Weak interactions

Neutral kaon decay and CP violation
Quark mixing and CKM matrix

• Observation: $\Delta S=1$ weak decay suppressed than $\Delta S=0$ weak process

\[
\begin{align*}
n & \rightarrow p + e^- + \bar{\nu}_e \quad (1) \\
\Sigma^- & \rightarrow n + e^- + \bar{\nu}_e \quad (2)
\end{align*}
\]

Process (2) is suppressed by a factor of 20 compared with (1)

Look at the diagrams at the quark level for these two processes (diagram on board)
In analogous to doublet, introduce quark doublet:

\[
\begin{pmatrix}
u \\ d \end{pmatrix}_{\text{weak}} = \begin{pmatrix}
u \\ d \end{pmatrix}_{\text{strong}}
\]

Then the effective coupling for

\[
n \rightarrow p + e^- + \bar{\nu}_e
\]

\[
\Sigma^- \rightarrow n + e^- + \bar{\nu}_e
\]

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**Cabibbo angle**

\[
20 = \left( \frac{\cos \theta_c}{\sin \theta_c} \right)^2 \implies \theta_c \approx 12.6^0
\]

In 1970, Glashow, Iiiopoulos and Maiani (GIM)

\[
\begin{pmatrix}
  u \\
  d'
\end{pmatrix} = \begin{pmatrix}
  u \\
  d \cos \theta_c + s \sin \theta_c
\end{pmatrix}
\quad \begin{pmatrix}
  c \\
  s'
\end{pmatrix} = \begin{pmatrix}
  c \\
  s \cos \theta_c - d \sin \theta_c
\end{pmatrix}
\]

Which can be expressed in matrix form (charm discovered in 1974)

\[
\begin{pmatrix}
  d' \\
  s'
\end{pmatrix} = \begin{pmatrix}
  \cos \theta_c & \sin \theta_c \\
  -\sin \theta_c & \cos \theta_c
\end{pmatrix}
\begin{pmatrix}
  d \\
  s
\end{pmatrix}
\]

Quark state in the weak interaction
Neutral current selection rule $\Delta S=0$

Only consider u, d, s quarks, the neutral current interaction (diagram on board):

$$ (u\bar{u} + d\bar{d} \cos^2 \theta_c + s\bar{s} \sin^2 \theta_c) + (s\bar{d} + d\bar{s}) \sin \theta_c \cos \theta_c $$

$\Delta S=0$

$$ (u\bar{u} + d\bar{d} \cos^2 \theta_c + s\bar{s} \sin^2 \theta_c) + (s\bar{d} + d\bar{s}) \sin \theta_c \cos \theta_c $$

$\Delta S \neq 0$

To fix this, introduce a second quark doublet by having a new quark $c$

$$ (u\bar{u} + c\bar{c}) + (d\bar{d} + s\bar{s}) \cos^2 \theta_c + (d\bar{d} + s\bar{s}) \sin^2 \theta_c + (s\bar{d} + d\bar{s} - \bar{s}d - s\bar{d}) \sin \theta_c \cos \theta_c $$
With six quark flavors, the weak currents will be described by unitary transformations among three quark doublets

\[
\begin{pmatrix}
  d' \\
  s' \\
  b'
\end{pmatrix}
= V
\begin{pmatrix}
  d \\
  s \\
  b
\end{pmatrix}
\]

Where the transformation matrix $V$ is 3 by 3 unitary
Cabbibo, Kobayashi, Maskawa Matrix

- Standard model:
  - The CKM matrix: the complex phase of CKM matrix leads to CP violation

\[
\begin{pmatrix}
    d' \\
    s' \\
    b'
\end{pmatrix} =
\begin{pmatrix}
    V_{ud} & V_{us} & V_{ub} \\
    V_{cd} & V_{cs} & V_{cb} \\
    V_{td} & V_{ts} & V_{tb}
\end{pmatrix}
\begin{pmatrix}
    d \\
    s \\
    b
\end{pmatrix} = \hat{V}_{CKM}
\begin{pmatrix}
    d \\
    s \\
    b
\end{pmatrix}
\]

\[\begin{array}{c}
\Rightarrow \\
\end{array} \]

For \( n \) generations:

- Weak eigenstates
- Mass eigenstates

\[
\begin{array}{c}
\text{Weak eigenstates} & \frac{n(n-1)}{2} \quad \text{angles} & 3 \\
\text{Mass eigenstates} & \frac{(n-1)(n-2)}{2} \quad \text{phases} & 1
\end{array}
\]
\[
\begin{pmatrix}
(d') \\
(s') \\
(b')
\end{pmatrix} =
\begin{pmatrix}
c_1 & s_1c_3 & s_1s_3 \\
-s_1c_2 & c_1c_2c_3 - s_2s_3e^{i\delta} & c_1c_2c_3 + s_2e^{i\delta} \\
-s_1s_2 & c_1s_2c_3 + c_2s_3e^{i\delta} & c_1s_2c_3 - c_2e^{i\delta}
\end{pmatrix}
\begin{pmatrix}
d \\
s \\
b
\end{pmatrix}
\]

\[c_i = \cos(\theta_i)\]
\[s_i = \sin(\theta_i)\]
\[i = 1, 2, 3\]

**Charged hadronic weak current**

\[J_\lambda^+ = (\bar{u}, \bar{c}, \bar{t})\gamma_\lambda (1 - \gamma_5) \times \begin{pmatrix}
c_1 & s_1c_3 & s_1s_3 \\
-s_1c_2 & c_1c_2c_3 - s_2s_3e^{i\delta} & c_1c_2c_3 + s_2e^{i\delta} \\
-s_1s_2 & c_1s_2c_3 + c_2s_3e^{i\delta} & c_1s_2c_3 - c_2e^{i\delta}
\end{pmatrix}
\begin{pmatrix}
d \\
s \\
b
\end{pmatrix}\]
The phase enters the wavefunction as \( \exp[i(\omega t + \delta)] \) not invariant under \( t \rightarrow -t \). This phase introduces the important Possibility of T-violation, i.e., CP violation.

The various elements of the matrix have been determined in a range of experiments

\[
|V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2 = 1
\]

\[
V_{CKM} = \begin{pmatrix}
V_{ud} = 0.975 & V_{us} = 0.221 & V_{ub} = 0.005 \\
V_{cd} = 0.221 & V_{cs} = 0.974 & V_{cb} = 0.04 \\
V_{td} = 0.01 & V_{ts} = 0.041 & V_{tb} = 0.999 
\end{pmatrix}
\]
Wolfenstein parametrization -1983

- Expansion in terms $\lambda = \sin \theta_c$:
  
  $V_{us} = 0.22 = \lambda \rightarrow V_{ud} = (1 - \lambda^2 / 2)$
  
- From B-lifetime: $V_{cb} = 0.04 \sim 0.06 = A\lambda^2$

- CP violation effects smaller third order: $A\lambda^3 (\rho - i\eta)$

- Keep $V_{ud}, V_{us}, V_{ts}$ and $V_{tb}$ real;
  
  Use unitarity to calculate $\sum_+$

\[
\begin{pmatrix}
1 - \frac{\lambda^2}{2} & \lambda & A\lambda^3 (\rho - i\eta + i\eta \frac{\lambda^2}{2}) \\
-\lambda & 1 - \frac{\lambda^2}{2} - i\eta A\lambda^4 & A\lambda^2 (1 + i\eta \lambda^2) \\
A\lambda^3 (1 - \rho - i\eta) & -A\lambda^2 & 1
\end{pmatrix}
\]
Neutral $K$ mesons

- Two isospin doublets

$\begin{align*}
    s &= +1 \\
    &\begin{pmatrix}
        K^+ (u\bar{s}) \\
        K^0 (d\bar{s})
    \end{pmatrix}
\end{align*}$

$\begin{align*}
    s &= -1 \\
    &\begin{pmatrix}
        \bar{K}^0 (\bar{d}s) \\
        K^- (s\bar{u})
    \end{pmatrix}
\end{align*}$

They are particle and anti-particle and by the CPT theorem have the same mass. Experimentally we find:

$| (m_{K^0} - m_{\bar{K}^0}) | / m < 9 \times 10^{-19}$

$K_1^0 = \frac{1}{\sqrt{2}} [ | K^0 > + | \bar{K}^0 > ]$, CP = -1

$| K^0 > = - | \bar{K}^0 >$

$| \bar{K}^0 > = - | K^0 >$

$K_2^0 = \frac{1}{\sqrt{2}} [ | K^0 > - | \bar{K}^0 > ]$, CP = +1

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Weak decay $\Delta S=\pm 1$ to non-strange mesons, leptons

$$K^0 \rightarrow \pi^+\pi^- \quad K^+ \rightarrow \pi^+\pi^0$$

$K_S^0$ mean lifetime $(0.8923 \pm 0.0022) \times 10^{-10}$ (sec)

$K_S^0 \rightarrow \pi^+\pi^-, \pi^0\pi^0$, CP $= +1$

$K_L^0$ mean lifetime $(5.183 \pm 0.040) \times 10^{-8}$ (sec)

$K_L^0 \rightarrow \pi^0\pi^0\pi^0$, $\pi^+\pi^-\pi^0$, CP $= -1$
Strangeness Productions

\[ \pi^- + p \rightarrow \Lambda + K^0 \]
\[ \pi^+ + p \rightarrow K^+ + \bar{K}^0 + p \]

\[ K^0, \bar{K}^0 \quad \text{Eigen states of production} \]

\[ \gamma + p \rightarrow K^0 \Lambda^+ \quad \gamma + p \rightarrow K^0 \bar{K}^0 p \]
\[ \gamma + p \rightarrow K^+ \Lambda^0 \quad \gamma + p \rightarrow K^+ K^- p \]

Strong and EM interactions conserve strangeness
**Experimental observation**

Starting off with a beam of pure $K^0$ beam one could end up after a few meters, with a beam of mixed strangeness $K^0$ and $\bar{K}^0$

$$|K(t)\rangle = \alpha(t) |K^0\rangle + \beta(t) |\bar{K}^0\rangle$$

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Object decay by weak interactions are eigenstates of CP, not of Strangeness```

*Strangeness eigenstates* \[\leftarrow\] *production*

*CP eigenstates*

*Mass eigenstates* \[\leftarrow\] *decay*
**Strangeness oscillation**

The weak interaction kaon eigenstates are those concerned when kaons propagate through space. Because $K_L$ and $K_S$ have different life times and decay modes, they can be expected to have a difference in mass, which will alter the relative phase of these two eigenstates.
The amplitude of the state $K_s$ as a function of time, measured in
the particle rest frame, with $E=m$ is

$$A_s(t) = A_s(0)e^{-(\Gamma_s / 2 + im_s)t}$$

$$\Gamma_s = \frac{\hbar}{\tau_s}$$

Is the width of the $K_S$ state and $m_s$ is its rest mass.

Similarly for $K_L$

$$A_L(t) = A_L(0)e^{-(\Gamma_L / 2 + im_L)t}$$

Assume at $t=0$, we start with pure $K^0$ beam

$A_L(0) = A_s(0) = \frac{1}{\sqrt{2}}$

At time $t$ the $K^0$ intensity
\[ |K^0 > = \frac{1}{\sqrt{2}} [ |K_L^0 > + |K_S^0 > ] \]

\[ |\bar{K}^0 > = \frac{1}{\sqrt{2}} [ |K_L^0 > - |K_S^0 > ] \]

\[ I(K^0) = \frac{1}{2} [A_S(t) + A_L(t)][A_S^*(t) + A_L^*(t)] \]

\[ = \frac{1}{4} [e^{-\Gamma_S t} + e^{-\Gamma_L t} + 2e^{-(\Gamma_S + \Gamma_L)\frac{t}{2}} \cos \Delta m t] \]

\[ \Delta m = m_L - m_S \]

\[ I(\bar{K}^0) = \frac{1}{4} [e^{-\Gamma_S t} + e^{-\Gamma_L t} - 2e^{-(\Gamma_S + \Gamma_L)\frac{t}{2}} \cos \Delta m t] \]

\[ \Delta m = (3.491 \pm 0.009) \times 10^{-12} \text{ MeV} \]
Strangeness Oscillations

the probability that a beam initially consisting of $K^0$’s contains $K^0$’s at a later time:

$$\left|\langle \bar{K}^0 | K^0(t) \rangle \right|^2 = \frac{1}{4} \left( e^{-\Gamma_s t} + e^{-\Gamma_L t} - 2e^{-\frac{t}{2}(\Gamma_s + \Gamma_L)} \cos(\Delta m t) \right)$$

We can measure the strangeness content of a beam as a function of time (distance) by putting some material in the beam and counting the number of strong interactions that have $S=+1$ in the final state vs. the number with $S=-1$.

$$K^0 p \rightarrow K^+ n \quad S=+1$$

$$\bar{K}^0 p \rightarrow \Lambda p \quad S=-1$$

Strangeness oscillations for an initially pure $K^0$ beam. A value of $(m_2-m_1)\tau_s=0.5$ is used.
**CP Violation**

In 1964, Christenson, Cronin, Fitch and Turlay discovered at BNL that the long-lived neutral K meson with CP=-1 could decay occasionally to $\pi^+\pi^-$ with CP=+1 about once every 500 decays

\[ K^0_L \rightarrow \pi^+\pi^-\pi^0 \quad K^0_L \rightarrow \pi^+\pi^- \]

- CP violations in nuclear systems
- More CP violations in experiments: B meson decays, SLAC and KEK
CP-violation in neutral kaon system

• $\pi^0 \pi^0$, $\pi^+ \pi^-$
  – Bose statistics $1 \Leftrightarrow 2$ the total wavefunction is symmetric
  – Spin 0 $1 \Leftrightarrow 2 = \text{operation of } C \text{ followed by } P \Rightarrow CP=+1$

• $\pi^0 \pi^+ \pi^-$
  CP = +1 for $\pi^+ \pi^-$

• Neutral kaon decay has very small Q-value, 3 pions in relative $S$-state

• $\pi^0 : P=-1$, $C=+1 \Rightarrow \pi^0 \pi^+ \pi^- = -1$
Neutral Kaons and CP violation

Since CP is not conserved in neutral kaon decay it makes more sense to use mass (or lifetime) eigenstates rather than $|k_1>$ and $|k_2>$:

Short lifetime state with $\tau_S \approx 9\times 10^{-11}$ sec.

$$|K_S> = \frac{1}{\sqrt{1 + |\varepsilon|^2}} (|K_1> + \varepsilon |K_2>)$$  "K - short"

Long lifetime state with $\tau_L \approx 5\times 10^{-8}$ sec.

$$|K_L> = \frac{1}{\sqrt{1 + |\varepsilon|^2}} (|K_2> + \varepsilon |K_1>)$$  "K - long"

$\varepsilon$ is a (small) complex number that allows for CP violation through mixing.

There can be two types of CP violation in $K_L$ decay:

- indirect ("mixing"): $K_L \rightarrow \pi\pi$ because of its $K_1$ component
- direct: $K_L \rightarrow \pi\pi$ because the amplitude for $K_2$ allows $K_2 \rightarrow \pi\pi$

It turns out that both types of CP violation are present and indirect $>>$ direct!

Slides 18, 21-24 adapted from notes by Richard Kass at OSU
Neutral Kaons and CP violation

The $K_L$ and $K_S$ states are not CP (or S) eigenstates since:

$$CP |K_s >= \frac{1}{\sqrt{1+|\epsilon|^2}} (CP |K_1 >= \epsilon CP |K_2 >) = \frac{1}{\sqrt{1+|\epsilon|^2}} (|K_1 > - \epsilon |K_2 >) \neq |K_s >$$

$$CP |K_L >= \frac{1}{\sqrt{1+|\epsilon|^2}} (CP |K_2 >+ \epsilon CP |K_1 >) = \frac{1}{\sqrt{1+|\epsilon|^2}} (-|K_2 >+ \epsilon |K_1 >) \neq |K_L >$$

In fact these states are not orthogonal!

$$< K_S | K_L > = < K_S | K_L > = \frac{2 \text{Re} \epsilon}{\sqrt{1+|\epsilon|^2}}$$

If CP violation is due to mixing (“indirect” only) then the amplitude for $k_L \rightarrow \pi\pi$ is:

$$< K_1 | K_L > = \frac{1}{\sqrt{1+|\epsilon|^2}} (< K_1 | K_2 > + \epsilon < K_1 | K_1 >) = \frac{\epsilon}{\sqrt{1+|\epsilon|^2}}$$

While the amplitude for $k_L \rightarrow \pi\pi\pi$ is:

$$< K_2 | K_L > = \frac{1}{\sqrt{1+|\epsilon|^2}} (< K_2 | K_2 > + \epsilon < K_2 | K_1 >) = \frac{1}{\sqrt{1+|\epsilon|^2}}$$

From experiments we find that $|\epsilon| = 2.3 \times 10^{-3}$.

However, the standard model predicts a small amount of direct CP violation too!
Neutral Kaons and CP violation

The standard model predicts that the quantities $\eta_{+-}$ and $\eta_{00}$ should differ very slightly as a result of direct CP violation (CP violation in the amplitude).

$$\eta_{+-} = \frac{Amp(K_L \to \pi^+\pi^-)}{Amp(K_s \to \pi^+\pi^-)} \quad \eta_{00} = \frac{Amp(K_L \to \pi^0\pi^0)}{Amp(K_s \to \pi^0\pi^0)}$$

CP violation is now described by two complex parameters, $\epsilon$ and $\epsilon'$, with $\epsilon'$ related to direct CP violation. The standard model estimates $\text{Re}(\epsilon'/\epsilon)$ to be $(4-30) \times 10^{-4}$.

Experimentally what is measured is the ratio of branching ratios:

$$\frac{\text{BR}(K_L \to \pi^+\pi^-)}{\text{BR}(K_s \to \pi^+\pi^-)} = \left| \frac{\eta_{+-}}{\eta_{00}} \right|^2 = \frac{|\epsilon + \epsilon'|^2}{|\epsilon - 2\epsilon'|^2} \approx 1 + 6\text{Re}\left(\frac{\epsilon'}{\epsilon}\right)$$

After many years of trying (starting in 1970’s) and some controversial experiments, a non-zero value of $\text{Re}(\epsilon'/\epsilon)$ has been recently been measured (2 different experiments):

$$\text{Re}(\epsilon'/\epsilon) = 17.2 \pm 1.8 \times 10^{-4}$$

At this point, the measurement is more precise than the theoretical calculation! Calculating $\text{Re}(\epsilon'/\epsilon)$ is presently one of the most challenging HEP theory projects.
Neutral Kaons, Flavor Oscillations, & CP Violation

From an experimentalist's point of view a good quantity to measure is the yield (as a function of proper time, time measured in the rest frame of the K) of $\pi^+\pi^-$ decays from a beam that is initially $K^0$. Since we are measuring the sum of the square of the amplitude $|K_L\rightarrow\pi^+\pi^+\rangle + |K_S\rightarrow\pi^+\pi^-\rangle$ there will be an interference term in the number of $\pi^+\pi^-$ decays/time ($\equiv l(t)$). The yield of $\pi^+\pi^-$ decays is given by:

$$I_{\pi^+\pi^-}(t) = I_{\pi^+\pi^-}(0) \left( e^{-\frac{t}{\tau_L}} + \left|\eta_{+-}\right|^2 e^{-\frac{t}{\tau_S}} + 2\left|\eta_{+-}\right|^2 e^{-\frac{t}{2\tau_S} \left(\frac{1}{\tau_S} + \frac{1}{\tau_L}\right)} \cos\left(\frac{t}{\hbar} (m_2 - m_1) + \phi_{+-}\right)\right)$$

Thus by measuring this yield we gain information on the mass difference as well as the CP violation parameters $\eta_{+-}$ and $\phi_{+-}$. 

Event rate for $\pi^+\pi^-$ decays as a function of proper time. The best fit requires interference between the $K_L$ and $K_S$ amplitudes.

$\Delta m = 3.491 \pm 0.009 \times 10^{-6}$ eV
\[ |\eta_{+-}| = (2.29 \pm 0.01) \times 10^{-3} \]
\[ \phi_{+-} = (43.7 \pm 0.6)^0 \]
$\tau_S = 0.893 \times 10^{-10}$ sec
$\tau_L = 0.517 \times 10^{-7}$ sec