Optimized $\gamma$-multiplicity-based spin assignments of $s$-wave neutron resonances

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Abstract

The multiplicity of $\gamma$-ray emission following neutron capture at isolated resonances carries valuable information on the resonance spin. Several methods utilizing this information have been developed. The latest method was recently introduced for analyzing the data from time-of-flight measurements with $4\pi$ $\gamma$-calorimetric detection systems. The present paper describes a generalization of this method. The goal is the separation of the $\gamma$-emission yields belonging to the two neutron capturing state spins of isolated (or even unresolved) $s$-wave neutron resonances on targets with non-zero spin. The formalism for performing this separation is described and then tested on artificially generated data. This new method was applied to the $\gamma$-multiplicity data obtained for the $^{147}$Sm($n,\gamma$)$^{148}$Sm reaction using the DANCE detector system at the LANSCE facility at Los Alamos National Laboratory. The analyzing power of the upgraded method is supported by combined dicebox and GEANT4 simulations of the fluctuation properties of the $\gamma$ multiplicity distributions.

Key words: neutron resonances, spin assignment, $\gamma$ multiplicity, neutron time-of-flight method, $4\pi$ BaF$_2$ $\gamma$ detectors


1. Introduction

Determining spins for neutron resonances in odd-$A$ samples can be very difficult. Unlike even-even samples, where $s$-wave resonances all have the same spin, two $s$-wave spins are possible for odd-$A$ samples, and differentiating between the two can be very difficult using traditional techniques, especially in cases of weak and/or overlapping resonances. The most successful methods in this regard have been based on using information contained in the multiplicity distribution of $\gamma$ rays following neutron capture. For example, in a pioneering effort [1], it was shown that the ratio of singles to coincidence counts in a pair of $\gamma$-ray detectors could be used to determine spins for a number of resonances in several odd-$A$ nuclides. Later [2], it was demonstrated that the average multiplicity measured with a $4\pi$ $\gamma$-ray detector could be used to assign spins for many resonances.

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in $^{147,149}$Sm+n. However, even these two techniques are limited; the former by the relatively low efficiency of using only two detectors, and the latter by the inherently reduced sensitivity of using the average multiplicity and problems related to the resolution of closely-spaced resonances.

In a recent paper [3], a new method for using the information contained in multiplicity distributions of $\gamma$ rays following neutron capture was used to assign resonance spins in $^{147}$Sm+n. It was demonstrated that this technique is superior to using only the average $\gamma$ multiplicity for assigning spins, especially in cases of closely spaced resonances. In addition, it was shown that the DANCE $4\pi$ BaF$_2$ detector crystal ball installed at the LANSCE [4, 5, 6] spallation pulsed neutron source at Los Alamos National Laboratory is a nearly ideal facility for this application.

In the present paper we deal with a refinement of this approach. The emphasis is on obtaining the capture yields belonging to the different spin values of $s$-wave resonances of target nuclei with a non-zero spin $I$. However, we also focused our efforts on estimating these yields at neutron energies in between the resonances, where the counting statistics is, as a rule, very low. In these conditions the standard statistical methods as, e.g. the method of least squares, may not provide unbiased estimates of the sought yields. The knowledge of the spin makeup of the yield from the distant-resonance capture will certainly facilitate the $R$-matrix based decomposition of the neutron capture cross sections from future neutron time-of-flight experiments on DANCE-type $\gamma$-calorimetric systems.

We will describe the algorithm for obtaining the yields for the two spins of $s$-wave resonances and test this algorithm on well-behaved, artificially generated data on multiplicities of $\gamma$ of rays following resonance neutron capture. To test how the new technique works on real experimental data we applied our algorithm to the data on the $^{147}$Sm(n,$\gamma$)$^{148}$Sm reaction [3].

2. Description of the method

2.1. Mathematical formulation

Let the quantity $y_m(E_n)$ represents a background-free detector yield with $\gamma$ multiplicity $m$ at neutron energy $E_n$ with an uncertainty characterized by variance

$$\text{Var}[y_m(E_n)] = \sigma^2_m(E_n).$$

(1)

In real conditions the yields $y_m(E_n)$ are deduced from the following background elimination

$$y_m(E_n) = y_m^{(\text{in})}(E_n) - B y_m^{(\text{out})}(E_n).$$

(2)

Here, $y_m^{(\text{in})}(E_n)$ and $y_m^{(\text{out})}(E_n)$ stand for the number of detected events with $\gamma$ multiplicity $m$ recorded from two the separate “sample-in” and “sample-out” measurements, while $B$ is the ratio of live times, $B = T^{(\text{in})}/T^{(\text{out})}$, of these measurements. As variances $\sigma^2_m(E_n)$ are not a priori known, only their estimates are available, specifically

$$\hat{\sigma}^2_m(E_n) = y_m^{(\text{in})}(E_n) + B^2 y_m^{(\text{out})}(E_n).$$

(3)

Eq. (3) is valid under the assumption that non-negative discrete random variables $y_m^{(\text{in})}(E_n)$ and $y_m^{(\text{out})}(E_n)$ are governed by separate Poisson distributions with unknown expectation values. This condition is satisfied, as statistics of counts is, indeed, governed by a Poisson distribution. In
addition, it is tacitly assumed that for \( m \neq m' \) and/or \( E_n \neq E'_n \) the following expectation values are equal to zero
\[
E\left[y_m^{(\text{in})}(E_n)\ y_{m'}^{(\text{in})}(E'_n)\right] = 0, \quad E\left[y_m^{(\text{out})}(E_n)\ y_{m'}^{(\text{out})}(E'_n)\right] = 0,
\]
where \( E[x] \) stands for expectation value of a random variable \( x \).

The yield \( y_m(E_n) \) can be expressed as the sum of contributions of resonances with two spin values.
\[
y_m(E_n) = q^+(E_n) \mu^+_m + q^-(E_n) \mu^-_m + \delta y_m(E_n),
\]
where \( \mu^\pm_m \) represent the probabilities for observing multiplicity \( m \) at well-isolated resonances with the corresponding spin values \( J = I \pm 1/2 \), and \( \delta y_m(E_n) \) - random perturbations due to counting statistics uncertainties. It is assumed that the first and second moments of \( \delta y_m(E_n) \) are
\[
E[\delta y_m(E_n)] = 0 \quad \text{and} \quad E[\delta y_m^2(E_n)] = \sigma_m^2(E_n).
\]

Hereafter, the sets \( \{ \mu^\pm_m \} \) will be referred to as prototypical multiplicity distributions. It is tacitly assumed that all resonances with a fixed \( J \) have the same multiplicity distribution. In other words, for resonances of the same spin the corresponding values of \( \mu^+_m \) or \( \mu^-_m \) will not display variations from resonance to resonance due to Porter-Thomas fluctuations [7] of partial radiation widths. It should be stressed that under realistic conditions the influence of these fluctuations on the multiplicity distributions do cause some difficulties. This is discussed in Sec. 4.3.

We ignore the negligible role of very high multiplicities \( m > m_{\text{max}} \). In the general case the following probability normalization condition should hold:
\[
\sum_{m=m_{\text{min}}}^{m_{\text{max}}} \mu^+_m = 1.
\]

If \( m_{\text{min}} = 1 \) the energy-dependent quantities \( q^\pm(E_n) \) in Eq. (5) will represent the true total yields belonging to resonances with the indicated spin at energy \( E_n \).

The true total capture yield can be expressed as
\[
q(E_n) = q^+(E_n) + q^-(E_n).
\]

If the lower summation bound cannot be chosen to be equal to 1 (in DANCE experiments the most significant background problems are with multiplicity one), the quantities \( q^\pm(E_n) \) and \( q(E_n) \) will represent only partial yields.

With the sets \( \{ \mu^\pm_m \} \) together with a set \( \{ y_m(E_n) \} \) and a set \( \{ \sigma^2_m(E_n) \} \) for the individual energies \( E_n \) and \( m = m_{\text{min}}, m_{\text{min}} + 1, \ldots, m_{\text{max}} \), we can extract estimates of the yields \( q^\pm(E_n) \).

We construct the quantities \( \nu^+_m \) and \( \nu^-_m \) that satisfy the conditions
\[
\sum_m \nu^+_m \mu^-_m = \sum_m \nu^-_m \mu^+_m = 0
\]

\(^1\)For simplicity, in summing over \( m \) we do not show the lower and upper bounds \( m_{\text{min}} \) and \( m_{\text{min}} \).
and
\[ \sum_m \nu_m^\pm = 1. \] (10)

Then one can show that the estimates of \( q^\pm (E_n) \) and of their variances \( \text{Var} [ q^\pm (E_n) ] \) can be expressed as\(^2\)
\[ \hat{q}^\pm = \left( \sum_{m'} \nu_{m'}^\pm \mu_{m'}^\pm \right)^{-1} \sum_m \nu_m^\pm y_m \] (11)
and
\[ \text{Var} [ \hat{q}^\pm ] = \left( \sum_{m'} \nu_{m'}^\pm \mu_{m'}^\pm \right)^{-2} \sum_m \nu_m^\pm 2 \sigma_m^2. \] (12)

There exist many sets of quantities \( \{ \nu_m^\pm \} \) satisfying the conditions imposed by Eqs. (9) and (10). Our goal is to choose the optimum sets \( \{ \nu_m^\pm \} \) leading to conditional minima of \( \text{Var} [ \hat{q}^\pm ] \). In pursuing this goal the standard Lagrange approach fails, as it leads to a very complicated system of equations for the quantities \( \{ \nu_m^\pm \} \) and the auxiliary Lagrange parameters. Fortunately, our objective can be achieved by means of a tedious, but conceptually simple Monte Carlo-based approach.

We emphasize, that the optimum sets \( \{ \nu_m^+ \} \) and \( \{ \nu_m^- \} \) leading to the minima of \( \text{Var} [ \hat{q}^+ ] \) and \( \text{Var} [ \hat{q}^- ] \), respectively, are to be determined separately for each energy \( E_n \). In the previous paper [3], analogs of the sets \( \{ \nu_m^\pm \} \) were chosen on an intuitive basis and assumed to be energy independent.

2.2. Optimization by Monte Carlo trials

In our approach optimum sets \( \{ \nu_m^\pm \} \) for a given energy \( E_n \) were determined in the following way. A set of a large number of points, randomly distributed with a constant density on the surface of an \( n \)-dimensional sphere (where \( n = m_{\text{max}} - m_{\text{min}} + 1 \)) with a unit radius and a center at the co-ordinate origin, was generated by the Monte Carlo technique. For this purpose an efficient algorithm, devised originally by G. W. Brown, as quoted in Ref. [8], was adopted. According to this algorithm an \( m \)-th coordinate \( \rho_m \) of a position vector \( \rho \) of a random point on the sphere can be given by
\[ \rho_m = \frac{\xi_m}{(\xi, \xi)^{\frac{1}{2}}}, \] (13)
where quantities \( \xi_{m_{\text{min}}}, \xi_{m_{\text{min}}+1}, \ldots, \xi_{m_{\text{max}}} \), forming \( n \) coordinates of a vector \( \xi \), represent a sample of size \( n \) drawn at random from a normal distribution with a zero mean and a unit variance.

For each vector \( \rho \) an additional vector \( r \) was calculated, specifically
\[ r = \rho - \frac{(\rho, \mu^+)}{\mu^+} \mu^+, \] (14)

\(^2\)Quantities \( y_m, \sigma_m^2, \hat{q}, \) and \( q^\pm \), as well as their estimates are \( E_n \)-dependent. Hereafter, for the sake of brevity, this dependence is explicitly indicated only when needed.
where the vector \( \mu^+ \) is formed by the coordinates \( \{ \mu^+ \} \). It is easy to demonstrate that any vector \( r \) given by Eq. (14) is orthogonal to vector \( \mu^+ \). As a consequence, a vector \( \nu^- \), expressed as
\[
\nu^- = \frac{r}{(r, r)^{1/2}},
\]
satisfies the conditions imposed by Eqs. (9) and (10).

From a large number of vectors \( \nu^- \), generated according to the outlined algorithm, the one which yields a minimum value of \( \text{Var} \left[ \hat{q}^- \right] \) was accepted for the estimate of capture yield \( q^- \) according to Eq. (11). Obtaining an optimum vector \( \nu^+ \) for the extraction of \( q^+ \) followed a similar path.

Usually a very high number of trials is needed to guarantee that the minima of \( \text{Var} \left[ \hat{q}^- \right] \) found are close enough to the true minima. In our case, to avoid excessive CPU time, we did not generate an \textit{a priori} fixed number of random vectors \( \rho, \) but proceed in the following way. In each Monte Carlo trial we created the sequence of vectors \( \xi, \rho, r \) and \( \nu^- \) with a subsequent determination of estimate of \( \text{Var} \left[ \hat{q}^- \right] \) for \textit{a given energy} \( E_n \). If this estimate was lower than all estimates from previous trials, its value was pushed into a queue buffer of a prefixed size \( k \). As a criterion for terminating the search for the minimum of \( \text{Var} \left[ \hat{q}^- \right] \) we adopted a condition that each of the \( k \) values retained in the queue buffer represents the decrease of \( \text{Var} \left[ \hat{q}^- \right] \) by a factor \( d \), satisfying the relation \( d_{\text{crit}} < d < 1 \), where \( d_{\text{crit}} \) is a reasonably high prefixed lower limit. Nevertheless, as a consequence of Bellman’s curse of high dimensionality [9], for a small fraction of the cases, fulfilling these conditions, an unacceptably large number of Monte Carlo trials would have been required. To bypass this difficulty, the overall number of trials was limited. Naturally, the same criteria were applied also in searching for the minima of \( \text{Var} \left[ \hat{q}^+ \right] \).

2.3. \textit{Unbiasedness of the method}

Consider now a much simpler solution to our problem of best estimates \( \hat{q}^\pm \) of yields \( q^\pm \). If variances \( \{ \sigma^2_m \} \) are \textit{a priori} known, then, according to the method of least squares (MLSs), the best estimates of \( q^\pm \) should lead to a minimum of the sum of the weighted quadratic deviations
\[
S^2 = \sum_m \frac{1}{\sigma^2_m} \left( y_m - \sum_{\pi=\pm} \mu^\pi_m q^\pi \right)^2.
\]
(16)

Here, \( \pi \) takes “values” \( \pi = + \) and \( \pi = - \).

Let us introduce a \( 2 \times 2 \) matrix \( A \) with elements
\[
A_{\pi\pi'} = \sum_m \frac{1}{\sigma^2_m} \mu^\pi_m \mu'^\pi_m,
\]
(17)
a 2-dimensional vector \( \hat{q} \) formed by components \( \hat{q}^\pm \), and another vector \( b \) with components
\[
b_{\pi} = \sum_m \frac{1}{\sigma^2_m} y_m \mu^\pi_m.
\]
(18)
The best estimates $\hat{q}^\pm$ then follow from the expression containing the matrix resulting from inversion of $A$

$$\hat{q} = A^{-1} b$$

(19)

and estimates of variances $\text{Var} [\hat{q}^\pm]$ are

$$\text{Var} [\hat{q}^\pi] = A_{\pi\pi}^{-1}.$$  

(20)

From Eqs. (18) and (19) it straightforward that the best estimates $\hat{q}^\pm$ are given by linear combinations of the observables $\{y_m\}$. According to the well-known Gauss-Markov theorem, among all linear combinations yielding unbiased estimates of $q^\pm$ the combinations following from Eq. (19) lead to minimum variances of $\hat{q}^\pm$.

The method for estimating yields $q^\pm$ based on minimizing $S^2$ is computationally simple, but for its unbiasedness the knowledge of true variances $\{\sigma^2_m\}$ is needed. The replacement of $\{\sigma^2_m\}$ by $\{\hat{\sigma}^2_m\}$ according to Eq. (3) leads to satisfactory results on condition that the total yields $q$ are high enough. However, if the method is applied to get estimates of $q^\pm$ at neutron energies where the total yields $q$ are low, this replacement will lead to biased estimates of $q^\pm$. If some value $y_m$ is very close to zero (which may happen at very low or very high $m$), the bias may become intolerably high. When $y_m = 0$ for some $m$, the method of least squares collapses whatsoever.

Our novel method does not suffer from these problems. If the constraint imposed on vectors $\nu^\pm$ by Eqs. (9) and (10) is satisfied, then the expectation values of estimates $\hat{q}^\pm$ are

$$E [\hat{q}^\pm] = \left( \sum_{m'} \nu^\pm_{m'} \mu^\pm_{m'} \right)^{-1} \sum_m \nu^\pm_m E [y_m]$$

$$= \left( \sum_{m'} \nu^\pm_{m'} \mu^\pm_{m'} \right)^{-1} \sum_m \nu^\pm_m \left( q^+ \mu^+_m + q^- \mu^-_m \right) = q^\pm.$$  

(21)

which implies that the estimates $\hat{q}^\pm$ are unbiased.

Knowing that

$$E \left[ \sigma^2_m \right] = E \left[ y^{(in)}_m \right] + B^2 E \left[ y^{(out)}_m \right],$$

(22)

it is easy to show that

$$E \left[ \text{Var} \left[ \hat{q}^\pm \right] \right] = \text{Var} \left[ \hat{q}^\pm \right].$$

(23)

The estimate of variance $\text{Var} \left[ \hat{q}^\pm \right]$ is thus unbiased too.

The unbiasedness embodied by Eqs. (22) and (23) is guaranteed also in conditions when vectors $\nu^\pm$ are arbitrarily far from their optimum choice leading to a minimum of $\text{Var} \left[ \hat{q}^\pm \right]$.

To conclude, the proposed method is by no means a reinvention of the well-known method of least squares and a labored implementation of this reinvention. For low counting statistics our method is superior.
3. Validation of the method

In order to test both the correctness and the sensitivity of the method described in Secs. 2.1 and 2.2, we applied it to artificial data, resembling the data from a real experiment, and checked whether the extracted estimates $\hat{q}^{\pm}$ agree with what had been postulated while creating these data. For this purpose we postulated functions $q^{\pm}$, formed by contributions of several resonances of Breit-Wigner shape, and two prototypical multiplicity distributions, characterized by the sets $\{\mu_{m}^{\pm}\}$ illustrated in Fig. 1. A background has also been considered. Then we constructed the functions $\{y_{m}\}$ and perturbed them randomly by uncertainties given by variances $\{\sigma_{m}^{2}\}$ according to Eq. (3). The values of the original and perturbed total capture yield $q$ are plotted in panel (a) of Fig. 2 as a smooth curve and points, respectively. In panels (c) and (e) the values of postulated functions $q^{\pm}$ are plotted as smooth curves. The postulated resonance energies for two spin groups are indicated by arrows in the same panels. Knowing $\{\mu_{m}^{\pm}\}$, $\{y_{m}\}$ and $\{\sigma_{m}^{2}\}$, with the aid of our method we extracted the estimates $\hat{q}^{\pm}$ for each energy value. These estimates are plotted by open circles in panels (c) and (e). To assess the quality of these estimates, we introduced residuals

$$\delta^{\pm} = \frac{\hat{q}^{\pm} - q^{\pm}}{\sqrt{\text{Var}[\hat{q}^{\pm}]}} \quad (24)$$

Values of $\delta^{\pm}$ are plotted in panels (b) and (d) of Fig. 2.

As is evident from Fig. 2, the method works: (i) the quadruplet (Q), triplet (T) and doublet (D) structures are correctly decomposed, and (ii) within experimental uncertainties the extracted estimates $\hat{q}^{\pm}$ agree well with the postulated functions $q^{\pm}$, although the assumed distributions of multiplicities $m$ display a very small difference between their average values, $\langle m^{+} \rangle - \langle m^{-} \rangle = 0.24$, see Fig. 1.

However, it should be emphasized that these results were reached under the simplifying condition that the contribution of each resonance of the same spin group to $y_{m}(E_{n})$ according to Eq. (2) is governed by common multiplicity distributions represented by fixed vectors $\mu^{+}$ and $\mu^{-}$ for $J = I + 1/2$ and $J = I - 1/2$, respectively. In reality, a set of partial radiation widths $\{\Gamma_{\lambda\gamma f}\}$ for transition from resonance $\lambda$ to any fixed final level $f$ of the product nucleus fluctuates from resonance to resonance. This phenomenon, known as Porter-Thomas fluctuations [7], means that the vectors $\mu^{\pm}$ also fluctuate. The encouraging results illustrated in Fig. 2 thus represent only the solution of the idealized problem formulated in Sec. 2.1. As follows from the detailed analysis in Sec. 4.3, prior a reliable use of the above-outlined method for estimating the yields $q^{\pm}$ and spin determination of neutron resonances the influence of Porter-Thomas fluctuations is to be assessed.

For the sake of comparison, we also analyzed the same artificial data using a simplified option of our method based on the standard least square technique, as outlined in Sec. 2.3. In this case the estimates of $q^{\pm}$ and $\text{Var}[\hat{q}^{\pm}]$ were calculated using Eqs. (19) and (20), respectively. The results of this analysis are illustrated in Fig. 3. Here, all point and line plots have the same meaning as in Fig. 2. Positions of gray vertical bars in panels (c) and (e) of Fig. 3 indicate neutron energies at which the least square technique collapsed due to zero values of some of the estimates $\sigma_{m}^{2}$, see the discussion in Sec. 2.3. The comparison between the data plotted in Figs. 2 and 3 indicates that at neutron energies characterized by a low total capture yield $q(E_{n})$, e.g. at $E_{n} = 65 - 74$ eV, the least square technique made it possible to estimate $q^{+}(E_{n})$ and $q^{-}(E_{n})$ only at about 40% of the
neutron-energy bins. On the other hand, at energies occupied by strong neutron resonances this technique not only does not fail, but it leads to the same estimates of $q^\pm (E_n)$. This ascertainment is documented by virtually identical values of residuals $\delta^\pm$ plotted in Figs. 2 and 3 at these energies.

4. Application of the method to the $\gamma$-multiplicity data from the $^{147}$Sm(n,$\gamma$)$^{148}$Sm reaction

4.1. Sm data

In order to test the method under realistic conditions, we applied it to the data from the $^{147}$Sm(n,$\gamma$)$^{148}$Sm reaction measured at the Los Alamos DANCE facility in the neutron energy range $E_n = 38 - 700$ eV. The experimental setup and measurement conditions are described elsewhere [3]. Here we give only the details specific for the testing of the method.

The DANCE $4\pi$ detector system consists of 160 BaF$_2$ scintillation crystals forming a compact spherical shell with outer diameter of 64 cm and thickness of 15 cm. The sample of Sm$_2$O$_3$ was enriched to 97.93% $^{147}$Sm, with a thickness of $5.32 \times 10^{19}$ atoms of $^{147}$Sm per cm$^2$. The set of spectra $\{y_m(E_n)\}$, belonging to individual multiplicities $m$, were constructed from off-line scanning of the accumulated event-mode data under the condition that the full energy $E_{\gamma \text{sum}}$ deposited in the detector system in each event of the detected neutron capture satisfied condition $3.0 \text{ MeV} < E_{\gamma \text{sum}} < 8.3 \text{ MeV}$. While constructing these spectra, the sample-out background was eliminated, as well as the background due to neutrons scattered off the sample and subsequently captured in Ba nuclei in the scintillation crystals. The size of the background caused by the scattered neutrons was determined using data from an auxiliary run with a carbon sample. It turned out that this background was much smaller than the sample-out background.

The multiplicities $m$ which the background-corrected spectra $\{y_m(E_n)\}$ refer to are cluster multiplicities. Here $m$ represents the number of spatially separated detector clusters in which $\gamma$-ray energy is deposited from a neutron capture event. Each cluster is defined as formed by a contiguous 3D space region filled in only by those BaF$_2$ crystals that respond to the following $\gamma$ rays:

- $\gamma$ rays following the capture of a neutron in the sample and
- $\gamma$ rays following the capture of a neutron in some of the Ba isotopes in the scintillation crystals after its scattering from the sample or additional single or multiple scattering from the sample and/or detector material.

The cluster multiplicity is close to the genuine physical multiplicity distribution. This is not the case for the crystal multiplicity which represents the overall number of detector modules that produce simultaneously signals above a given threshold.

The prototypical distributions of cluster multiplicities $\{\mu_m^\pm\}$ for $J = 3$ and $J = 4$ resonances of $^{147}$Sm were determined from background-corrected experimental yields $\{y_m(E_n)\}$ in energy intervals covered by strong, well isolated resonances at 58.09 and 49.36 eV, see Fig. 4. The spin assignments of these two resonances are believed to be $J = 3$ and $J = 4$, respectively.

To suppress the background we excluded multiplicity one by setting $m_{\text{min}} = 2$. As we did not observe statistically meaningful values of $\{y_m\}$ for $m > 7$, we set $m_{\text{max}} = 7$ in our analysis.
4.2. Analysis

The results of decomposition of the experimentally observed yield $\sum_{m} y_{m}$ from the DANCE experiment to the estimates of yields $q^{+}$ and $q^{-}$ are illustrated in Fig. 5. This illustration covers the neutron energy range $38-520$ eV. While decomposing the yield $q$ the size of the queue buffer, mentioned in Sec. 2.2, was adjusted at $k = 5$, the critical attenuation factor was set at a value $d_{\text{crit}} = 0.99$ and the number of Monte Carlo trials should not exceed $2 \times 10^{6}$.

On the whole our method seems to work very well: as demonstrated in Fig. 5, an unambiguous spin assignment can be made for an overwhelming majority of the observed resonances and even for close multiplets. In addition, without the application of this method the doublet structure of some of the apparently single resonances could not have been revealed.

Before detailed inspection of the results, it is important to outline briefly what can be expected when the sets of probabilities $\{\mu_{m}^{\pm}\}$ fluctuate from one resonance to another of the same spin. In other words, what happens if the probabilities $\{\mu_{m}^{\pm}\}$ for individual resonances differ from the probabilities of the prototype resonances. As already mentioned in Sec. 3, in terms of the statistical model of the nucleus these fluctuations are caused exclusively by Porter-Thomas fluctuations [7] of partial radiation widths for primary transitions. The statistical behavior of secondary transitions influences only the expectation values of probabilities $\{\mu_{m}^{\pm}\}$, not the fluctuations of these probabilities from resonance to resonance.

If the expected fluctuations of $\{\mu_{m}^{\pm}\}$ are not negligible, the extracted yield, say, $\hat{q}^{+}$, belonging to a real isolated resonance $\lambda$ at energy $E_{n} = E_{\lambda}$, may be accompanied by a false (positive or negative) resonance-like yield $\hat{q}^{-}$ at the same energy $E_{\lambda}$ and of the same shape as the parent real resonance $\lambda$ with spin $J = I + 1/2$. These distorted yields are referred to as ghost resonances.

In the general case, if positive resonance-like structures occur simultaneously in estimates of both yields, $\hat{q}^{\pm}$, at virtually the same energy $E_{\lambda}$, the question will arise whether there is indeed a doublet formed by resonances with different spin assignments at this energy, or a single parent resonance with its ghost. As will be apparent from results of simulations described in Sec. 4.3, if the sizes of the estimates $\hat{q}^{+}$ and $\hat{q}^{-}$ are comparable, it is highly probable that the resonance-like structures are a signature of a doublet.

With these considerations in mind, inspection of Fig. 5 leads to the following conclusion: from the large and comparable sizes of the yields $\hat{q}^{\pm}$ near $E_{n} = 140.05$ eV we infer a very close doublet formed by $J = 3$ and $J = 4$ resonances. Similar conclusions are reached for the yields $\hat{q}^{\pm}$ near energies 290.2 and 486.4 eV.

If the energy dependence of one of the estimates $\hat{q}^{\pm}$ appears to show a peak and the other one a dip, the latter has to be definitely a ghost. In this case its position and shape must be identical to those of the parent resonance.

Consider now the case when the decomposition leads to positive estimates $\hat{q}^{+}$ and $\hat{q}^{-}$ with the following properties: (i) their energy dependence displays a resonance-like shape, (ii) they are not necessarily comparable in the size and, (iii) their maxima are situated at close, but, in terms of statistical significance, different resonance energies, say, $E_{X}^{+}$ and $E_{X}^{-}$. In this instance the conclusion is that at these energies there resides a doublet formed by resonances with different $J$ assignments. As an example, for the estimates $\hat{q}^{\pm}$ displaying in Fig. 5 maxima near 350.0 eV this conclusion can be made at a statistical significance level of 99.9%.

Another example of close resonances with different spin assignment seems to be the peak at
39.66 eV. As is evident from the upper panel of Fig. 6, the undershoot of $q^-$ near the strong $J = 4$ resonance at 39.66 eV cannot be accounted for as a ghost of this resonance, as its position and shape differ from what one expects for a ghost resonance. Indeed, from the $\chi^2$ value obtained for the best fit – specifically $\chi^2 = 115.4$ for the number of degrees of freedom $\nu = 76$, see Fig. 6 – this conclusion is made at a statistical significance level of 99.8%. However, the problem of fitting the undershoot of the yield $q^- (E_n)$ can be easily overcome. This is evident from the lower panel of Fig. 6 where the data are fitted with a superposition formed by (i) a ghost of the $J = 4$ resonance at 39.66 eV of unknown amplitude, (ii) the left wing of a nearby $J = 3$ resonance at 40.72 eV, and (iii) an additional $J = 3$ resonance with energy $E^\prime_n$ and neutron width $\Gamma^\prime_n$ as free parameters. Indeed, as seen from the lower panel of Fig. 6, the best fit obtained with this parametrization ($\Gamma^\prime_n \approx 1.6$ meV and $E^\prime_n = 39.52$ eV) is in good agreement with the data. It should be pointed out that the resonance at 39.66 eV has a very large neutron width, $\Gamma_{\lambda n} = 70.2$ meV, see Ref. [10].

This example demonstrates not only the ability of the proposed method for spin determination of very weak neutron resonances, but also the capability of resolving very close resonance doublets.

Negative ghosts accompanying parent $J = 4$ resonances near energies 171.8 and 206.03 eV may indicate a role of Porter-Thomas fluctuations. In order to learn more about the statistical behavior of these ghosts, we selected 30 well-isolated peaks, assuming that they represent 14 and 16 singlet $s$-wave resonances with $J = 3$ and $J = 4$, respectively. Knowing the estimates $\hat{q}^+ \lambda$ and $\hat{q}^− \lambda$, we calculated the relative sizes of ghost resonances, specifically the values of the ratio $R$ of the area under a ghost resonance to the area under the total yield of the parent resonance. The results are shown in the upper panel of Fig. 7. From them we estimated the rms uncertainty $\sigma(R)$ characterizing fluctuations of $R$ from resonance to resonance around the estimates of the corresponding expectation values $E[R]$. We obtained a value $\sigma(R) = 0.056 \pm 0.010$ from properly weighted values for individual spins $J = 3$ and $J = 4$, see the upper panels of Fig. 7. The values of $E[R]$ depend on the choice of a pair of resonances from which prototypical distributions $\{\mu_m^\pm\}$ are extracted. By definition these resonances should not be accompanied by ghost resonances. We found that any change of the choice of the resonance for the prototypical distribution leads to a change of the estimate $E[R]$ of $R$ for the resonances of the corresponding spin, while the value of $\sigma(R)$ remains unchanged.

Inspecting the data plotted in Fig. 5, it is evident that out of 34 well-isolated and strong-enough resonances in the energy region of 38-520 eV the vast majority of them (30) display ghost resonances characterized by the absolute value $|R - \bar{E[R]}| < 0.15$. The exceptions are represented by resonances at 140.05, 290.2, 486.5 and $\approx 514$ eV for which we found the values of $R - \bar{E[R]}$ equal to 0.36, 0.48, 0.36 and 0.49, respectively. The relatively wide interval 0.15–0.36 that has no values of $|R - \bar{E[R]}|$ suggests that the distribution of the statistical variable $R$ actually is as narrow as shown in the previous paragraph and that the high values of $|R - \bar{E[R]}|$ for the resonances at 140.05, 290.2, 486.5 and $\approx 514$ eV are artifacts of their composite structure.

In this context it is worth noting that the maximum of the yield $q^− (E_n)$ of the resonance near 140.05 eV differs by its position from the maximum of $q^+(E_n)$ by about 50 meV, which makes it possible to reject the hypothesis of a singlet resonance at 140.05 eV at a statistical significance level of 95%. A detailed statistical test indicated that the relative size of the yield $q^+(E_n)$ at 140.05 eV together with relative sizes $R$ of the ghosts of the 14 analyzed $J = 3$ resonances cannot belong to a common normal distribution. This finding leads to the exclusion of a singlet structure at a much higher statistical significance level, specifically at 99.7%. On the basis of a similar argument, it is
clear that also the peak at ≈514 keV represents a doublet formed by resonances with a $J = 3$ and $J = 4$ assignment.

To conclude, the experimental data accumulated from the DANCE experiment are strongly suggestive of a narrow distribution of the ratio $R$, characterizing the relative size of ghost resonances. However, before thorough analysis a firmer statement regarding the statistical behavior of $R$ cannot be made. To be more specific, strictly from the data themselves it cannot be concluded that the fluctuations of statistical variable $R$ originate from Porter-Thomas fluctuations.

4.3. Simulations of residual fluctuations of $\{\mu_m^\pm\}$

In order to check whether the sizes and fluctuation behavior of the observed ghost resonances are in reasonable quantitative agreement with the predictions of the extreme statistical model, we undertook a series of simulations focused on estimating fluctuations of the probabilities $\{\mu_m^\pm\}$ and the associated distortions of the extracted yields $q^\pm$. We simulated $\gamma$ cascades following neutron capture in $^{147}$Sm with the aid of a customized algorithm DICEBOX [11] in combination with the CERN particle transport toolkit GEANT4.

The main part of the DICEBOX algorithm generates a complete decay scheme of a fictitious nucleus that would display a similar $\gamma$-decay pattern as the actual nucleus of interest.

Below some critical energy, $E_{\text{crit}}$, all the characteristics of the decay scheme of this nucleus, i.e., energies, spins and parities of levels, as well as branching intensities of their $\gamma$-decay, are taken from existing experimental data. The choice of $E_{\text{crit}}$ should be made with care to guarantee that all of the information for energies below $E_{\text{crit}}$ is complete. We took the required data from Ref. [12] and adopted $E_{\text{crit}} = 2.2$ MeV.

Above $E_{\text{crit}}$ the nuclear level system is generated by Monte Carlo techniques using a level density function $\rho(E, J, \pi)$. Decay properties of the created level system, i.e., the branching intensities for the $\gamma$ decay of any level, are artificially generated using the photon strength functions (PSFs). Following the statistical model, the partial radiation width $\Gamma_{a\gamma b}$ for a transition between an initial level $a$ and a final level $b$, is given by

$$\Gamma_{a\gamma b} = \sum_{XL} \frac{\eta_{XL}^2 f^{(XL)} E_{\gamma}^{2L+1}}{\rho(E_a, J_a, \pi_a)},$$

where $E_\gamma$ is the $\gamma$-ray energy, $f^{(XL)}$ stands for the PSF for transitions of type $X$ (electric or magnetic) and multipolarity $L$, and $\eta_{XL}$ is a random number drawn from a normal distribution with zero mean and unit variance. This random number ensures that the corresponding term on left-hand side of Eq. (25) fluctuates according to the Porter-Thomas distribution [7]. In the general case the summation in Eq. (25) should go over all allowed types $X$ and multiplicities $L$ of the transition $a \rightarrow b$, but we confined ourselves to multipolarities $E1$, $M1$, and $E2$.

The $E1$ PSF we used in DICEBOX simulations was taken according to the model of Kadmenskij, Markushev and Furman [13]. The $M1$ PSF was assumed to be formed by the sum of a single-particle constant $f_{SP}^{(M1)} = 4.0 \times 10^{-9}$ MeV$^{-3}$ and a term originating from the scissors-mode resonance [14] at 3.0 MeV with a damping width $\Gamma_{SR} = 0.6$ MeV and a peak photoabsorption cross section $\sigma_0 = 0.15$ mb, assuming that the scissors resonance is built on each excited level of the residual nucleus [15]. For the $E2$ PSF we used the single-particle model and chose $f^{(E2)}(E_\gamma) = Q_{SP}$, where $Q_{SP} = 5 \times 10^{-11}$ MeV$^{-5}$. 
The level density $\rho(E_a, J_a, \pi_a)$ was assumed to be described by the well-known back-shifted-Fermi-gas (BSFG) formula given explicitly in Ref. [16] together with its recommended parameters for $^{148}\text{Sm}$.

In accordance with the notation introduced in Ref. [11], the system of all levels and all their decay branching intensities is called a nuclear realization (NR). Due to the Porter-Thomas and the level-spacing fluctuations there is an infinite number of NRs that differ from each other, even for fixed models of PSFs and level density. The real nucleus is assumed to be characterized by one of these NRs.

However, for our purpose we had to work with NRs forming specific supersets: each NR of a fixed superset differs from each other only by sets of branching intensities, $\{I_{c+\gamma b}\}$ and $\{I_{c-\gamma b}\}$ characterizing the $\gamma$ decay of neutron capturing levels $c^\pm$ with spin $J = I \pm 1/2$ to all levels $b$ below the neutron threshold. The $\gamma$ cascades obtained from many supersets of NRs carry information about the set of expectation values $\{E[\mu^+_m]\}$, while the NRs belonging to a fixed superset provide us with information about fluctuations of $\{\mu^+_m\}$ when one proceeds from resonance to resonance. Because of the large computing time, we generated only one superset consisting of 12 NRs and for each of them we simulated $5 \times 10^4$ $\gamma$ emission events, i.e., sequences of energies $E_\gamma^1, E_\gamma^2, \ldots, E_\gamma^m$ of individual $\gamma$-cascade steps. The number of steps $m$ varies from cascade to cascade.

Having these sequences it was easy to construct the distribution of the physical multiplicity separately for each NR. However, for each NR we needed to determine the distributions of the cluster multiplicities $\{\mu^+_m\}$ and $\{\mu^-_m\}$ characterizing the decay of neutron capturing states $c^+$ and $c^-$, respectively. For this purpose the response of the DANCE detector system to DICEBOX-generated $\gamma$ cascades was simulated for each of the $5 \times 10^4$ $\gamma$ emission events belonging to a given NR. To determine the number of detector clusters responding to $\gamma$ rays of a cascade, we employed the CERN package GEANT4. In order to reach realistic simulations of the cluster multiplicity distributions $\{\mu^+_m\}$, the spatial characteristics of all active and passive material in the DANCE detector system were carefully prepared as the input to the GEANT4 code. Conditions imposed on the total $\gamma$-ray energy deposited from a cascade in the whole detector system were also taken into account in the GEANT4 simulations. Further details of the GEANT4 simulations of the DANCE detector array can be found in Ref. [17].

One of the characteristics of the distributions $\{\mu^+_m\}$ is represented by the average multiplicity $\langle m^\pm \rangle$, where, in our conditions,

$$\langle m^\pm \rangle = \sum_{m=2}^7 \mu^\pm m. \quad (26)$$

Simulated distributions of $\langle m^\pm \rangle$-values obtained for two values of resonance spin are plotted in Fig. 8 and compared with the corresponding distributions, deduced from the DANCE data obtained from the $^{147}\text{Sm}(n, \gamma)^{148}\text{Sm}$ measurements [3] for 30 well-isolated resonances mentioned in Sec. 4.2.

The results plotted in Fig. 8 indicate that expectation values of the experimental average cluster multiplicities differ from their simulated counterparts by only $\approx 0.04$. Even more surprising is the difference between the average values of $\langle m^+ \rangle$ and $\langle m^- \rangle$ obtained from simulations: this difference is 0.247 and differs from the difference determined from the experimental data by only 0.012, see Fig. 8.

The estimates of the rms values characterizing the fluctuations of the experimental average cluster multiplicities $\langle m^+ \rangle$ and $\langle m^- \rangle$ are higher by a factor of 1.24 compared to estimates from simulated average multiplicities. Using Fisher’s $F$-test the hypothesis that the true rms values of
all four distributions plotted in Fig. 8 are equal can only be refuted at a low statistical significance level of 85%. The size of the observed fluctuations of \( \langle m^\pm \rangle \) is thus compatible within about “one \( \sigma \) deviation” with what is expected from the fluctuations of intensities of primary transitions depopulating the neutron resonant capturing states.

From each neutron energy, from the simulated distributions of \( \langle m^\pm \rangle \) we constructed the observables \( \{y_{m}^\pm(E_n)\} \), where \( y_{m}^\pm(E_n) = \hat{q}(E_n)\mu_{m}^\pm \). Here, the function \( \hat{q}(E_n) \) represents the shape and size of a typical isolated resonance seen from DANCE measurements at an energy near 200 eV with 6000 counts per 0.2 eV bin at the maximum. To take into account the role of counting-statistics uncertainties, the values of \( y_{m}^\pm(E_n) \) for each energy bin and each \( m \) were additionally perturbed by random values taken from the normal distributions with a zero mean and variances equal to \( y_{m}^\pm(E_n) \). In this way we simulated the outcomes of DANCE measurements of 12 pairs of isolated resonances with \( J = 3 \) and \( J = 4 \) with fluctuating multiplicity distributions \( \{\mu_{m}^\pm\} \).

As a last step, from the simulated cluster multiplicity distributions \( \{\mu_{m}^\pm\} \) we chose two of them as prototypical for resonances with spin \( J = 3 \) and \( J = 4 \). Having 12 pairs of artificially generated sets \( \{y_{m}^\pm(E_n)\} \) and \( \{y_{m}^\pm(E_n)\} \) we applied on each of these sets the method described in Sec. 2 to extract from each pair \( \{y_{m}^\pm(E_n)\} \) and \( \{y_{m}^\pm(E_n)\} \) the yields \( q^\pm(E_n) \). In an analogous way as for the experimental data, we determined the relative ghost size \( R \) for each of the 24 simulated resonances. The empirical distributions of the values of \( R \) obtained in this way for \( J = 3 \) and \( J = 4 \) resonances are plotted in the lower panels of Fig. 7.

It should be stressed again that estimates of the expectation values of \( R \) are of no importance, since for a large number of resonances they can be adjusted to zero by a suitable choice of prototypical multiplicity distributions. The relevant quantities are the estimates of the rms values characterizing the size of fluctuations of \( R \). From our simulations we arrived at values \( \bar{\sigma}(R) \) equal to 0.057 and 0.056 for \( J = 3 \) and \( J = 4 \) s-wave resonances, respectively, see the lower panels of Fig. 7. These values agree with the respective values of \( \bar{\sigma}(R) = 0.063 \) and \( \bar{\sigma}(R) = 0.048 \), obtained from experimental data, better than what one would expect from the accuracy of 25% following from the limited number of simulated and experimentally observed resonances. We conclude that the observed fluctuations of the relative ghost sizes \( R \) originate from Porter-Thomas fluctuations of the intensities of primary transitions depopulating neutron resonances.

Assuming a normal distribution for the random variable \( R \), it is straightforward to show that a value of \( R \) satisfying the condition \( R - \bar{E}[R] > 3.12 \bar{\sigma}(R) \) rules out a singlet character of a given resonance at the statistical significance level of 99.9%. For the \( ^{147}\text{Sm}(n,\gamma)^{148}\text{Sm} \) reaction this condition reads \( R - \bar{E}[R] > 0.18 \). This finding supports our conclusion in Sec. 4.2 that isolated peaks for which \( R - \bar{E}[R] > 0.36 \) are, in fact, close doublets.

4.4. Capture yields belonging to resonances in sample impurities

Although the sample used in the DANCE experiment contained 97.93% of \( ^{147}\text{Sm}_2\text{O}_3 \), even with this enrichment the strongest neutron resonances of neighboring odd-A \( ^{149}\text{Sm} \) nuclei (the most prominent sample impurity), are seen from the experimental capture yields \( q_{\text{exp}}(E_n) \) and estimates of \( q^\pm(E_n) \). This is apparent from Fig. 5. The allowed \( J^\pm \) values of s-wave resonances of \( ^{149}\text{Sm} \) are \( 3^- \) and \( 4^- \), as in the case of \( ^{147}\text{Sm} \), but there is no guarantee that multiplicity distributions \( \{\mu_{m}^\pm\} \) for \( ^{149}\text{Sm} \) are identical to those for \( ^{147}\text{Sm} \).

In spite of these difficulties the estimates of \( q^\pm(E_n) \), obtained from constructing of vectors \( \nu^\pm(E_n) \) for the \( ^{147}\text{Sm} \) data, may still serve for spin determination of \( ^{149}\text{Sm} \) resonances. From
simplified DICEBOX simulations it follows that the values of the ratio \( Q^+(E_n) = q^+(E_n)/q(E_n) \) belonging to \( J = 4 \) resonances should be on average higher compared to the values of the same ratio for \( J = 3 \) resonances. Using this criterion we were able to assign spin values to 8 resonances of \(^{149}\text{Sm}\), as summarized in Table 1.

Inspecting the plots in the uppermost panel of Fig. 5 and the data in Table 1, it is evident that the proposed method of spin determination works even in the extreme conditions when the effect from the \( J = 4 \) resonance of \(^{149}\text{Sm}\) at 45.06 eV represents only \( \approx 0.2\% \) of the effect of the neighboring \( J = 4 \) resonance of \(^{147}\text{Sm}\) at 38.66 eV.

A detailed analysis of the energy dependence of the yields \( q^\pm(E_n) \) revealed also statistically significant effects from relatively strong \( J^\pi = 1/2^+ \) resonances of \(^{148}\text{Sm}\), \(^{152}\text{Sm}\) and \(^{154}\text{Sm}\) at 94.90, 87.7 and 93.01 eV, respectively, see Ref. [10]. These resonances display negative relative yields \( Q^+(E_n) \) fluctuating around \( Q^+(E_n) \approx -1.0 \), which speaks for a much lower average multiplicities of their \( \gamma \) decay compared to the decay of \( J = 3 \) and \( J = 4 \) resonances of \(^{147}\text{Sm}\).

4.5. Main results

The spin values which we assigned to 80 resonances of \(^{147}\text{Sm}\) in neutron energy region 38–520 eV are listed in Table 2 together with the values proposed in Refs. [3, 10].

Five new resonances were established and, in addition, for six previously observed resonances with a tentative spin assignment a firm assignment was made. It should be noted, that all assignments made in [3] agree with the assignments resulting from application of the proposed method, see Table 2. However, in cases of unique spin assignments \( J = 4 \) and \( J = 3 \) made in Ref. [3] for resonances at 227.9 and 446.9 eV, respectively, we were able to assign these spin values only tentatively, see Table 2.

5. Convergence of the method

From the mathematical formulation of the method in Sec. 2.1 it is evident that the vectors \( \nu^\pm \) should vary with varying neutron energy. In view of the fact that estimates of these vectors are obtained from data on sets \( \{y_m\} \) that are subject to statistical errors it is natural to expect that the components of vectors \( \nu^\pm \) should exhibit additional variations. However, it is not clear how strong the overall variations are in practical application of the method to real experimental data and to what extent these variations are related to the local spin composition of the total capture yield \( q \) and to the statistical precision of a set of experimental values of \( \{y_m\} \). In addition the question arises as to whether the adopted criterion for terminating trials in searching for the minima of estimates \( \text{Var} \left[ \hat{q}^\pm \right] \), as described in Sec. 2.2, is sufficient to guarantee a satisfactory precision of the deduced vectors \( \nu^\pm \).

In Fig. 9 the values of the components \( \nu_3^\pm \) and \( \nu_5^\pm \) of vectors \( \nu^\pm \) as functions of neutron energies within the range of 114–215 eV are plotted. We selected these components because they display the strongest dependence on neutron energy. From the plots shown the following conclusions were drawn:

- The values of \( \nu_3^\pm \) and \( \nu_5^\pm \) display sizable variations. As an example, the component \( \nu_3^- \) changes within the interval \((-0.70, -0.12)\). The directions of vectors \( \nu^+ \) and \( \nu^- \) in the \( n \)-dimensional space thus vary markedly. In contrast, the modulus of the difference between these vectors
displays relatively small, weakly fluctuating values, see the lowest panel of Fig. 9. It implies that the angle between these vectors in n-dimensional space, \( \varphi = \arccos(\nu^+,\nu^-) \), is small and also weakly fluctuating. From our data we found \( \varphi = 11.0 \pm 2.5^\circ \).

- Provided that \( \pi \neq \pi' \) and/or \( m \neq m' \), pairs \( [\nu^n_m(E_n),\nu^n_{m'}(E_n)] \) for various energies \( E_n \) are correlated; here, \( \pi \) stands for sign + or −. Moreover, these pairs exhibit a strong linear regression. E.g., the data points represented by the co-ordinates \( [\nu_3^-(E_n),\nu_3^+(E_n)] \) are almost ideally distributed along their regression line: the rms deviation of values \( \nu_3^+(E_n) \) from this line in y-direction displays a relatively low value of 0.032.

- As seen from a comparison of the shapes of the energy-smoothed values of \( \nu_3^+ \) and \( \nu_5^+ \) obtained with the aid of the Savitzky-Golay digital filter [18] with the shapes of the plotted energy dependence of \( q^\pm(E_n) \), variations of \( \nu^\pm(E_n) \) are to a large extent systematic in the sense that they reflect the energy-dependent spin makeup of the total yield \( q(E_n) \). Indeed, looking at energy-smoothed values of \( \nu_3^\pm(E_n) \), it is evident that these values are low at energies for which the total yield \( q(E_n) \) is high, i.e., at neutron resonances. At the same time at resonances with a \( J = 4 \) assignment the values of \( \nu_3^\pm(E_n) \) are systematically lower compared to \( J = 3 \) resonances. In view of the strong anti-correlation between \( \nu_3^+(E_n) \) and \( \nu_5^+(E_n) \), the values of the components \( \nu_5^+(E_n) \) behave in an opposite manner.

- At the same time the variations of \( \nu^\pm \) also display statistical behavior as the values of components of these vectors fluctuate randomly around the curves obtained from the above-mentioned smoothing. In case of the components \( \nu_3^\pm \) and \( \nu_5^\pm \) the rms values of these fluctuations are equal, respectively, to 0.050 and 0.049. The rms value for \( \nu_5^+ \) is thus appreciably higher compared to the rms value characterizing the deviation of values of \( \nu_3^+ \) from the regression line. This assessment suggests indirectly that the influence of the finite number of trials in the search for minima of \( \text{Var}[q^\pm] \) on the precision of determining the vectors \( \nu^\pm \) is not as significant as it might seem from the fluctuations of the estimates of \( \nu_3^\pm \) and \( \nu_5^\pm \) around the energy-smoothed values.

- Fluctuations of \( \nu_3^\pm \) and \( \nu_5^\pm \) around smooth values are smaller at neutron resonances and larger at energies in between the resonances.

In spite of these conclusions the question about the degree of convergence of our algorithm for obtaining optimum vectors \( \nu^\pm \) remains unanswered. In order to shed more light on this problem, we undertook a set of additional calculations. For a fixed value of neutron energy selected between well separated resonances we applied our method for estimates of \( q^\pm \) and \( \nu^\pm \) from the experimental values \( \{y_m\} \) repeatedly and independently, in this specific case 5000 times. The results of these calculations are shown in Fig. 10, where distributions of the estimates of \( \nu_3^\pm \) and \( \nu_5^\pm \) for the fixed energy \( E_n = 163.77 \text{ eV} \) are plotted. As is apparent from these plots, the rms uncertainties of the estimates due to a limited number of trials in searching for the minimum of \( \text{Var}[q^\pm] \) are by a factor of \( \approx 2 \) lower compared with the global rms value characterizing deviations of the estimates of \( \nu_3^\pm \) and \( \nu_5^\pm \) from the energy-smoothed values, see Figs. 9 and 10. The rms uncertainties belonging to independent sources are to be added quadratically. This is also the case of the rms value due to the finite statistical precision of the experimental data and the rms value resulting from limited precision of our algorithm. As a consequence, the limited degree of convergence of our method increases the fluctuations of estimates of the components of vectors \( \nu_3^\pm \) and \( \nu_5^\pm \) by a factor less
than 1.2. For the components $\nu_3^-$ and $\nu_5^-$ the upper limit of this factor is even lower. This finding leads us to conclusion that the uncertainty in estimates of the vectors $\nu^\pm$ due to our criterion for terminating the search for minima of $\text{Var} \left[ q^\pm \right]$ is small. This uncertainty can be further reduced by adopting a more stringent criterion for convergence, but at the price of prolonged CPU time.

As far as the major part of the observed fluctuations of estimates of $\nu_3^\pm$ and $\nu_5^\pm$ around their smoothed values is concerned, it evidently originates from perturbation of $\{y_m(E_n)\}$ by counting statistics errors.

As mentioned above, the vectors $\nu^+$ and $\nu^-$ are almost parallel and at the same time they satisfy conditions given by $(\nu^\pm, \mu^\pm) = 0$. This implies that $(\nu^\pm, \mu^\pm) \ll 1$. As the right-hand side of Eq. (12) is proportional to $(\nu^\pm, \mu^\pm)^{-2}$, it is evident that even small experimental errors in the measurement of $\{y_m\}$ lead to relatively large uncertainties of the deduced capture yields $q^\pm$.

6. Summary and concluding remarks

The $\gamma$ multiplicity based method for determination of spin of $s$-wave neutron resonances proposed in Ref. [3] has been upgraded by replacing the *ad hoc* chosen spin identifiers with a Monte Carlo based algorithm providing estimates of energy-dependent $\gamma$ yields $q^+(E_n)$ and $q^-(E_n)$ belonging to neutron capturing states with spin $J = I + 1/2$ and $J = I - 1/2$, respectively. From the application of the proposed method to artificial data and to the data from DANCE measurements of the $^{147}\text{Sm}(n,\gamma)^{148}\text{Sm}$ reaction, it has been demonstrated that by analyzing these yields not only the spin of individual resonances can be assigned, but even the multiplet makeup of unresolved resonance-like structures in the neutron-energy spectra can be quantitatively specified.

The method has only minor limitations caused by the Porter-Thomas fluctuations of partial widths $\Gamma_{\lambda\gamma f}$ from resonance to resonance of the same spin. This type of fluctuation leads to the appearance of ghost resonances. From dedicated DICEBOX simulations of $\gamma$ cascades following neutron capture in $^{147}\text{Sm}$ and subsequent simulations of the response of the $4\pi$ detector system to individual $\gamma$ cascades with the aid of the code GEANT4, we demonstrated that the predictions of ghost resonances from the extreme statistical model agree quantitatively with the results of the application of the proposed method to the DANCE experimental data.

The ability of the method to reveal a weak resonance at 39.52 eV demonstrates not only the capacity of the method for spin determination of weak neutron resonances, but also its power to resolve close resonance doublets – even in extreme conditions when the neutron width $\Gamma_{\lambda n}$ of the stronger resonance $\lambda$ is higher by a factor of $\approx 50$ compared to the neutron width $\Gamma_{\lambda' n}$ of the other resonance of the doublet and in conditions when the spacing between these resonances is as small as 10% of the total width $\Gamma_{\lambda t}$ of the stronger resonance. The novel method thus seems to be a promising tool not only for spin determination of resonances, but also for complete neutron resonance spectroscopy.

In principle, the above method can be generalized for identifying $p$-wave resonances and determining their spins. This would be of particular importance for solving the perennial problem of separating strong $p$-wave neutron resonances from weak $s$-wave ones and hence obtaining pure and complete samples of $s$-wave resonances for zero-spin samples. Such sequences are invaluable
for studying quantum chaos [19] and related topics in well-defined conditions.

Acknowledgments

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References


Table 1: Spin assignment of parasitic $^{149}$Sm resonances seen from the DANCE data accumulated from the $^{147}$Sm(n,$\gamma$)$^{148}$Sm experiment. Values of energies are taken from the ENDF/B-VII.0 database, see Ref. [10].

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Table 2: Spin assignment of $^{147}$Sm resonances. Values of $J$ obtained from previous analysis of the same $^{147}$Sm(n,γ)$^{148}$Sm experimental data [3] are also listed together with recommended values in ENDF/B-VII.0 database, see Ref. [10]. Except for cases referred to a footnote the energies of the resonances are taken from Ref. [3].

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Table 2 – continued from previous page

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$^a$The energy of a resonance deduced from our analysis.
$^b$According to present data this resonance does not belong to $^{147}$Sm.
$^c$If this resonance exists, its spin will be $J = 3$.
$^d$The resonances at 486.4 and 486.6 eV have a comparable strength.
Figure captions

Figure 1: (Color online.) Postulated multiplicity distributions \( \{ \mu_m^{\pm} \} \) for testing the method on artificial data.

Figure 2: (Color online.) Results of testing the method on artificial data. The meaning of the individual plots is described in the text.

Figure 3: (Color online.) Results of testing the simplified option of the method based on the use of the least-square technique. The meaning of the individual plots is described in the text.

Figure 4: (Color online.) Prototypical cluster multiplicity distributions \( \{ \mu_m^{\pm} \} \) deduced from DANCE measurements of the \( ^{147}\text{Sm}(n,\gamma)^{148}\text{Sm} \) reaction. They were obtained from yields \( \{ y_m(E_n) \} \) at isolated \( J = 3 \) and \( J = 4 \) resonances at 58.09 and 49.36 eV, respectively.

Figure 5: (Color online.) Decomposition of the experimental capture yield from the \( ^{147}\text{Sm}(n,\gamma)^{148}\text{Sm} \) reaction into estimates of \( q^{\pm} \) in the neutron-energy range 38–520 eV. The histogram drawn by solid line – the total experimental capture yield; full and opened circle points – the extracted estimates \( \hat{q}^{-} \) and \( \hat{q}^{+} \), respectively. In the sections of the plots with a blown up vertical scale by a factor of 50 the strongest resonances of \( ^{149}\text{Sm} \) clearly indicated. Their positions are indicated by arrows.

Figure 6: (Color online.) Decomposition of a multiple resonance structure near 40 eV. The thick solid curve in the upper panel represents the best fit of the \( J = 3 \) estimate \( \hat{q}^{-} \) under the assumption that only a ghost of the \( J = 4 \) resonance at 39.98 eV and the tail of the \( J = 3 \) resonance at 40.72 eV contribute to \( q^{-} \). The best fit illustrated in the lower panel was achieved by postulating an additional \( J = 3 \) resonance at 39.52 eV. In both panels the ghost resonance and the tail of the resonance at 40.72 eV are plotted by thin dashed and thin dash-dotted curves, respectively. The estimate \( \hat{q}^{+} \) plotted is scaled down by a factor of 5.

Figure 7: (Color online.) Upper panels: distributions of relative sizes \( R \) of ghost resonances accompanying 14 and 16 well-isolated parent resonances with \( J = 3 \) and \( J = 4 \), respectively, as observed from the DANCE data on the \( ^{147}\text{Sm}(n,\gamma)^{148}\text{Sm} \) reaction. Lower panels: simulated distributions of relative sizes of ghost resonances accompanying isolated parent resonances of \( ^{147}\text{Sm} \) with spin \( J = 3 \) and \( J = 4 \); these results were obtained from 12 nuclear realizations belonging to one superset. In both panels the estimates of expectation values \( E[R] \) and their rms uncertainties are indicated. The dashed plots represent the fits by normal distributions.

Figure 8: (Color online.) Upper panel: distributions of average cluster multiplicities \( \langle m \rangle \) for 14 and 16 enough-resolved resonances with \( J = 3 \) and \( J = 4 \), respectively, as observed from the DANCE data. Lower panel: analogous distributions deduced from simulations using DiceBox/Geant4 codes for one superset consisting of 12 NRs. Estimates of expectation values \( E[\langle m \rangle] \) together with their rms uncertainties, characterizing the fluctuations of \( \langle m \rangle \), are indicated.

Figure 9: (Color online.) Upper panel: a segment of the plot shown in Fig. 5. Two middle panels: dependence of values of selected components \( (m = 3, m = 5) \) of vectors \( \nu^{-} \) and \( \nu^{+} \) on \( E_n \); solid
white and black curves shown were obtained from smoothing, see the main text. Lower panel: the modulus of the difference $\nu^+ - \nu^-$ as a function of $E_n$. Rms values characterizing fluctuations of $\nu_3^+$ and $\nu_5^+$ around the smooth curves are indicated; these values were deduced from the whole set of 4400 data points in neutron-energy region 38–520 eV.

**Figure 10:** (Color online.) Distributions of estimates of selected components ($m = 3$ and $m = 5$) of vectors $\nu^\pm(E_n)$ that were obtained from 5000 independent application of the proposed method to experimental values $\{y_m(E_n)\}$ at energy $E_n = 163.77$ eV. The deduced expectation and rms values of these estimates are indicated.
Fig. 1
Fig. 2
Fig. 3
Fig. 4
Fig. 5
Doublet

\[ \chi^2 = 115.4, \nu = 76 \]

Triplet

\[ \chi^2 = 83.6, \nu = 74 \]

Fig. 6
Relative ghost size $R$

Number of resonances per bin

$J = 3$

$0.047 \pm 0.063$

$0.028 \pm 0.057$

$0.002 \pm 0.048$

$0.018 \pm 0.056$

DANCE

DICEBOX/GEANT4

Fig. 7
Fig. 8
Yield (counts per bin)

ν +

ν −

\[ \nu = 3 \]

\[ \nu = 5 \]

\[ J = 3 \]

\[ J = 4 \]

Neutron energy (eV)

120 140 160 180 200

Fig. 9
Fig. 10

Components of vectors $\nu^\pm$