omega$, and the centroid $x_0$ and has the form

$$I(x) = Ae^{\left(\frac{x-x_0}{\omega}\right)^2}. \quad (4.33)$$

The variables $x$ and $I(x)$ were the channel number (or energy) and the number of counts at a given $x$. To determine the total counts under the peak after background subtraction, the function was integrated from $-\infty$ to $\infty$ and has the solution of

$$I = \int_{-\infty}^{\infty} Ae^{\left(\frac{x-x_0}{\omega}\right)^2} dx = \sqrt{2\pi}\omega A. \quad (4.34)$$
Table 4.4: The coefficient $c$ used in the exponential-Gaussian fit in PAW. Different values were used to make the best fit depending on the angle.

<table>
<thead>
<tr>
<th>$\theta_{lab}$</th>
<th>$c$</th>
<th>erf($\frac{c}{\sqrt{2}}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>54°</td>
<td>1.2</td>
<td>0.7698</td>
</tr>
<tr>
<td>56°</td>
<td>1.1</td>
<td>0.7286</td>
</tr>
<tr>
<td>58°</td>
<td>1.0</td>
<td>0.6826</td>
</tr>
<tr>
<td>60°</td>
<td>0.9</td>
<td>0.6318</td>
</tr>
<tr>
<td>62°</td>
<td>0.75</td>
<td>0.5468</td>
</tr>
<tr>
<td>64°</td>
<td>0.6</td>
<td>0.4516</td>
</tr>
</tbody>
</table>

The total counts turn out to be independent of the position of the peak ($x_0$). The parameters in the fit are correlated, and the correlation was taken into account in calculating for the uncertainty in $I$. The equation for the error is

$$ \Delta I = |I| \left[ \left( \frac{\Delta A}{A} \right)^2 + \left( \frac{\Delta \omega}{\omega} \right)^2 + \frac{2}{\omega A} \Delta \omega A \right]^{\frac{1}{2}}. $$

The quantity $\Delta \omega A$ is the covariance and was calculated from the fit minimization by PAW using the error matrix [Bev92].

For the backward angles, the $^1\text{H}(d,d)^1\text{H}$ peak is approaching the tail of the detector noise which is also exponential in shape. An added feature of the backward angle spectra was the non-Gaussian shape to the peak. The best fit was a Gaussian with an exaggerated exponential tail on the low-energy side. The fit had the following functional form:

$$ I(x) = A e^{\left( \frac{(x-x_0) + \omega^2}{\tau} \right)}; \quad x < x_0 - c\omega $$

$$ I(x) = A e^{\left( \frac{x-x_0}{\omega} \right)^2}; \quad x \geq x_0 - c\omega. $$

The parameter $c$ was chosen to quantify the distance from the centroid to the point where the two curves meet. Table 4.4 lists what value of $c$ [Wea64] was needed at each angle. Again, the total number of counts is the area under the peak. For this case, the integration was separated into two integrals, each
corresponding to a specific region

\[
I = \int_{-\infty}^{x_0-\sigma_\omega} Ae\left[\frac{x-x_0}{\sigma}\right]^2 dx + \int_{x_0-\sigma_\omega}^{\infty} Ae\left[\frac{x-x_0}{\sigma}\right]^2 dx
\]

\[
= \frac{\omega A}{c} e^{-\frac{c^2}{2}} + \omega A \sqrt{\frac{\pi}{2}} \left[1 + erf\left(\frac{c}{\sqrt{2}}\right)\right].
\]

(4.37)

Since \( c \) is a constant in the equation 4.37, \( \Delta I \) is the same as in equation 4.35 with a different form of \(|I|\). The error function \( erf\left(\frac{c}{\sqrt{2}}\right) \) cannot be solved analytically, but numerical values can be found in almost any book of mathematical tables.

Figure 4.5 shows the fit to the \(^1\text{H}(d, p)^2\text{H}\) scattering at \( \theta_{\text{lab}} = 60^\circ \). The \( \chi^2 / \text{datum} \) for the exponential-Gaussian fits ranged from 1.5 to 2.0 while the \( \chi^2 / \text{datum} \) for the purely Gaussian fits were two orders of magnitude larger. The exponential-Gaussian fits the peak very well while the pure Gaussian cannot reproduce the spectrum adequately.

With all of the corrections made to the data as well as the optimization of the background subtraction, the \( \sigma_{\text{rel}}(\theta) \) was determined. Figure 4.6 is a plot of \( \sigma_{\text{rel}}(\theta) \) with respect to \( \theta_{\text{c.m.}} \). The normalization procedure is the last step.
Figure 4.6: Relative cross-section for $^1\text{H}(d,d)^1\text{H}$ scattering at $E_{c.m.} = 667$ keV. The solid curve is the variational calculations with AV18+UR. The data are normalized to the theoretical calculations at $\theta_{c.m.} = 92.4^\circ$.

4.3 Absolute $\sigma(\theta)$ Normalization

As it was stated in Section 3.1.2, the absolute $\sigma(\theta)$ for d-p scattering was normalized to p-p scattering. Again, the reader is referred to equation 3.2 for the cross-section with a detector efficiency of 1.0 which is repeated here:

$$\sigma(\theta) = \frac{I_0(\theta)}{N_b n_t \Delta \Omega}. \quad (4.38)$$

The number of target nuclei per area $n_t$ was the difficult quantity to measure. The incident deuterons sputtered the target $^1\text{H}$-atoms at an appreciable amount [Gei98] over an extended time period. To avoid measuring the target thickness continuously throughout the experiment, the targets were studied with a proton beam, and the d-p scattering $\sigma_{ref}(\theta)$ was normalized to the absolute cross-section for p-p scattering. The following equations are for the d-p scattering and p-p scattering cross-sections, $\sigma_{dp}(\theta)$ and $\sigma_{pp}(\theta)$, respectively

$$\sigma_{dp}(\theta) = \frac{I_0(dp)}{N_b(dp)n_t \Delta \Omega}, \quad \text{and} \quad \sigma_{pp}(\theta)$$
\[ \sigma_{pp}(\theta) = \frac{I_0(pp)}{N_b(pp)n_t \Delta \Omega}. \] (4.40)

Since the same detector arrangement was used, \( \Delta \Omega \) was the same for each beam. The target thickness \( n_t \) should have also been the same for both beam runs because short runs were taken. Actually, the target deterioration was linear and will be discussed in the next paragraph. If equation 4.39 is divided by equation 4.40, and \( \sigma_{pp}(\theta) \) is moved to the right-hand side, \( \sigma_{dp}(\theta) \) becomes

\[ \sigma_{dp}(\theta) = \frac{I_0(dp)}{N_b(dp)} \times \frac{N_b(pp)}{I_0(pp)} \times \sigma_{pp}(\theta). \] (4.41)

The number of counts detected divided by the number of incident beam particles is just the yield, so this equation has the simpler form of

\[ \sigma_{dp}(\theta) = Y_{dp} \left[ \frac{\sigma_{pp}(\theta)}{Y_{pp}} \right]. \] (4.42)

The d-p scattering and p-p scattering yields were measured, and \( \sigma_{pp}(\theta) \) was obtained using the Nijmegen phase-shift analysis [Ren97]. The reliability of the Nijmegen phase-shift analysis was tested by comparing \(^1\text{H}(p,p)^1\text{H} \) scattering calculations with data at energies close to \( E_p = 4.0 \text{ MeV} \). The previous data considered were those of Knecht, Dahl, and Messalt [Kne66] at \( E_p = 3.037 \text{ MeV} \) and Imai et al. [Ima75] at \( E_p = 4.978 \text{ MeV} \). With these data sets, the comparisons span the energy of our experiment and are in close proximity. The data of Knecht, Dahl, and Messalt are known to 0.3\%, and the measurements of Imai et al. have statistical errors of \( \approx 0.3\% \) and systematic errors of 0.2\%. With these precise measurements, the calculations of the Nijmegen phase-shift analysis agree within 0.5\%.

Before the normalization was determined, we had to convince ourselves that the deuteron beam and proton beam interacted with the target in the same manner. This was done by comparing the ratio of the measured yields from the scattering of both beams to the calculated cross-sections for both scattering processes. This statement is true mathematically if equation 4.42 is rearranged...
Table 4.5: Ratio of measured $^1\text{H}(p, p)^1\text{H}$ yields to either $^1\text{H}(d, d)^1\text{H}$ or $^1\text{H}(d, p)^2\text{H}$ measured yields. The errors contain the experimental uncertainties of both d-p and p-p scattering yields.

<table>
<thead>
<tr>
<th>Target</th>
<th>$\frac{Y_{pp}(25^\circ)}{Y_{dd}(25^\circ)}$</th>
<th>$\frac{Y_{pp}(25^\circ)}{Y_{dp}(25^\circ)}$</th>
<th>$\frac{Y_{pp}(35^\circ)}{Y_{dp}(35^\circ)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.233 ± 0.005</td>
<td>0.443 ± 0.002</td>
<td>0.581 ± 0.006</td>
</tr>
<tr>
<td>2</td>
<td>0.238 ± 0.005</td>
<td>0.447 ± 0.003</td>
<td>0.579 ± 0.005</td>
</tr>
<tr>
<td>3</td>
<td>0.238 ± 0.004</td>
<td>0.453 ± 0.003</td>
<td>0.580 ± 0.003</td>
</tr>
</tbody>
</table>

as

$$\frac{\sigma_{pp}(\theta)}{\sigma_{dp}(\theta)} = \frac{Y_{pp}}{Y_{dp}}. \quad (4.43)$$

If the ratio of the measured yields agree with the ratio of the calculations, it is safe to assume that both beams scattered from the target in a reproducible manner. For $\sigma_{dp}(\theta)$, the calculations were those of Keivsky et al. [Kie98b]. From the comparison with the data of Kocher and Clegg [Koc69], the assumption is made that the calculations are reliable to 1.5%. Figure 4.7 are plots of yield versus run number where the actual run number is replaced with the numbers 1 through 7. The number 1 is the first run on a target, and 7 is the last run. The odd sequence numbers relate to an experiment with the deuteron beam, and the even numbers are for runs using the proton beam. Since the deuteron beam struck the target before the proton beam, the target did not have the same thickness for both beams. The beam was only on the target for short periods of time. When data were not being taken, a beam stop was in place to prevent the loss of target material. Linear fits were made to the data, and the yields were interpolated for the corresponding run numbers. The fits reproduced the data to $\approx 0.3\%$ which was well below the 1% statistical uncertainty of the measured yields. The $^1\text{H}(p, p)^1\text{H}$ yields were divided by $^1\text{H}(d, d)^1\text{H}$ and $^1\text{H}(d, p)^2\text{H}$ yields, respectively, for each run of each target. Table 4.5 lists the average ratio over all of the runs for each target. Table 4.6 provides cross-sections for $^1\text{H}(d, d)^1\text{H}$ calculated using the variational method with AV18+UR and for $^1\text{H}(p, p)^1\text{H}$ by
Figure 4.7: Target tests for the absolute cross-section normalization. The circles are the yields and the solid lines are linear fits to the data.

the Nijmegen phase-shift analysis. The calculated ratios are in close agreement with the measured ratios.

With the target deterioration understood, the p-p scattering relative cross-section which was measured during the target tests was normalized to the calculated absolute cross-section at $\theta_{c.m.} = 50.1^\circ$ ($\theta_{lab} = 25^\circ$). The cross-section value provided by the Nijmegen phase-shift analysis was divided by the measured $^1\text{H}(p, p)^1\text{H}$ scattering relative cross-section to give a normalization factor. The absolute cross-section for $^1\text{H}(d, d)^1\text{H}$ scattering was determined by multi-
Table 4.6: Ratio of \(^1\text{H}(p, p)^1\text{H}\) cross-sections to either \(^1\text{H}(d, d)^1\text{H}\) or \(^1\text{H}(d, p)^2\text{H}\) cross-sections. The \(^1\text{H}(p, p)^1\text{H}\) cross-sections were calculated from Nijmegen phase-shift analysis, and the \(^1\text{H}(d, d)^1\text{H}\) and \(^1\text{H}(d, p)^2\text{H}\) cross-sections were determined from the variational method with the AV18+UR potentials. From kinematics, \(^1\text{H}(d, d)^1\text{H}\) scattering does not occur at \(\theta_{lab} > 30^\circ\).

<table>
<thead>
<tr>
<th>(\theta_{lab})</th>
<th>(\sigma_{pp}) ((\text{mb sr}))</th>
<th>(\sigma_{dd}) ((\text{mb sr}))</th>
<th>(\sigma_{dp}) ((\text{mb sr}))</th>
<th>(\frac{\sigma_{pp}}{\sigma_{dd}})</th>
<th>(\frac{\sigma_{pp}}{\sigma_{dp}})</th>
</tr>
</thead>
<tbody>
<tr>
<td>25°</td>
<td>407 ± 2</td>
<td>1760 ± 26</td>
<td>930 ± 14</td>
<td>0.231 ± 0.004</td>
<td>0.437 ± 0.007</td>
</tr>
<tr>
<td>35°</td>
<td>383 ± 2</td>
<td>-</td>
<td>656 ± 10</td>
<td>NA</td>
<td>0.584 ± 0.009</td>
</tr>
</tbody>
</table>

Applying the same normalization factor to the d-p scattering relative cross-sections from the normalization tests according to equation 4.42. The results were converted to the center-of-mass system and are shown in Figure 4.8. The normalized d-p scattering data are in excellent agreement with the variational calculations at these three angles. Finally, the entire cross-section angular distribution was normalized.

So far, the errors listed for the cross-section measurement are purely from counting statistics. The uncertainty in the normalization is also a factor in the measurement. The uncertainty in Nijmegen phase-shift determinations was discussed earlier in this section and was about 0.5% from the comparison of previous data [Kne66, Ima75]. Another source of systematic uncertainty was the determination of the beam-current integration. From the \(^{197}\text{Au}(d, d)^{197}\text{Au}\) and \(^{197}\text{Au}(p, p)^{197}\text{Au}\) scattering measurements (see Section 3.1.2), 0.6% was set as the uncertainty in beam-current integration. These two systematic errors were summed in quadrature to provide 0.8% in the normalization uncertainty. Another source of uncertainty was in the the angle settings. However, the uncertainty in angular position is dependent on how quickly the distribution changes with respect to angle. To investigate this effect, the angular distribution was fit with a 10th order polynomial. Next, the cross-section was calculated for a 0.1° change
Figure 4.8: Normalization of $\sigma(\theta)$ for $^1\text{H}(d,d)^1\text{H}$ from target testing at $\theta_{\text{c.m.}} = 82.7^\circ$ and $110.0^\circ$. The solid curve is the variational calculation including the AV18 and UR potentials.

in either direction. For the $\theta_{\text{lab}} = 9.0^\circ$ data point, the cross-section would differ by 5% in the $-0.1^\circ$ direction and $3.5\%$ in the $+0.1^\circ$ direction. In a flatter region of the distribution such as the point corresponding to $\theta_{\text{lab}} = 27.0^\circ$, the difference is $0.3\%$ in either direction. The final sources of uncertainty in the experimental procedure were the knowing the incident beam energy. With the analyzing magnet after the accelerator, the uncertainty was $\approx 0.4\%$ [Fis97]. Uncertainty due to energy loss in the target was approximately $0.3\%$ as described in Section 3.1. The uncertainty in beam energy does not cause the cross-section data to scale as in the case of the normalization errors. Instead, different regions of the angular distribution shift by different amounts. For example, there would be a larger shift at forward angles where the Coulomb interaction dominates than at $\theta_{\text{c.m.}} = 90.0^\circ$ where the nuclear interaction is the primary interaction.
Table 4.7: Summary of uncertainties in absolute cross-section measurement. The error in the angle setting is for the data around $\theta_{lab} = 27.0^\circ$ ($\theta_{c.m.} = 92.2^\circ$)

<table>
<thead>
<tr>
<th>Source</th>
<th>Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Counting statistics</td>
<td>0.5</td>
</tr>
<tr>
<td>Normalization to p-p $\sigma(\theta)$</td>
<td>0.5</td>
</tr>
<tr>
<td>Beam current integration</td>
<td>0.6</td>
</tr>
<tr>
<td>Angle setting</td>
<td>0.3</td>
</tr>
<tr>
<td>Energy loss</td>
<td>0.3</td>
</tr>
<tr>
<td>Beam energy</td>
<td>0.4</td>
</tr>
</tbody>
</table>

Table 4.7 summarizes the contributions to the uncertainty of the absolute cross-section measurement at $\theta_{lab} = 27.0^\circ$ ($\theta_{c.m.} = 92.2^\circ$).

A comparison between the absolute cross-section data and the theoretical predictions is reserved for Chapter 6.
Chapter 5

Phase-Shift Analysis

To gain a greater understanding of the differences between the experimental data and theoretical calculations of d-p scattering, something more must be done beyond comparing the values of the observables. In the previous chapter, measurements are compared to the theory and obvious discrepancies were observed for $A_y$ and $iT_{11}$. There are smaller, yet still significant differences in $T_{20}$, $T_{21}$, and $T_{22}$ as well. The objective of this project is to locate the underlying nuclear physics (the partial-wave contributions of the system) for which the theory has not accounted. A single-energy phase-shift analysis was conducted for the data set taken at $E_{c.m.} = 667$ keV to do just that.

5.1 Theoretical Calculations of Phase Shifts

Most quantum mechanics textbooks [Mer98, Mor53] introduce the phase shift in the solution of the scattering of a particle from a potential. The incident particle is considered to be a plane wave, and the outgoing particle is represented by a spherical wave. In the asymptotic region ($r \to \infty$), the potential goes to zero which is a valid assumption for the nuclear potential since it has a finite range (see Section 1.1). In essence, the scattered particle no longer feels the effect of the other particles in the collision. For $^1\text{H}(d,d)^1\text{H}$ and $^2\text{H}(p,p)^2\text{H}$ scattering, this statement is not entirely true since the Coulomb force is present. However,
the Coulomb repulsion is well understood and is neglected to focus on the nuclear interaction. With the potential neglected, the initial state wavefunction is equated to the final state or scattered wavefunction. The difference between the two wavefunctions is described by the phase shift. The scattered wavefunction is the same as the incident wavefunction but is shifted by a specific amount due to the interaction with the potential.

In the introductory derivation, the scattering system has been solved without including spin and/or isospin. A realistic solution with spin and isospin follows the same guidelines of the simpler case and can be found in the literature ([Mer98, Kie97a]). To simplify the discussion, I will proceed directly to the solution. The asymptotic wavefunction from the nuclear interaction of N-d scattering is written as [Kie97a]

\[
\psi^\lambda_{LSJ}(\vec{x}_i, \vec{y}_i) = \sum_{L=0,2} \psi_{L=0,2}(\vec{x}_i) R^\lambda_L(\vec{y}_i) \times \\
\times \left\{ \left[ (Y_{L=0,2}(\vec{x}_i) s_j^k \right]_1 \otimes s_l^r \otimes Y_L(\vec{y}_i) \right\}_{JJ_z} \left[ t_j^k t_l^r \right]_{TT_z},
\]

where \( \psi_{L=0,2}(\vec{x}_i) \) is the radial component of the deuteron wavefunction with the nucleons separated by \( x_i \) and \( R^\lambda_L(\vec{y}_i) \) describes the other radial component (\( y_i \) is the separation of the third nucleon and the deuteron). The quantity in the \( \left\{ \right\}_{JJ_z} \) carries the angular momentum coupling, and the isospin components are found in \( \left[ \right]_{TT_z} \). This asymptotic wavefunction has two solutions, a real (\( \lambda = R \)) and an imaginary (\( \lambda = I \)). With the real solution, \( R^\lambda_L(\vec{y}_i) \) takes the functional form of the regular solution \( F_L(\vec{y}_i) \) [Arf95]. With the imaginary solution, \( R^\lambda_L(\vec{y}_i) \) is in the form of the irregular solution \( y_i^{L+1} G_L(\vec{y}_i) \) [Arf95]. The asymptotic solution can be generalized as a sum of the incoming and outgoing waves

\[
\psi_{LSJ}(\vec{x}_i, \vec{y}_i) = \psi^0_{LSJ}(\vec{x}_i, \vec{y}_i) + \sum_{L' S' L'S'L'J} U_{L'S'L'S'L'J} \psi^1_{L'S'L'S'L'J}(\vec{x}_i, \vec{y}_i),
\]

where \( \psi^0_{LSJ}(\vec{x}_i, \vec{y}_i) \) and \( \psi^1_{LSJ}(\vec{x}_i, \vec{y}_i) \) are combinations of the regular and irregular solutions

\[
\psi^0_{LSJ}(\vec{x}_i, \vec{y}_i) = i \psi^R_{LSJ}(\vec{x}_i, \vec{y}_i) - \psi^I_{LSJ}(\vec{x}_i, \vec{y}_i)
\]
\[
\psi_{L,SJ}^I(x_i, y_i) = i\psi_{L,SJ}^R(x_i, y_i) + \psi_{L,SJ}^J(x_i, y_i). \tag{5.3}
\]

The important result is the inclusion of the collision matrix \( U \) in the outgoing function. The collision matrix contains all of the phase shifts from the nuclear interactions. For spin-\( \frac{1}{2} \times \text{spin-1} \) scattering, the \( U \)-matrix is a \( 3 \times 3 \) matrix and which can be parameterized as \( [\text{Sey69}] \)

\[
U^{J\pi} = (u^{J\pi})^\dagger e^{2i\delta^{J\pi}} u^{J\pi}. \tag{5.4}
\]

The \( \delta^{J\pi} \)-matrix contains all of the phase shifts

\[
\delta^{J\pi} = \begin{pmatrix}
\delta_{J=\frac{5}{2}} & 0 & 0 \\
0 & \delta_{J=\frac{3}{2}} & 0 \\
0 & 0 & \delta_{J=\frac{1}{2}}
\end{pmatrix}. \tag{5.5}
\]

The partial waves are allowed to interfere with one another so there should be a mechanism to calculate the "mixing". That is where the \( u^{J\pi} \) matrix enters. This matrix is a combination of three rotation matrices \( u^{J\pi} = v^{J\pi} w^{J\pi} x^{J\pi} \), where each matrix appears as

\[
v^{J\pi} = \begin{pmatrix}
1 & 0 & 0 \\
0 & \cos\epsilon_{J\pi} & \sin\epsilon_{J\pi} \\
0 & -\sin\epsilon_{J\pi} & \cos\epsilon_{J\pi}
\end{pmatrix} \tag{5.6}
\]

\[
w^{J\pi} = \begin{pmatrix}
\cos\zeta_{J\pi} & 0 & \sin\zeta_{J\pi} \\
0 & 1 & 0 \\
-\sin\zeta_{J\pi} & 0 & \cos\zeta_{J\pi}
\end{pmatrix} \tag{5.7}
\]

\[
x^{J\pi} = \begin{pmatrix}
\cos\eta_{J\pi} & \sin\eta_{J\pi} & 0 \\
-\sin\eta_{J\pi} & \cos\eta_{J\pi} & 0 \\
0 & 0 & 1
\end{pmatrix}. \tag{5.8}
\]

The three angles \( \epsilon_{J\pi}, \zeta_{J\pi}, \) and \( \eta_{J\pi} \) are the mixing parameters which give the strength of the mixing. Table 5.1 displays the quantities that are related to each
Table 5.1: List of angular momentum mixing and the relations to the mixing parameters.

<table>
<thead>
<tr>
<th>$\epsilon_{J\pi}$</th>
<th>$\zeta_{J\pi}$</th>
<th>$\eta_{J\pi}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\vec{S}$</td>
<td>mixing</td>
<td>no mixing</td>
</tr>
<tr>
<td>$\vec{L}$</td>
<td>no mixing</td>
<td>mixing</td>
</tr>
</tbody>
</table>

mixing parameter. This parameterization makes the $U$-matrix to be a unitary, symmetric matrix. With all of these tools, the phase shifts and the new quantities (the mixing parameters) can be found.

To calculate the observables from the phase shifts, the transition matrix $M$ is constructed. An in-depth discussion is given in [Sey69]. For an incident wavefunction with $S$ and $L$ and an outgoing wavefunction with $S'$ and $L'$, the elements of the $M$-matrix are

\[
M_{SS'LL'} = \frac{\sqrt{\pi}}{k} \left[ -C(\theta)\delta_{SS'}\delta_{S_zS'_z} + i \sum_{JLL'} (2L+1)^{1/2} (SJS_0) (SLS_0) \times \\
\times (S'J'S_z - S'_z J'S_z) e^{i(\omega_L + \omega_{L'})} (\delta_{SS'}\delta_{L'L} - U_{S'S'}^{J'} SL) \times \\
\times Y_{S_z}^{S_z'}(\theta, 0) \right]. \quad (5.9)
\]

where $\delta_{SS'}$, $\delta_{LL'}$ and $\delta_{S_zS'_z}$ are Kronecker delta functions, $Y_L^m(\theta, \phi)$ are normalized spherical harmonics, $C(\theta)$ is the Coulomb scattering amplitude, $\omega_L$ and $\omega_{L'}$ are Coulomb corrections, and $U^{J'}$ is the collision matrix defined above. The Coulomb scattering amplitude allows for p-d and n-d scattering solutions. The observables can be calculated with the following relations

\[
\sigma(\theta) = \frac{1}{6} Tr\{MM^\dagger\} \quad (5.10)
\]

\[
A_y = \frac{Tr\{M\sigma_y M^\dagger\}}{2Tr\{MM^\dagger\}} \quad (5.11)
\]

\[
iT_{11} = \frac{\sqrt{3}Tr\{MS_y M^\dagger\}}{2Tr\{MM^\dagger\}} \quad (5.12)
\]

82
\[ T_{20} = \frac{Tr \{ M(3S_yS_z - 2M^1) \}}{\sqrt{2} Tr \{ MM^1 \}} \]

(5.13)

\[ T_{21} = -\frac{3Tr \{ MS_xS_zM^1 \}}{\sqrt{3} Tr \{ MM^1 \}} \]

(5.14)

\[ T_{22} = -\frac{1}{\sqrt{3}} \left( \frac{T_{20}}{\sqrt{2}} + \frac{Tr \{ M(3S_yS_z - 2M^1) \}}{Tr \{ MM^1 \}} \right) \]

(5.15)

The explicit forms of these matrices \( S_x, S_y, S_z \), and \( \sigma_y \) can be found in the paper by Seyler [Sey69].

### 5.2 Previous Phase-Shift Analyses

In the past ten years, there have been two single-energy PSA performed for d-p scattering below the breakup threshold. The first was the work of Knutson et al. [Knu93] at \( E_{c.m.} = 2 \) MeV. The data set included angular distributions for \( \sigma(\theta), A_y, iT_{11}, T_{20}, T_{21}, \) and \( T_{22} \). In all, 24 phase shifts and mixing parameters were allowed to vary with partial waves of \( L > 3 \) that were fixed by theory. The theoretically-calculated phase shifts were based on a solution of the Faddeev equations with a NN potential and an approximation of the Coulomb interaction [Ber90b]. The PSA found that the largest discrepancies were with \( ^2S_{\frac{1}{2}} \) and \( ^1D_{\frac{1}{2}} \), which differ from the calculations by approximately 30% and 320%, respectively. These differences were attributed to the underbinding of the \(^3\)He by the calculations. The P-P mixing parameters from the fits disagreed with the calculated parameters by small amounts (\( \approx 0.5\% \)), but greatly influenced the fits to the analyzing powers.

The PSA of Kievsky et al. [Kie96] was also performed at \( E_{c.m.} = 1.67 \) MeV and 2 MeV where there exists complete data sets of cross section and all of the aforementioned analyzing powers. For these fits, all of the phases of \( L \leq 3 \) (32 total) were varied and the phases of higher \( L \) were held constant. The variational calculations with the AV18+UR potentials were employed to provide
the predicted phase shifts. These PSA agreed with the analysis of Knutson et al. in concluding that $^2S_1$ is overestimated if the calculations only include a NN interaction. It was also discovered that $A_y$ changed by approximately 30% when the $^4P_1$ phase shift was adjusted by 3%. Finally, the fit to $iT_{11}$ was improved by the variation in the mixing parameter $\epsilon_{\frac{3}{2}}$. Another PSA was performed at $E_{c.m.} = 667$ keV and 1.33 MeV by Kievsky et al. [Kie96] where there is only cross section and $A_y$ data. The fits to $A_y$ were influenced primarily by $^4P_1$ phase shift while fits to $\sigma(\theta)$ were improved by the inclusion of $^4S_{\frac{3}{2}}$.

### 5.3 Phase Shifts from Experimental Data

The phase-shift analysis was performed by taking theoretically-calculated phase shifts and mixing parameters [Kie96] as starting values, allowing some of them to freely vary, calculating the observables by the equations previously described, and fitting the observables to the data. This process was employed with the complete set of measurements at $E_{c.m.} = 667$ keV. The FORTRAN program PSA.F [Bru00] was used for the phase-shift analysis. Phase shifts were included up to a maximum angular momentum of $L=4$. With the low energy of the system, higher partial waves do not contribute.

With the original phase shifts and mixing parameters as well as experimental values read in, the fitting process was begun. The fit consisted of a $\chi^2$ minimization. The $\chi^2$ is a function which gives a measure of the difference between experiment and theory while accounting for the error in the measurement. The expression is

$$\chi^2 = \sum_i \left[ \frac{x_i(\theta) - y_i(\theta)}{\Delta x_i(\theta)} \right]^2, \quad (5.16)$$

where $x_i(\theta)$ is the $i^{th}$ experimental value with an error of $\Delta x_i(\theta)$. The corresponding theoretical value is $y_i(\theta)$. For the PSA, the theoretical value was the value calculated from the phase shifts. The $\chi^2$ is then a sum over all $N$ data
Table 5.2: The $\chi^2_N$ for each observable after each trial of the phase-shift analysis. The total $\chi^2_N$ is the sum of $\chi^2$ for each observable divided by 143 (the total number of data points).

<table>
<thead>
<tr>
<th>Trials</th>
<th>Free Parameters</th>
<th>$\sigma(\theta)$</th>
<th>$A_y$</th>
<th>$iT_{11}$</th>
<th>$T_{20}$</th>
<th>$T_{21}$</th>
<th>$T_{22}$</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>AV18+UR</td>
<td>None</td>
<td>1.2</td>
<td>190.5</td>
<td>61.4</td>
<td>1.0</td>
<td>2.5</td>
<td>0.7</td>
<td>13.9</td>
</tr>
<tr>
<td>1</td>
<td>$^{4}P_{\frac{1}{2}}, \epsilon_{\frac{1}{2}-}$</td>
<td>1.1</td>
<td>0.6</td>
<td>1.8</td>
<td>1.7</td>
<td>1.2</td>
<td>0.7</td>
<td>1.2</td>
</tr>
<tr>
<td>2</td>
<td>$^{4}P_{\frac{1}{2}}, \epsilon_{\frac{3}{2}-}$</td>
<td>1.2</td>
<td>0.5</td>
<td>5.3</td>
<td>2.2</td>
<td>5.7</td>
<td>0.7</td>
<td>2.2</td>
</tr>
<tr>
<td>3</td>
<td>$^{4}P_{\frac{1}{2}}, \epsilon_{\frac{3}{2}-}$</td>
<td>1.3</td>
<td>0.5</td>
<td>1.9</td>
<td>0.8</td>
<td>1.8</td>
<td>0.7</td>
<td>1.2</td>
</tr>
<tr>
<td>4</td>
<td>$^{4}P_{\frac{1}{2}}, ^{4}P_{\frac{3}{2}}$, $^{4}P_{\frac{3}{2}}, \epsilon_{\frac{1}{2}-}$</td>
<td>1.1</td>
<td>0.5</td>
<td>1.8</td>
<td>0.9</td>
<td>1.2</td>
<td>0.7</td>
<td>1.0</td>
</tr>
</tbody>
</table>

points. If the difference is equal to the error for every data point, the $\chi^2$ is equal to $N$. A useful calculation to do is $\bar{\chi}^2_N$ or $\chi^2_N$. A $\chi^2_N$ larger than one corresponds to a comparison which is outside of a number of the experimental errors. If $\chi^2_N < 1$, the uncertainty in the data is overestimated. Each time a parameter was varied, the observables were calculated, and the $\chi^2$ was determined. The value of the parameter was shifted until a minimal $\chi^2$ was found. Without changing any parameters, I calculated the $\chi^2_N$ between the theoretical and experimental values. The first line of Table 5.2 lists the initial $\chi^2_N$ for each observable and for the set as a whole. Previous phase-shift analyses [Kie96] have linked the VAPs problem to the $^{4}P_J$-wave splitting. The first parameters to be varied were $^{4}P_{\frac{3}{2}}$ and $\epsilon_{\frac{1}{2}-}$. Table 5.2 shows the $\chi^2_N$ of the fit. By varying only two parameters, a good fit was produced with a most dramatic improvement in the VAPs. The $\chi^2_N$ reduced by a factor of approximately 400 for $A_y$ and a factor of 30 for $iT_{11}$. By making separate trials with the pairs $^{4}P_{\frac{3}{2}}$ and $\epsilon_{\frac{1}{2}-}$ and $^{4}P_{\frac{3}{2}}$ and $\epsilon_{\frac{3}{2}-}$, similar results were achieved with the $\chi^2_N$ although $^{4}P_{\frac{3}{2}}$ and $\epsilon_{\frac{1}{2}-}$ gave the best fit. The separate trials are listed in Table 5.2. When all three $^{4}P_J$ and $\epsilon_{\frac{1}{2}-}$ were varied together, the improvement in $\chi^2_N$ was in $T_{21}$ in addition to $iT_{11}$ and $A_y$. While the $\chi^2_N$
was greatly improved, the fit needed to be fine tuned. The extra refining was accomplished by including a few more parameters. Whenever new parameters were varied the previously-free parameters were held constant with their best-fit values. The two groups of parameters were switched from being free to constant until a reproduceable $\chi^2_N$ was found. After these iterations produced the same $\chi^2_N$, all of the trial parameters were varied together. In this manner, local minima in the parameter space were avoided.

The fine tuning beyond the $^4P_J$ phase shifts consisted of a fit with the least number of free parameters and achieving $\chi^2_N = 1.0$ without the parameters diverging to unphysical values. Since a two parameter fit with $^4P_\frac{1}{2}$ and $\epsilon_{\frac{3}{2}-}$ had the most effect on the fit, a third parameter was included to test the effect of varying other parameters. The choice of beginning with $^4P_\frac{1}{2}$ and $\epsilon_{\frac{3}{2}-}$ is somewhat arbitrary, but these parameters provided a good starting point. The same result for the final fit was found when other $^4P_J$ phase shifts were employed in the first step of the analysis. In the end, seven free parameters were varied to give the best fit. The remaining phases and mixing parameters were taken from the theoretical calculations [Kie96]. As stated earlier, the three $^4P_J$ phase shifts improved the fits to $T_{21}$. The $^4S_\frac{3}{2}$ phase shift primarily affected the $\chi^2_N$ for $\sigma(\theta)$ with a small effect on the fits to $T_{21}$. The mixing parameter $\eta_{\frac{1}{2}+}$ helped lower the $\chi^2_N$ of $T_{22}$. Finally, $^4D_\frac{1}{2}$ was included to reduce the $\chi^2_N$ of $iT_{11}$ from 1.8 to 1.0. The final values for the parameters are given in Table 5.3, and the $\chi^2_N$ for the fits is listed in Table 5.4. A comparison of the fits to the experimental data will be shown in Chapter 6 with a more detail discussion of the results.

Errors were calculated for specific parameters by calculating the total $\chi^2 + 1$. Once I was satisfied with a fit, the parameter in question was shifted by a small amount and held constant. The fit was run again with the remaining free parameters, and a total $\chi^2$ was calculated. The parameter was incremented again, and the process was repeated. Figure 5.1 has four examples of plots of total $\chi^2$ versus a given parameter. Each graph has the characteristic parabolic
shape of a $\chi^2$ study. The value of the uncertainty was extracted by finding the
difference between the values of the parameter at the minimum and $\chi^2 + 1$. The
errors in a fit such as this are dependent on the number of free parameters. The
more parameters are allowed to vary leads to a larger error in each parameter.
The uncertainties in phase shifts and mixing parameters are given in Table 5.3.

The implications of this analysis will be discussed in the next chapter along
with a possible solution to the “$A_y$ Puzzle”.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>AV18+UR (°)</th>
<th>Final (°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^4S_{\frac{3}{2}}$</td>
<td>-36.930</td>
<td>-36.983(24)</td>
</tr>
<tr>
<td>$^4P_{\frac{1}{2}}$</td>
<td>9.199</td>
<td>9.071(30)</td>
</tr>
<tr>
<td>$^4P_{\frac{3}{2}}$</td>
<td>10.690</td>
<td>10.663(27)</td>
</tr>
<tr>
<td>$^4P_{\frac{5}{2}}$</td>
<td>10.070</td>
<td>10.087(26)</td>
</tr>
<tr>
<td>$^4D_{\frac{3}{2}}$</td>
<td>-0.768</td>
<td>-0.786(07)</td>
</tr>
<tr>
<td>$\eta_{\frac{1}{2}+}$</td>
<td>1.600</td>
<td>1.762(57)</td>
</tr>
<tr>
<td>$\epsilon_{\frac{3}{2}^-}$</td>
<td>-1.060</td>
<td>-1.272(14)</td>
</tr>
</tbody>
</table>
Table 5.4: Values of $\chi^2_N$ from final phase-shift analysis.

<table>
<thead>
<tr>
<th>Data</th>
<th>$\chi^2_N$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>AV18+UR</td>
</tr>
<tr>
<td>$\sigma(\theta)$</td>
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</tr>
<tr>
<td>$A_y$</td>
<td>190.5</td>
</tr>
<tr>
<td>$iT_{11}$</td>
<td>61.4</td>
</tr>
<tr>
<td>$T_{20}$</td>
<td>1.0</td>
</tr>
<tr>
<td>$T_{21}$</td>
<td>2.5</td>
</tr>
<tr>
<td>$T_{22}$</td>
<td>0.7</td>
</tr>
<tr>
<td>Total</td>
<td>13.9</td>
</tr>
</tbody>
</table>

Figure 5.1: Study of $\chi^2 + 1$ for selected parameters from phase-shift analysis.
Chapter 6

Discussion

This chapter is divided into two sections. The first part is a comparison of the data to the theoretical calculations using the variational method. A $\chi^2$ analysis will be shown for each observable to provide more quantitative comparisons. The second part is a discussion of the results and limitations of the phase-shift analysis (PSA).

6.1 Comparison to Theory

6.1.1 The Absolute Cross-Section

Figure 6.1 provides the comparison between the absolute $\sigma(\theta)$ and the theoretical predictions which have been calculated with and without the UR-3N potential. To make a comparison with greater sensitivity, Figure 6.2 plots the ratio of experimental data to the theoretical cross-section versus $\theta_{c.m.}$. The data show excellent agreement with the calculations with a few data points above 1.0 in the forward angles. None of the data are farther than two standard deviations away from 1.0.

Even though the $^2\text{H}(p,p)^2\text{H}$ data are relative measurements, a qualitative comparison with the $^1\text{H}(d,d)^1\text{H}$ scattering cross-section measurements can be made. The ratio of experiment to theory for the $^2\text{H}(p,p)^2\text{H}$ data was normalized
Figure 6.1: Absolute differential cross-section for $^1\text{H}(d,d)^1\text{H}$ at $E_{c.m.} = 667$ keV. The solid and dashed curves are calculations with the AV18 and AV18+UR potentials, respectively. The graph shows no visible difference between the data and the calculations or between the calculations themselves.

Figure 6.2: Ratio of experimental cross-section for $^1\text{H}(d,d)^1\text{H}$ scattering to variational calculations with the AV18 and UR potentials.
to the results of the $^1$H$(d,d)^1$H measurement at $\theta_{c.m.} = 87.3^\circ$ and is displayed in Figure 6.3. The two measurements are within errors of each other and have the same overall trend. Linear fits were made to both cross-section distributions in Figure 6.3 for a more quantitative comparison. The slope and the intercept for the fit to the $^1$H$(d,d)^1$H scattering data were $-6 \times 10^{-5} \pm 2 \times 10^{-5}$ and $1.009 \pm 0.002$. In the case of the $^2$H$(p,p)^2$H scattering data, the slope and intercept were $-14 \times 10^{-5} \pm 2 \times 10^{-5}$ and $1.014 \pm 0.002$. The magnitudes of the slopes are very small and differ by almost two standard deviations. The intercepts compare well between the two sets of data and to the theoretical predictions (an intercept of 1.000). The similarity in data sets confirms that the absolute measurement has the correct angular shape.

The important comparison is between these data and the two previous, disparate measurements. The $\sigma(\theta)$ data of Kocher and Clegg [Koc69] and Huttel et al. [Hut83a] were mentioned in Chapter 1 as a motivation for the cross-section measurements. Figure 6.4 is a plot of all three data sets. The forward-angle data
Figure 6.4: Comparison of three $\sigma(\theta)$ data sets normalized to the variational calculations with the AV18 and UR potentials. The triangles are the work of Kocher and Clegg [Koc69], and the squares are the measurements by Huttel et al. [Hut83a]. The circles are the data of this thesis.

have the trend of the Kocher and Clegg data and do not show the positive slope of the Huttel et al. data, and the Kocher and Clegg measurements are within errors of the current data. The general conclusion is that the new data do have slightly larger uncertainties than the previous measurements but double the total number of data points at $E_{c.m.} = 667$ keV over a much larger angular range. The overall agreement with the Kocher and Clegg data leads one to conclude that it is the more reliable measurement and that there exists a systematic problem in the Huttel et al. data.

6.1.2 The Tensor Analyzing Powers

The results for $T_{20}$, $T_{21}$, and $T_{22}$ measurements are plotted in Figure 6.5. A plot of $A_{yy}$ is included with $T_{22}$ since $A_{yy}$ was measured directly, and $T_{22}$ is a combination of $A_{yy}$ and $T_{20}$. From the graphs, the agreement between the data
Figure 6.5: Angular distributions for TAPs for $^1\text{H}(\vec{d},d)^1\text{H}$ at $E_{\text{c.m.}} = 667$ keV. The solid and dashed curves are calculations with the AV18 and AV18+UR potentials, respectively. The dot-dashed line marks zero on the graphs. The angular distribution for $A_{yy}$ is included since it was a directly-measured quantity unlike $T_{21}$ which is a combination of $T_{20}$ and $A_{yy}$.
and the theory is good. The calculations which include a 3N force reproduce the
data better than the ones with the NN force alone. A list of the data points is
given in Tables C.1 and C.2.

6.1.3 The Vector Analyzing Powers

After the raw data were reduced as described in Chapter 4, the \( A_y \) and \( iT_{11} \)
angular distributions took the shape shown in Figure 6.6. The numerical values
of the \( iT_{11} \) and \( A_y \) data points are listed in Tables C.3 and C.4, respectively.

From the plots, it is evident that there is a large discrepancy between theory and experiment. The inclusion of the UR 3N potential does not fill in the
gap. The “\( A_y \) Puzzle” [Wit94] remains at this energy. There is a previous measure-
ment [Hut83a] of \( A_y \) at this energy, and Figure 6.7 is a comparison with the
current data. Both sets of data show good agreement with each other. The previous
measurements have more points than our data set; however, the present data
have smaller errors and less scatter amongst points in the angular distribution.

6.1.4 \( \chi^2 \) Calculations

The entire 667 keV data set of cross-section and analyzing powers has been
displayed throughout this chapter. From the different graphs, the qualitative
comparisons between theory and experiment lead to the following conclusions.
The cross-sections show very good agreement, the TAPs have good agreement
and the VAPs have large discrepancies. To obtain a quantitative comparison,
the \( \chi^2 \) per datum was calculated.

For the present data set, there is a total number of 143 data points. Table 6.1
lists the number of data points in each angular distribution. The \( \chi^2_X \) for each
observable is also listed in Table 6.1 as well as the total for the entire data
set. The calculations were completed with both the AV18 potential and the
AV18+UR potentials. None of the data have been renormalized in order to find
Figure 6.6: Angular distributions for $iT_{11}$ and $A_y$ for p-d scattering at $E_{c.m.} = 667$ keV. The errors include the uncertainty in the beam polarization as well as statistical uncertainties. The solid and dashed curves are calculations with the AV18 and AV18+UR potentials, respectively.
Figure 6.7: Comparison with previous $A_y$ data at $E_{cm.} = 667$ keV. The triangles are the earlier work [Hut83a] and the circles are data of the author. The solid and dashed curves are calculations with the AV18 and AV18+UR potentials, respectively.

Table 6.1: The $\chi_N^2$ for each observable. The theoretical calculations include the AV18 and AV18+UR potentials.

<table>
<thead>
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<th></th>
<th>AV18</th>
<th>AV18+UR</th>
<th>Number of points</th>
</tr>
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<tr>
<td>Data</td>
<td>$\chi_N^2$</td>
<td>$\chi_N^2$</td>
<td></td>
</tr>
<tr>
<td>$\sigma(\theta)$</td>
<td>45.20</td>
<td>1.15</td>
<td>56</td>
</tr>
<tr>
<td>$A_y$</td>
<td>275.80</td>
<td>190.46</td>
<td>7</td>
</tr>
<tr>
<td>$iT_{11}$</td>
<td>112.35</td>
<td>61.39</td>
<td>8</td>
</tr>
<tr>
<td>$T_{20}$</td>
<td>3.54</td>
<td>0.98</td>
<td>24</td>
</tr>
<tr>
<td>$T_{21}$</td>
<td>4.46</td>
<td>2.52</td>
<td>24</td>
</tr>
<tr>
<td>$T_{22}$</td>
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<td>0.66</td>
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</tr>
<tr>
<td>Total</td>
<td>39.29</td>
<td>13.91</td>
<td>143</td>
</tr>
</tbody>
</table>
a minimal $\chi^2_N$. From $\chi^2_N$ calculations, it is possible to draw conclusions similar to those from the visual comparisons; the $\sigma(\theta)$ is best reproduced by theory, and the VAPs are the worst. However, all of the $\chi^2_N$ are improved when a three-nucleon force is included. For $\sigma(\theta)$, $\chi^2_N$ improves by an order of magnitude. The $\chi^2_N$ is reduced by $\frac{1}{3}$ to $\frac{1}{2}$ for the TAPs. The VAPs also show an improvement with UR; however, the $\chi^2_N$ remains in the hundreds. The VAPs problem is not solved with the inclusion of the UR interaction.

### 6.2 Results of PSA

The final fits of the PSA outlined in Chapter 5 are displayed in Figures 6.8 and 6.9. For the cross-section, the data are divided by the fit values to enhance
the sensitivity of the comparison. All of the fits reproduce the data very well. Including more free parameters reduced the $\chi^2_N$ below 1.0, but the resulting values of the parameters became unphysical. The phase shifts did not converge to a single value when more free parameters were added (such as the S-wave or $^2P_J$ phase shifts). Their values would change erratically as each new parameter was included. When all of the S-, P-, and D-wave phase shifts and their associated mixing parameters (20 total) were allowed to vary freely, the $\chi^2_N$ for each observable was well below 1.0. However, the parameters which were expected to be close to their theoretical value, such as the D-waves, deviated by many degrees. Additional parameters beyond a four parameter fit varied by large amounts forcing the $^4P_J$ and $\epsilon_+$ parameters to over-compensate to find the lowest $\chi^2_N$. This divergence of the parameters leads to the conclusion that the present number of

Figure 6.9: Fits to $A_y$ and $iT_{11}$ data by the PSA. The solid curve is the fit.
VAP data points is not enough to constrain the fits, and there are an infinite number of solutions in the fitting process. To remedy the situation, most of the parameters should be fixed by the theory. The price to pay is model dependence of the fixed phase shifts and mixing parameters. The least number of possible free parameters were varied to give reasonable results as in the case of the seven parameter fit.

The single-energy PSA produced very good fits to the data set with sensible results. The fit to the cross-section data was sensitive to the $^4S_\frac{3}{2}$ phase shift which was the similar result of the PSA of Kievsky et al. [Kie96]. The phase-shift analyses at $E_{cm.} = 2$ MeV [Knu93, Kie96] differ with this result in that the cross-section fits depended on $^2S_\frac{1}{2}$. The greatest influence to the $A_y$ and $iT_{11}$ fits came from the $^4P_\frac{1}{2}$ and $\epsilon_\frac{3}{2}$- parameters which was noticed in the earlier analyses [Knu93, Kie96]. In fact, $\epsilon_\frac{3}{2}$- was increased by $\approx 20\%$ over what was predicted even if the UR-3N potential was included in the predictions. This increase in the mixing parameter is about twice the enhancement required by the 2 MeV PSA. The theoretical calculations also appear to underestimate the splitting of the $^4P_J$ phase shifts by $0.1 - 0.2^\circ$. This result is lower in the case of the PSA at $E_{cm.} = 2$, MeV where the splitting was on the order of $0.5^\circ$.

With the cause of the “$A_y$ Puzzle” isolated to the $^4P_J$ phase shifts and $\epsilon_\frac{3}{2}$-, the next step is to find a physical interaction which will reproduce these parameters. If the interaction in the 3N system comes from NN forces only, one solution may lie in the values of the $^3P_J$ phase shifts in the NN potential models. Section 1.4 described the interplay between the $^3P_J$ phases of the NN interaction and the $^4P_J$ phases in N-d scattering VAP. Since the $^4P_\frac{1}{2}$ and $\epsilon_\frac{3}{2}$- parameters proved the greatest improvement to the VAP fits, the deficiency may lie in the $^3P_0$ and $^3P_1$ phase shifts, respectively, as indicated by Tornow et al. [Tor98]. The strength of $^3P_1$ may be too large causing an underestimation for $\epsilon_\frac{3}{2}$- in the theoretical calculations.

Another answer may be the construction of a new 3N force and not the
alteration of the NN potentials. Hüber and Friar [Hüb98] have suggested that a possible candidate should be a spin-dependent 3NF since the VAP is a difference between polarized cross-sections. One possible candidate is a phenomenological 3N $\bar{L} \cdot \vec{S}$ force [Kie99] of the form

$$V_{3N}^{ls} = \sum_{i<j} \frac{1}{2} \left[ w_{11}^{ls}(r_{ijk}) \bar{L}_{ij} \cdot \vec{S}_{ij} + \bar{L}_{ij} \cdot \vec{S}_{ij} w_{11}^{ls}(r_{ijk}) \right] P_{11}(i,j),$$

where $P_{11}(i,j)$ is the projector onto the spin-isospin states ($S = 1$, $T = 1$) and $r_{ijk}$ is a scalar function of the three interparticle distances. The 3N interaction has been proposed as

$$w_{11}^{ls}(r_{ijk}) = v_{11}^{ls}(r_{ij}) + W_0 e^{-\alpha \rho},$$

where $\rho$ is the hyperradius. The parameters $W_0$ and $\alpha$ are the strength and range of the force. Figure 6.10 shows a comparison of calculations using this potential with measured $A_y$ and $iT_{11}$ of the present data set. Three calculations with the different strength and range parameters are shown. The dotted curve represents a long-range interaction ($W_0 = -1$ MeV, $\alpha = 0.7$ fm$^{-1}$), the dashed curve is for a medium-range ($W_0 = -10$ MeV, $\alpha = 1.2$ fm$^{-1}$), and the dot-dashed curve indicates a short-range interaction ($W_0 = -20$ MeV, $\alpha = 1.5$ fm$^{-1}$) [Kie99]. This force influences the splitting of the $^4P_J$ phases and the magnitude of $\epsilon_5^-$ without appreciably affecting the other parameters. Moreover, when the deuteron and spectator nucleon are separated by a large distance, the force reduces to the NN $\bar{L} \cdot \vec{S}$ potential $v_{11}^{ls}(r_{ij})$. The addition of the 3N spin-orbit force has provided much better agreement than previous calculations. Since the best agreement is found with either the medium- or long-range 3N forces, one could speculate that the deficiency is in the $2\pi$ exchange potential. A modification such as an effective pion-mass in the presence of the third nucleon would have more physical meaning than the phenomenological potential provided here. One last observation is that the range and strength parameters needed to best reproduce one VAP do not give the best values for the other. The prediction with the medium-range force best
Figure 6.10: Comparison between $A_y$ and $iT_{11}$ and calculations with phenomenological spin-orbit forces. The solid curve is the calculations with AV18+UR. The dotted, dashed, and dot-dashed curves are calculations which include long-, medium-, and short-range 3N $\vec{L} \cdot \vec{S}$ forces, respectively.
describes the $iT_{11}$ measurement while the calculations with the long-range force agrees the best for the $A_y$ data. It would be interesting to know if there is some physical reason for this difference in describing $A_y$ versus $iT_{11}$ or if the cause is somewhere in the phenomenological nature of the new 3N spin-orbit force. Needless to say, the use of a phenomenological 3N $\vec{L} \cdot \vec{S}$ force has yielded agreement with the VAPs, and its refining may provide more insight into the role of the $^4P_J$ in the "$A_y$ Puzzle".
Chapter 7

Conclusions

The aim of this last chapter is to summarize what has been accomplished and to draw some conclusions. From there, suggestions will be made for future work.

7.1 Summary

Angular distributions for cross-section, $A_y$, $iT_{11}$, $T_{20}$, $T_{21}$, and $T_{22}$ have been measured with high precision for d-p scattering at the center-of-mass energy of 667 keV. The cross-section data were measured with a statistical uncertainty of approximately 0.4% and an error of 0.9% in the absolute normalization. Even though the analyzing powers were very small (on the order of $10^{-2}$ for the TAPS and $10^{-3}$ for the VAPs) the measurements were determined on average with a 3% statistical uncertainty. The data were compared with the most recent variational calculations [Kie98b]. Two sets of calculations were made, one employing only a 2N potential and the other with both 2N and 3N potentials. The cross-section measurements show excellent agreement with the variational predictions with a 3N force included. When the 3N force was used in the variational calculations, the $\chi_N^2$ lowered by an order of magnitude to 1.2. The TAPs data agree well with small differences in the $T_{20}$ and $T_{21}$ distributions ($\chi_N^2$ was lowered by a factor of 2 or 3 when the 3N force was utilized in the predictions). The VAPs comparison shows large discrepancies ($\approx 40\%$ at the maximum of the distributions). If both
the AV18 and UR potentials were included in the theoretical calculations, the \( \chi^2_N \) was 190.5 for \( A_y \) and 61.4 for \( iT_{11} \). The “\( A_y \) Puzzle” exists at low energies, and in fact, the difference between theory and experiment grows as the energy is lowered. A study of the \( \chi^2_N \) quantified these statements but also confirmed that the inclusion of a 3N potential moved the calculations closer to the experimental values. The improvement was greatest with the cross-section, showing a better description when the potential reproduces the \( ^3\text{He} \) binding energy. The VAPs calculations with a 3N potential still falls far from reproducing the data. The agreement between the VAPs data and the theoretical calculations improves when the phenomenological spin-orbit 3NF [Kie99] is included in the calculations.

With the complete set of analyzing powers and cross-section, a single-energy phase-shift analysis was performed. By varying two parameters (\( \epsilon_{\pm}, \ 4P_{\pm} \)) and maintaining all of the other parameters as determined from theory, an almost perfect fit was achieved for \( A_y \). For \( iT_{11} \), the \( \chi^2_N \) reduced by more than an order of magnitude to approximately 1.7. To determine a more realistic fit, the number of free parameters were increased to obtain \( \chi^2_N \) of 1.0. From this analysis, it is evident that the theoretical calculations underestimate the splitting of the \( 4P_J \) phase shifts. Furthermore, the mixing between \( 2P_{\pm} \) and \( 4P_{\pm} \) is underestimated by \( \approx 20\% \). The inclusion of a phenomenological 3N spin-orbit force into the calculations improves the description of the VAP. Further development of such a force is needed to understand the root of the “\( A_y \) Puzzle”.

### 7.2 Future Work

To produce a more stringent phase-shift analysis, there need to be more data points in the angular distributions. The VAPs distributions could be fit better if there are more data points to define the shape. The present work, optimized with the coincidence technique, could measure points to fill in the gaps in the distributions. This, however, would be inefficient. More work by both theorists
and experimentalists is necessary to investigate the 3N system further, although
at this point there is a sufficient set of experimental data to keep the theorists
busy for quite some time.

7.2.1 Theoretical Inquiries

As stated in Section 6.2, the increase in \( \epsilon_{3} \) from the variational calculations
may be due to a too large strength of \( ^3P_{\frac{1}{2}} \) in the NN potential. To confirm
this, calculations with NN potentials using modified \( ^3P_{\frac{1}{2}} \) phase shifts need to
be made until the predictions reproduce the VAP data at \( E_{c.m.} = 667 \text{ keV} \).
This work would be similar to the calculations at higher energies by Witala and
Glöckle [Wit91]. It would be interesting to see if the \( ^3P_{\frac{1}{2}} \) phase shifts need to be
modified by the same amount as those in the calculations that reproduced the
VAP data at \( E_{c.m.} \geq 2 \text{ MeV} \).

The more likely candidate for solving the VAP problem is the inclusion of a
3NF. The evidence comes from the overall improvement in the \( \chi^2_N \) for all of the
observables when the UR potential was included in the variational calculation.
However, the current 3NFs do not solve the “\( A_y \) Puzzle”. Newer 3N potentials
are needed such as the 3N \( \bar{L} \cdot \bar{S} \) force of Kievsky [Kie99]. Further refinements
may include investigations of the other available operators in the 3N interactions
similar to the \( \bar{L} \cdot \bar{S} \) case.

Another analysis that would prove helpful is an energy-dependent PSA. If fits
are made to data over a range of energies, greater constraints could be placed
on the role of the \( ^4P_J \) phase shifts and \( \epsilon_{\frac{3}{2}} \) in the 3N system. Knowledge of
the energy dependence of these parameters would aid in the investigation of
questions such as why the difference between the VAP predictions and the data
increases as the energy of the system decreases. Such an analysis is in progress by
Black [Bla00]. Our data set at \( E_{c.m.} = 667 \text{ keV} \) will add weight to the low-energy
region of the analysis.

105
7.2.2 Additional Experiments

The $A_y$ and $iT_{11}$ distributions could be enlarged by employing a different target that does not contain carbon. The $^{12}\text{C}(d,d)^{12}\text{C}$ tail in the singles spectra creates a background for the peaks of interest which can not be adequately subtracted for such small analyzing powers. That necessitated the use of the coincidence method. However, the kinematics of the coincidence method restricted the measurements to a smaller angular range than an arrangement employing single detectors. The solution to the target problem is a gas-jet target [Bit79a] which recently has been refurbished at TUNL. In this apparatus, a jet of hydrogen gas is made by injecting the gas at a high pressure into a vacuum chamber while pumping the gas away with a series of vacuum pumps. The region between the injector and the gas catcher provides a high-density target of pure hydrogen. The experiments would not be restricted by the difficulties of pile-up and background subtraction created by $^{12}\text{C}(d,d)^{12}\text{C}$ scattering. Furthermore, the d-p scattering could be detected at forward angles (such as $10^\circ$) without the presence of a $^{12}\text{C}(d,d)^{12}\text{C}$ scattering peak. The problem which cannot be avoided in this new experiment is the slower count rate for a $10^{-3} \rightarrow 10^{-4}$ analyzing power experiment run in singles mode. Each data point would require at least a day of beamtime. The coincidence method may have to be employed even if the target contaminants are eliminated.

Other measurements that might be more fruitful would be the determination of spin-transfer coefficients for d-p scattering. In such an experiment, the polarization of the scattered particle is measured after a polarized beam was incident on the target. Both experiments would required extensive hardware development such as a polarimeter that could be moved to different angle settings in the scattering chamber. However, these data would illuminate partial wave contributions beyond what the present phase-shift analysis produced. Calculations of the transfer coefficients $K_{yz}^y$ and $K_{ax}^{x'y'}$ can change by 20% to 50% over calculations with the Bonn potential when the $^3P_J$ NN-phase shifts are switched.
off individually, similar to the case of $A_y$ [Wit91]. To calculate more accurately the value of the phase shifts and mixing parameters from the data, more spin observables beyond the analyzing powers are needed.

To investigate the $^3P_J$ NN phase shifts further, $A_y$ for p-p scattering could be measured at low energies. The lack of polarization observables below $E_p = 6$ MeV translates to greater uncertainty to higher order partial waves used in the current NN potential models. TUNL has an intense polarized proton source, the gas-jet target would provide a hydrogen target. The magnitude of $A_y$ would be on the order of $10^{-4}$ to $10^{-5}$ and great care must be taken so that there are no false asymmetries. With analyzing powers of this small magnitude, the beam time per data point could range from a week to ten days.

Finally, the “$A_y$ Puzzle” and 3NF questions can be examined by studying the 4N system. With the recent development of variational calculations to solve the p-$^3$He scattering observables, comparison to previous data has shown the VAPs problem to exist in this system as well [Viv00]. This system would be an interesting candidate for further investigation at lower energies. Since $^3$He is more tightly bound than the deuteron, the nucleons are closer together leading one to believe the 3N force effects might be greater. This system would be a good extension to the study of the phenomenological spin-orbit 3NF. The p-$^3$He scattering measurements could tell us whether the new $3N \vec{L} \cdot \vec{S}$ force is an anomaly of the 3N system or does it have merit for higher $A$ systems. Again, there are hardware problems to overcome, primarily with the targets. Pure $^1$H or $^3$He targets are essentially found in gaseous form. Gas cells, with entrance and exit windows, produce too much energy loss for elastic scattering. The logical solutions are a gas-jet target or gas-filled chamber. Measurements of p-$^3$He scattering will be of great worth to the understanding of 3N interactions and are a natural progression in the study of the structure of light nuclei.
Appendix A

Targets

The choice of target can make or break an experiment. Many factors influence this decision such as thickness, stability, availability of materials, and reproducible fabrication. For $^1\text{H}(d, d)^1\text{H}$ and $^2\text{H}(p, p)^2\text{H}$ scattering, a gas-cell would seem to be an ideal first choice. It has a constant thickness and monoisotopic gases can be used. Hydrogen and deuterium gases are commercially available. Fabrication involves gluing onto the gas cell a foil window such as Havar which can survive a particle beam and hold a few atmospheres of gas. The problem is energy loss through the foil windows. Enough energy is lost through the entrance and exit windows in these low energy elastic-scattering experiments that the peaks corresponding to the detected particles are too spread out to be resolved.

A thin, solid target solves any energy-loss concerns. The difficulty is that molecular hydrogen and deuterium at standard temperature and pressure are gases, liquid targets are much too costly, and H$_2$ and D$_2$ cannot be made into solids. Instead of molecular hydrogen and deuterium, a compound with these elements could be made into a thin film. The Plasma Assisted Chemical Vapor Deposition (PACVD) developed by Black [Bla95] at TUNL does exactly that. The PACVD converts methane gas into a plasma and fabricates a hydrocarbon film. The foils contain twice as many carbon atoms as hydrogen atoms. Cylinders of methane (CH$_4$) and deuterated methane (CD$_4$) are commercially available.
The PACVD process has very few parts to it which helps in the reproducibility of fabrication. A variety of thicknesses can be produced by running the PACVD for a variable length of time. The longer the apparatus is run the thicker the film becomes. A beam of deuterons at 2.0 MeV will lose only 8.0 keV through a hydrogenated carbon foil with $1 \times 10^{18}$ H/cm$^2$ and $2 \times 10^{18}$ C/cm$^2$. The disadvantage is the carbon contamination. Elastically-scattered particles in the spectrum have an order of magnitude higher count rates compared to the d-p scattering of interest. The elastic scattering on carbon can be reduced as a problem by veto electronics or the coincidence method. With deuteron beams, excited states of $^{12}\text{C}(d,p)^{13}\text{C}$ reaction do appear as well. While the excited states are a nuisance, angular distributions of observables from $^1\text{H}(d,d)^1\text{H}$ scattering can be measured with small gaps. Overall, the hydrogenated and deuterated carbon foils from the PACVD are reliable and sturdy targets.

### A.1 Fabrication Procedure

The manufacture of thin films with the PACVD is a simple process. Figure A.1 displays the PACVD assembly. A plasma is made from the dissociation of a gas by two electrostatic plates at opposite potentials. After the plasma has been struck, the electrons are attracted to the positively-biased plate and the positive ions collect on the plate at negative potential. If another conductive substrate is placed on the negatively-biased plate, the film could be removed from the PACVD and manipulated into a target. The section below is a list of instructions to produce a hydrogenated carbon foil.

#### A.1.1 PACVD Instructions

- Clean off as many ferrotype slides as needed to make a sufficient amount of targets with distilled water first, then alcohol. Quickly, pass the slides through the flame of a propane torch to remove any residue water. A
ferrotype slide is a chrome covered steel plate used in photographic etching. One side has a dull finish while the chrome side has a mirror finish. The mirror finish is necessary for building up a uniform thickness of target material. Each slide was 1” × 3” similar to a microscope slide. Ferrotype slides are employed since the deposition surface put in the PACVD must be conductive.

- Evaporate somewhere between 200 Å to 400 Å of NaCl onto the mirrored sides of the slides. The salt is applied for the removal of the target material
Figure A.2: A schematic of the PACVD gas manifold.

which will be discussed at the end of the procedure. A NaCl thickness of less than 200 Å is not sufficient to remove the targets from the slides and of more than 400 Å will cause the NaCl to spark in the plasma. The sparking leads to non-uniformities in target thickness. Store the salted slides under vacuum until they are used. The vacuum will keep water vapor from collecting on the slides and dissolving the salt.

- Vent the PACVD chamber. Close the valve for the roughing pump to the PACVD and open nitrogen valve on the gas manifold. A drawing of the gas manifold is in Figure A.2. Be sure that the bypass valve is open in order to allow nitrogen into the PACVD chamber. After the chamber is at atmospheric pressure, open up the PACVD and close the nitrogen valve.

- Put two or three slides with NaCl side up on the bottom plate (negatively-biased). Center the slides on the plate as best as possible. The electric field between the plates will be most uniform at the centers of the plates.
• Close the PACVD and pump out the chamber with the roughing pump.

• When a vacuum of \(< 30\) mTorr has been achieved, purge the methane lines from the gas cylinder to the chamber twice to eliminate any air that has leaked into the gas lines over time. The cleanest possible vacuum is desired since impurities in the chamber will be deposited on the slides. It is believed that these impurities lead to sparking of the deposited material. The sparking creates fractures in the targets causing them to be weaker. When a good vacuum has been restored, close the bypass valve. Open the methane valve so that methane has filled the gas lines up to the leak valve. Open the leak valve until the vacuum in the chamber is \(\approx 190\) mTorr. Above this vacuum, the plasma extinguishes itself.

• Turn on the power supply. After the supply has warmed up, turn up the voltage on the plates until a plasma has been struck\((\approx 450\) V\). Adjust the voltage until current on the plates read between 4.00 and 5.00 mA, and the plasma is stable. Make all the adjustments quickly since material is building up as soon as the plasma is struck. Less voltage can be applied, but this will provide less current and longer deposition time.

• Turn on the clock. Run until a desired amount of charge has been collected on the bottom plate. Minor adjustments to the voltage or gas pressure may be needed in order to keep the current as constant as possible.

• When the desired thickness has been obtained, turn down the voltage to zero and turn off the power supply. Close the leak and methane valves. The leak valve closes at 100. Open the bypass valve and pump out the system.

• Vent the chamber again with nitrogen and remove the slides. Clean off the plates with alcohol and lint-free tissues until the plates have a mirror
finish. The hydrogenated-carbon film is an insulator and will interfere with the collection of material on the slides.

- Scrap the 3" long sides of the slides with a razor blade to loosen the target material. Cut the target material into the desired sizes for targets with the razor blade. Slowly dip the slides into a pan of distilled water to remove the targets. The layer of NaCl between the film and the slide will dissolve in the distilled water, and the targets will float to the top of the water. Pick up the targets with target rings and place them in a secure box. Alcohol can be added to the pan of water to aid in the picking up the targets since alcohol has no surface tension.

After the excess water and alcohol from the floating process has evaporated, the targets are ready to be put in a beam. The targets were found to be quite strong at the thickness chosen for the experiments conducted at $E_{cm} = 667$ keV. The most beam current that was applied was approximately 300 nA due to dead-time limitations created by the $^{12}$C elastic scattering. When a target did break, a hole was created in the foil the size of the beam instead of the whole foil tearing.
Appendix B

Polarimetry

Five of the six experiments of this project involved polarized beams; therefore, it is beneficial to discuss how the beam polarizations were measured. The standard method comprises nuclear reactions. A good nuclear polarimeter should have a large figure of merit which implies a reaction of a large cross section and a large analyzing power. If both quantities have large magnitudes, the beam polarization can be measured in a very short period of time. If the beam polarization measurement took longer than the actual data measurements, the experiment would operate inefficiently. It is also convenient for the polarimeter to have smoothly varying cross section and analyzing powers with respect to energy. There might be measurement problems if the target produced an energy spread in the beam entering the polarimeter, and the analyzing power of the polarimeter varied wildly in this small energy region.

B.1 Tensor Polarimetry

In the energy region of $E_d = 2.00$ MeV, the reaction of choice for polarimetry was $^3\text{He}(d, p)^4\text{He}$. Fortunately, two polarimeters optimized for this reaction were built and calibrated prior to this project. For the $A_{zz}$ and $A_{yy}$ experiments, the Tonsfeldt polarimeter ([Ton80, Ton81]) was employed. This device consists of a pair of detectors symmetric with the beam direction, each at 24.5°, and a center
detector at $0^\circ$. The center detector can only be used for an $A_{zz}$ experiment when $\beta = 0^\circ$ (see Section 2). Each detector consisted of plastic scintillator and a photomultiplier tube (PMT). Plastic scintillators were used because the polarimeter was calibrated in the energy range of 1 MeV to 16 MeV, and reliable detectors to span that range. The target was a $^3$He gas-cell with a 0.25 mil Havar window. The gas-cell was pressurized to 1.5 atm of $^3$He gas. The polarimeter calibration is known to approximately 3% [Ton81].

The second polarimeter utilized for the $A_{xz}$ experiment was the Leonard Five-Detector Polarimeter (LFDP). The $A_{xz}$ experiment was the christening run for the LFDP since it was calibrated just prior to the experiment. The LFDP has a design similar to the Tonsfeldt polarimeter with the addition of two out-of-plane detectors in the up and down positions also at 24.5$^\circ$. The added advantage of five detectors is that $\beta$ and $\phi$ can be measured along with the beam polarization. As with the Tonsfeldt polarimeter, the target was a similar gas-cell with 1.5 atm of $^3$He gas, and the detectors were CsI crystals and PMTs. The uncertainty in the LFDP is $\approx 2\%$ [Leo00].

The measurement of $p_{zz}$ was performed using a simple technique involving many cross-checks. The polarized deuteron beam was produced from the Atomic-Beam, Polarized-Ion Source (ABPIS), situated in the low-energy bay of TUNL. It was accelerated through the FN tandem to its final energy, and tuned down the 52$^\circ$ beamline onto target. In the scattering chamber, the beam passed through an empty space on the target ladder and into the polarimeter behind the chamber. Each polarimeter has Faraday cup so that beam current was integrated. At the beginning of each experiment, the beam energy was greater than $E_d = 2.0$ MeV. The energy was chosen at a maximum of the analyzing power excitation function. These values are shown in Table B.1. At this energy, $p_{zz}$ was measured in just a few minutes. With the beam polarization established, the beam energy was lowered to $E_d = 2.0$ MeV and $p_{zz}$ was measured again. Since the excitation functions of the polarimeter analyzing powers are sloped at $E_d = 2.0$ MeV, we
Table B.1: Polarized deuteron beam energies where a maximum exists in the analyzing power excitation function for a $^3\text{He}(\vec{d}, p)^4\text{He}$ polarimeter.

<table>
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<th>Polarimeter TAP</th>
<th>$\beta$</th>
<th>$E_d$ (MeV)</th>
</tr>
</thead>
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<tr>
<td>$A_{zz}$</td>
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</tr>
<tr>
<td>$A_{yy}$</td>
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</tr>
<tr>
<td>$A_{xz}$</td>
<td>45°</td>
<td>8.0</td>
</tr>
</tbody>
</table>

returned to the higher energies two or three times within the span of two or three hours as a consistency check. With confidence in the $p_{zz}$ measurements, the beam energy was set for 2.0 MeV in the center of the target, and a target was put in the beam for data collection.

A final word should be mentioned about our method of polarimetry. Transmission targets were utilized for all of the experiments and made possible online polarimetry. Since the polarimeters sat behind the scattering chamber, the beam that did not react with the target material passed directly into the polarimeter. Thus, beam polarization was monitored continuously throughout the experiments. Figure B.1 shows $p_{zz}$ with respect to run number for the three TAP experiments.

### B.2 Vector Polarimetry

The experimental setup for vector polarized beam polarimetry was essentially the same as for the tensor polarized beam experiments. The polarimeters were placed behind the chamber, and $p_z$ was measured online throughout the experiments. Both $\vec{d}$ and $\vec{p}$ beams were created at the ABPIS, accelerated through the FN tandem, and tuned down the 52° beamline into the same scattering chamber as before. The difference came in the reactions used to measure $p_z$.

For vector-polarized deuteron beams, our first choice was the $^{12}\text{C}(\vec{d}, p)^{13}\text{C}$ reaction. This reaction was discovered to have an analyzing power which increased
Figure B.1: Consistency of $p_{zz}$ measurements over time using the $^3\text{He}(\vec{d}, p)^4\text{He}$ polarimeters. The data in graphs (a), (b), and (c) were taken for the $A_{zz}$, $A_{xz}$, and $A_{yy}$ experiments, respectively. The triangles are $p_{zz}$ (state 2) measurements, and the squares are $p_{zz}$ (state 3) measurements. Errors are smaller than the symbols.

from -0.273 at $E_d = 370$ keV to -0.441 at $E_d = 530$ keV [Gei98]. Further evidence of its merit as a polarimeter was its large cross section [Phi50] (on the order of 40 mb/sr at $\theta_{Lab} = 90^\circ$). In fact, this polarimeter was calibrated for $^1\text{H}(\vec{d}, d)^1\text{H}$ scattering at $E_d = 1.3$ MeV. Before the device was calibrated, the optimal angle for the detector pair needed to be determined. A vector-polarized deuteron beam was incident on a 20 μg/cm² carbon foil in the 52° scattering chamber. The beam polarization was measured with the $^3\text{He}(\vec{d}, p)^4\text{He}$ reaction at $E_d = 12.0$ MeV. The Tonsfeldt polarimeter was positioned behind the chamber as the initial polarization monitor. The beam polarization of the down and up states was measured to be -0.611 and 0.452, respectively, with a 3% error. Two pairs of 300 μm silicon-surface barrier detectors were set up in the chamber at 90° and 160°. The detectors remained stationary while $A_y$ for the $^{12}\text{C}(\vec{d}, p)^{13}\text{C}$
reaction was measured in the range of incident deuteron energies from 1.0 MeV to 2.0 MeV in 100 keV steps. From the energy excitation, $A_y(160^\circ)$ was very small ($\leq 0.2$). For the 90° detector, the largest $A_y$ values occurred at 1.1 MeV ($A_y = -0.581 \pm 0.014$) and 1.7 MeV ($A_y = 0.444 \pm 0.015$). With this information, $A_y$ angular distributions were measured at $E_d = 1.1$ MeV and 1.7 MeV. The results of the angular distributions is shown in Figure B.2. It is clear that at both energies the optimal angle is at $\theta_{Lab} = 90^\circ$.

Fortunately, a device exists which has a fixed pair of detectors at 90° [Bru97].
The only modifications were to add 38 $\mu$-thick mylar foils in front of the detectors, add a flange to the base plate, and put carbon foils on the target rod. The mylar foils eliminated $^{12}$C($d,d$)$^{12}$C scattering from the spectra. As in the earlier tests, 20 $\mu$g/cm$^2$ carbon foils were used as the targets. The beam polarization was measured again at $E_d = 12.0$ MeV and was within errors of its previous values.

The Tonsfeldt polarimeter was removed and the $^{12}$C($d,p$)$^{13}$C polarimeter was placed behind the chamber. The beam energy was lowered and the $^{12}$C($d,p$)$^{13}$C device was calibrated from 1.2 MeV to 1.8 MeV in 25 keV steps. Figure B.3 is a typical spectrum from the polarimeter. Steps of 5 keV were taken from 1.280 MeV to 1.335 MeV in order to map out the variation of $A_y$ in this region. Figure B.4 is the excitation function of the calibrated $^{12}$C($d,p$)$^{13}$C polarimeter. The values of $A_y$ and $\ell T_{11}$ of the calibration are listed in Table C.6. The polarimeter is useful in the regions from 1.25 MeV to 1.3 MeV and especially from 1.5 MeV to 1.7 MeV where $A_y$ has a sizable value. However, there are some disadvantages to the polarimeter. There are two resonances in the excitation function, at
Figure B.4: Energy excitation function for the calibrated $^{12}\text{C}(\vec{d}, p)^{13}\text{C}$ polarimeter. The errors are from counting statistics alone.

1.35 MeV and another at 1.4495 MeV. The 1.4495 MeV resonance is known to have a width of $7.0 \pm 0.5$ keV [Try75]. Also, the useful energy range is not very large. The analyzing powers become zero at approximately 1.2 MeV and 1.75 MeV. Since $A_y$ is so small at 2 MeV, the $^{12}\text{C}(\vec{d}, p)^{13}\text{C}$ reaction was not used for polarimetry for the $^1\text{H}(\vec{d}, d)^{1\text{H}}$ experiment at this energy. A new low-energy deuteron polarimeter was needed. The $^2\text{H}(\vec{d}, p)^{3}\text{H}$ reaction appeared to be suitable as a polarimeter [Gei98]. The Fisher polarimeter, consisting of a small gas-filled chamber and detectors, was calibrated a few months before the $^1\text{H}(\vec{d}, d)^{1\text{H}}$ experiment. The energy lost through a gas was sufficiently low so that energy peaks corresponding to the exiting particles were clearly resolved. A pair of 100 $\mu m$ silicon surface barrier detectors were positioned symmetrically at 90° in the reaction plane. The gas cell was pressurized to 1 atm of deuterium gas. The $^3\text{He}(\vec{d}, p)^{4}\text{He}$ reaction [Bit90] was employed to calibrate the polarimeter which provided an uncertainty of approximately 2% to the calibration. Figure B.5 is a plot of the $A_y$ excitation function.
Since the maximum of the excitation function occurs at $E_d = 2.000$ MeV, the $\vec{d}$ beam was tuned directly to that energy for the initial beam polarization measurement. Figure B.6 shows the stability of the $p_z$ over the course of the experiment. The drop in $p_z$ in the beginning resulted from a faulty transition unit at the ABPIS and validated the need for online polarimetry. This effect was taken into account in the offline analysis.

For vector-polarized proton beams at $E_p = 1.0$ MeV, there was no existing polarimeter. The $^6\text{Li}(\vec{p}, ^3\text{He})^4\text{He}$ reaction appeared to be a good candidate. In the proton energy range from 255 keV to 456 keV, $A_y(90^\circ)$ for the reaction increased smoothly from 0.235 to 0.317 [Bru97]. The task was to calibrate the low-energy polarimeter at 1.0 MeV. The $^6\text{Li}(\vec{p}, ^3\text{He})^4\text{He}$ polarimeter was calibrated with the well-known $^4\text{He}(\vec{p}, p)^4\text{He}$ reaction [Dod77]. A gas cell with a 2 $\mu$m Havar window was placed in the center of the 62 cm chamber on the 52$^\circ$ beamline and pressurized with 1 atm of $^4\text{He}$ gas. A 6 MeV polarized proton beam was incident on the chamber gas cell, and the elastically-scattered protons were detected in
Figure B.6: Consistency of $p_z$ measurements over time using the $^2\text{H}(\vec{d},p)^3\text{H}$ polarimeter. The circles and squares are $p_z$ measurements for states 2 and 3, respectively.

A value of 0.998 for $A_y$ for $^4\text{He}(\vec{p},p)^4\text{He}$ scattering was obtained from the literature [Dod77]. With this $A_y$, online calculations gave $p_z \approx \pm 0.47$ with an error of $\pm 0.01$. With $p_z$ established, the beam energy was lowered to 1.0 MeV, and the $^4\text{He}$ gas cell was lifted out of the beam so the beam was incident directly on the $^6\text{Li}(\vec{p},^3\text{He})^4\text{He}$ polarimeter behind the chamber. The polarimeter device consisted of a suppressor biased to -100 V, a target ladder with two LiF targets, a Faraday cup behind the targets, and a symmetric pair of detectors at 90°. The targets were a thickness of 500 Å on a 10 µg/cm² carbon backing. The target ladder was rotated 30° so as not to block the detectors. Silicon surface-barrier detectors, 100 µm thick, were used and had an angular acceptance of
\[ \Delta \theta = \pm 3.7^\circ. \] With \( p_z \) measured in the reaction in the chamber, \( A_y \) for the polarimeter was measured to be \( 0.600 \pm 0.018 \). Reproducibility was determined by cycling through \( p_z \) measurements with the chamber setup and \( A_y \) measurements with the polarimeter. Unfortunately, the calibration for the polarimeter did not extend to higher energies due to contamination peaks of some excited states of \( ^{19}\text{F}(\vec{p}, \alpha)^{16}\text{O} \) in the spectrum. The uncertainty in the calibration came from the analyzing powers of the \( ^4\text{He}(\vec{p}, p)^4\text{He} \) reaction [Dod77] which are known to 2\%. Future work could include an investigation of ways to eliminate these contamination peaks and expanding the calibration.
Appendix C

Data Tables

All analyzing powers for $^2\text{H}(\bar{p}, p)^2\text{H}$ and $^1\text{H}(\bar{d}, d)^1\text{H}$ scattering have been multiplied by $10^4$. The values and the respective errors are given in each column. The errors include the uncertainties resulting from counting statistics, background subtraction, and statistical uncertainty in beam polarization determination. The systematic uncertainties in the beam polarization determinations were 3% for the $T_{20}$ and $T_{22}$ measurements and 2% for the $T_{21}$, $iT_{11}$, and $A_y$ experiments.
Table C.1: Tensor analyzing powers $T_{20}$ and $T_{21}$ for $^1\text{H}(\vec{d},d)^1\text{H}$ scattering at $E_{c.m.} = 667$ MeV.

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Table C.2: Tensor analyzing powers $A_{yy}$ and $T_{22}$ for $^1\text{H}(\vec{d},d)^1\text{H}$ scattering at $E_{\text{c.m.}} = 667$ MeV.

<table>
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<th>$A_{yy} \times 10^4$</th>
<th>$\Delta A_{yy} \times 10^4$</th>
<th>$T_{22} \times 10^4$</th>
<th>$\Delta T_{22} \times 10^4$</th>
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</table>
Table C.3: Vector analyzing power for $^1\text{H}(\vec{d},d)^1\text{H}$ scattering at $E_{c.m.} = 667$ keV.

\begin{center}
\begin{tabular}{ccc}
$\theta_{c.m.}$ & $iT_{11} \times 10^4$ & $\Delta iT_{11} \times 10^4$\\
50.0 & 41.20 & 1.94 \\
54.8 & 47.89 & 1.72 \\
61.0 & 56.72 & 2.14 \\
74.0 & 66.44 & 2.19 \\
87.7 & 72.20 & 1.87 \\
107.5 & 56.60 & 1.93 \\
120.0 & 49.92 & 1.75 \\
138.0 & 26.72 & 2.36 \\
\end{tabular}
\end{center}

Table C.4: Vector analyzing power for $^2\text{H}(\vec{p},p)^2\text{H}$ scattering at $E_{c.m.} = 667$ keV.

\begin{center}
\begin{tabular}{cccccc}
$\theta_{c.m.}$ & $p_z^{[\text{i}] \text{ method}^1}$ & $\Delta A_y \times 10^4$ & $\vec{p}_z \text{ method}^1$ & $A_y \times 10^4$ & $\Delta A_y \times 10^4$ \\
51.9 & 86.19 & 3.62 & 84.33 & 3.92 \\
61.7 & 112.10 & 2.58 & 111.40 & 2.64 \\
75.5 & 137.80 & 2.89 & 137.60 & 2.91 \\
87.7 & 137.30 & 2.68 & 136.60 & 2.72 \\
109.7 & 107.90 & 2.76 & 106.60 & 2.85 \\
120.0 & 89.19 & 2.62 & 87.56 & 2.77 \\
140.9 & 52.97 & 3.24 & 53.19 & 3.65 \\
\end{tabular}
\end{center}

$^1$ see Section 2.5.2
Table C.5: Absolute cross section for $^1\text{H}(d,d)^1\text{H}$ scattering at $E_{c.m.} = 667$ keV.

<table>
<thead>
<tr>
<th>$\theta_{c.m.}$</th>
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<th>$\Delta\sigma(\theta)$</th>
<th>$\theta_{c.m.}$</th>
<th>$\sigma(\theta)$</th>
<th>$\Delta\sigma(\theta)$</th>
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Table C.6: Vector analyzing powers for calibrated $^{12}\text{C}(\vec{d}, p)^{13}\text{C}$ polarimeter.

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<th>$E_d$</th>
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REFERENCES


[Bla00] T. C. Black, 2000, private communication.


