Duke University
Physics Department

Physics 305
Assignment No. 1 (60 pts)
(due in class Jan-26-2010)

Problems:

1. [20 pts] The Lagrangian density for a vector field with mass $M$ is written as:

$$ L = -\frac{1}{4}(\partial^\nu A_\mu - \partial^\mu A_\nu)(\partial^\nu A^\mu - \partial^\mu A^\nu) + \frac{1}{2} M^2 A^\mu A_\mu $$

Use the Euler-Lagrange equation to derive the Proca equation:

$$ \partial_\nu (\partial^\nu A^\mu - \partial^\mu A^\nu) + M^2 A^\mu = 0 $$

Show Eqn. (2) can be written further as

$$ (\partial_\nu \partial^\nu + M^2) A^\mu = 0 $$

2. Verify explicitly that changing the Lagrangian density by a total divergence leaves the Euler-Lagrange equations unchanged. [20pts]

3. Use the requirement that the Lagrangian be invariant under a continuous symmetry to deduce the conserved quantity that corresponds to a particular transformation. Show that invariant under (i) translations in space, (ii) translations in time, (iii) spatial rotations implies conservation of (i) momentum, (ii) energy, (iii) angular momentum. [20pts]