Weak interactions and vector bosons

• What do we know now about weak interactions?
• Theory of weak interactions
  – Fermi's theory of weak interactions
  – V-A theory
  – Current - current theory, current algebra
  – W and Z bosons
  – Electroweak theory
• Neutral current reactions
Weak interactions

• Weak interactions occur for all fundamental particles except gluons and photons. Weak interactions involve the exchange or production of W or Z bosons.

• Weak forces are very short-ranged. In ordinary matter, their effects are negligible except in cases where they allow an effect that is otherwise forbidden. There is a number of conservation laws that are valid for strong and electromagnetic interactions, but broken by weak processes. So, despite their slow rate and short range, weak interactions play a crucial role in the make-up of the world we observe.

• Any process where the number of particles minus the number of antiparticles of a given quark or lepton type changes, is weak decay process and involves a W-boson. Weak decays are thus responsible for the fact that ordinary stable matter contains only up and down type quarks and electrons. Matter containing any more massive quark or lepton types is unstable. If there were no weak interactions, then many more types of matter would be stable.
Quarks and their weak decays

- Quark flavour never changes except by weak interactions, like beta decay, that involve W bosons.
- Any quark type can convert to any other quark type with a different electric charge by emitting or absorbing a W boson. Different possible transitions are shown schematically in the diagram.
**Weak interactions**

- Decay processes always proceed from a more massive quark to a less massive quark, because the reverse process would violate conservation of energy. Scattering processes can involve the reverse transitions, provided sufficient energy is available.

- Lepton number conservation rules
  Each charged lepton is converted only to its own neutrino type by emitting or absorbing a W boson. This leads to the three lepton number conservation laws.
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<td>$\nu_\mu e \rightarrow \mu\nu_e$</td>
<td>$\nu_e e \rightarrow \nu_e e$</td>
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<td>$\nu_\mu e \rightarrow \mu\nu_e$</td>
<td>$\bar{\nu}<em>\mu e \rightarrow \bar{\nu}</em>\mu e$</td>
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<td>$\tau \rightarrow l\nu\nu$</td>
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Fermi's theory of weak interactions

Beginning of the story:

• As an example Fermi considered $\beta$-decay of the neutron:
  \[ n \rightarrow p + e^- + \nu_e \]

• Assumption: at a single point in the space-time, the wavefunction of the neutron is transformed into that of the proton, and the wavefunction of incoming neutrino (equivalent to outgoing antineutrino which we actually see) is transformed into that of electron.

• The schematic diagram for this process:

Such a process is called four-fermion interaction.
**Relativistic interaction currents**

\[ J_{\mu}^{EM} = -\overline{\psi}_e \gamma_{\mu} \psi_e \]

electromagnetic interaction from QED

\[ H_{EM} = J_{\mu}^{EM} A^\mu = -e \overline{\psi} \gamma_{\mu} \psi A^\mu \]

Fermi (1934) introduced a weak hadronic current density

\[ V_{\mu}^{C+} = \overline{\psi}_P \gamma_{\mu} \psi_n \]

(charged weak hadronic current)

in analogy to \( J_{\mu}^{EM} \) and \( A^\mu \) (photon)

\[ V_{\mu}^{C} \] weak field \( W^\pm \) (\( W \) boson)

**Fermi constructed further**

\[ l_{\mu}^{C} = \overline{\psi}_e \gamma_{\mu} \psi_v \]

\[ H_\beta = \frac{G_\beta}{\sqrt{2}} \left( V_{\mu}^{C+} l_{\mu}^{C+} + l_{\mu}^{C} V_{\mu}^{C+} \right) \]
**V-A theory**

• The description of this process is given by the amplitude:

\[ M = G_F (\bar{\psi}_p \Gamma \psi_n)(\bar{\psi}_e \Gamma \psi_v) \]

where the factors \( \Gamma \) are responsible for particle transformation, and \( G_F \) is the very well known Fermi coupling constant which determines the strength of the weak interaction (the rate of the decay).

• In 1956 Feynman and Gell-Mann proposed that the interaction factors \( \Gamma \) are a mixture of vector and axial-vector quantities, to account for parity violation in weak interactions.
  – Vector - changes sign if rotated through 180°.
  – Axial vector - like vector under rotation, but opposite sign to vector under parity transformation.

• V-A theory was proven by many experiments at that time, in particular by the fact that neutrino is left-handed (helicity -1/2) - its spin is antiparallel to its momentum.
leptonic current density, which results in left-handed neutrinos:

\[ l^C_\mu = \bar{\psi}_e \gamma_\mu (1 - \gamma_5) \psi_v \]

hadronic current

\[ h^C_\mu^+ = \bar{\psi}_p \gamma_\mu (1 - c_A \gamma_5) \psi_n \] (V-A structure)

\(^{60}\text{Co}\) Parity-violation \(\beta\) decay

Pseudoscalar \(\left\langle \vec{S}_K \cdot \vec{P}_e \right\rangle\) \hspace{1cm} \(^{60}\text{Co} \rightarrow ^{60}\text{Ni} + e^- + \bar{\nu}_e\)

\(^{60}\text{Co}\) nucleus

Wu et al (57)
Vector and axial vector current

\[ j_\mu = \overline{\psi} \gamma_\mu \psi \]

\[ j_\mu^5 = \overline{\psi} \gamma_\mu \gamma_5 \psi \]

\[ i \partial_\mu j^\mu = i \partial_\mu (\overline{\psi} \gamma^\mu \psi) = i(\partial_\mu \overline{\psi}) \gamma^\mu \psi + i \overline{\psi} \gamma^\mu (\partial_\mu \psi) \]
\[ = -m \overline{\psi} \psi + m \overline{\psi} \psi \]

\[ i(\partial_\mu \overline{\psi}) \gamma^\mu = -m \overline{\psi} \]

\[ i \gamma^\mu (\partial_\mu \psi) = m \psi \]

\[ \therefore \partial_\mu j^\mu = 0 \]

Vector current is conserved
\[ i \partial_\mu j_\mu^5 = i(\partial_\mu \bar{\psi}) \gamma^\mu \gamma_5 \psi + i \bar{\psi} \gamma^\mu \gamma_5 (\partial_\mu \psi) \]
\[ = i(\partial_\mu \bar{\psi}) \gamma^\mu \gamma_5 \psi - i \bar{\psi} \gamma_5 \gamma^\mu (\partial_\mu \psi) \]
\[ = -m \bar{\psi} \gamma_5 \psi - m \bar{\psi} \gamma_5 \psi \]
\[ = -2m \bar{\psi} \gamma_5 \psi \]

The axial vector current is not conserved for \( m \neq 0 \)

\( m \): current mass of the quark
Current - current theory

• We can also describe weak interactions in terms of the interactions of two "currents". In β-decay one current converts a neutron into a proton, and the other creates an electron and its antineutrino (absorbs or destroys neutrino).

• For simplicity consider leptonic reactions which involves only leptons. As a result of lepton number conservation law, the absorption of an electron neutrino is always accompanied by the creation of an electron and vice versa. Equivalently, the creation of a positron should be accompanied by the creation of an electron neutrino (or the absorption/destroyp of electron antineutrino)

\[
L^W = \bar{\psi}_e \Gamma \psi_{\nu_e} + \bar{\psi}_\mu \Gamma \psi_{\nu_\mu} + \bar{\psi}_\tau \Gamma \psi_{\nu_\tau} \quad \bar{L}^W = \bar{\psi}_{\nu_e} \Gamma \psi_e + \bar{\psi}_{\nu_\mu} \Gamma \psi_\mu + \bar{\psi}_{\nu_\tau} \Gamma \psi_\tau
\]
the leptonic current is extended in an obvious way to the muonic & tau-lepton sectors:

\[ l^C_\mu = \overline{\psi}_e \gamma_\mu (1 - \gamma_5) \psi_{\nu_e} + \overline{\psi}_\mu \gamma_\mu (1 - \gamma_5) \psi_{\nu_\mu} + \overline{\psi}_\tau \gamma_\mu (1 - \gamma_5) \psi_{\nu_\tau} \]
**Leptonic currents**

\[ L^W \equiv \nu_e e^- + \nu_\mu \mu^- \]

\[ \bar{L}^W \equiv e^- \nu_e + \mu^- \nu_\mu \]

- The *leptonic current* should be multiplied by its antiworld partner to generate all observed weak interactions of leptons.
- The first-order amplitude of the all leptonic processes can be written as:

\[
M = G_F \bar{L}^W L^W \equiv e^- \nu_e + e^- \nu_e + \mu^- \nu_\mu + \mu^- \nu_\mu + \ldots
\]
**Hadronic current**

- There are also *semileptonic reactions* which involve both leptons and hadrons, such as neutron $\beta$-decay, and *hadronic weak reactions* which involve only hadrons, such as the decays of neutral kaons into 2 or 3 pions.

- Hadronic weak current can be characterised in terms of its effect on the quantum numbers of hadrons.

- The total weak current is a sum of leptonic and hadronic components:

  \[ J^w = L^w + H^w \]

- Then, the amplitude of weak interactions can be presented as:

  \[ G_F \bar{J}^w J^w = G_F (\bar{L}^w L^w + \bar{L}^w H^w + \bar{H}^w L^w + \bar{H}^w H^w) \]
Hadronic current and current algebra

• Similar to the leptonic current, hadronic current is a mixture of vector and axial-vector components, to ensure the correct parity-violating effects. But hadronic current should have also a part which conserves the strangeness of the participating hadrons and another part which changes the strangeness:

\[ H^W = h^\pm \cos \theta_C + s^\pm \sin \theta_C \], where \( \theta_C \) is the Cabibbo angle which governs the relative strength of the two components. This is for 4-quark scheme. For 6-quark scheme the \( \cos \theta_C \) and \( \sin \theta_C \) are replaced with a matrix.

• So, the hadronic weak current has four separate components

• The effect of different parts of the current on the quantum numbers:

\[ h^\pm \rightarrow (\Delta Y, \Delta I, \Delta I_3) = (0, 1, \pm 1) \quad \quad s^\pm \rightarrow (\Delta Y, \Delta I, \Delta I_3) = (\pm 1, 1/2, \pm 1/2) \quad (Y=B+S) \]

• Cabibbo angle was measured in the experiments: \( \cos \theta_C \approx 0.97, \theta_C \approx 12^\circ \).
Quark mixing and CKM matrix

• Observation: $\Delta S=1$ weak decay suppressed than $\Delta S=0$ weak process

$$n \rightarrow p + e^- + \bar{\nu}_e \ (1)$$

$$\Sigma^- \rightarrow n + e^- + \bar{\nu}_e \ (2)$$

*Process (2) is suppressed by a factor of 20 compared with (1)*

Look at the diagrams at the quark level for these two processes
\[
\begin{pmatrix}
u \\ d
\end{pmatrix}_{\text{weak}} = \begin{pmatrix}
u \\ d
\end{pmatrix}_{\text{strong}}
\]

In analogous to \( \begin{pmatrix} e \\ \nu_e \end{pmatrix} \) and \( \begin{pmatrix} u \\ \nu_\mu \end{pmatrix} \) doublet, **Introduce quark doublet:**

\[
\begin{pmatrix}
u \\ d \cos \theta_c + s \sin \theta_c
\end{pmatrix}
\]

**Then the effective coupling for**

\begin{align*}
n &\rightarrow p + e^- + \bar{\nu}_e \\
\Sigma^- &\rightarrow n + e^- + \bar{\nu}_e
\end{align*}

\[
G \cos \theta_c \\
G \sin \theta_c
\]
Cabibbo angle

\[ 20 = \left( \frac{\cos \theta_c}{\sin \theta_c} \right)^2 \implies \theta_c \approx 12.6^0 \]

In 1970, Glashow, Iiiopoulos and Maiani (GIM)

\[
\begin{pmatrix}
    u \\
d'
\end{pmatrix} = \begin{pmatrix}
    u \\
d \cos \theta_c + s \sin \theta_c
\end{pmatrix}
\]

\[
\begin{pmatrix}
    c \\
s'
\end{pmatrix} = \begin{pmatrix}
    c \\
    s \cos \theta_c - d \sin \theta_c
\end{pmatrix}
\]

Which can be expressed in matrix form (charm discovered in 1974)

\[
\begin{pmatrix}
    d' \\
s'
\end{pmatrix} = \begin{pmatrix}
    \cos \theta_c & \sin \theta_c \\
    -\sin \theta_c & \cos \theta_c
\end{pmatrix}
\begin{pmatrix}
    d \\
s
\end{pmatrix}
\]

Quark state in the weak interaction
Problems with Fermi's theory

- Low-energy weak interactions can be well explained in the framework of Fermi's theory (in the framework of the current-current theory in the V-A form).
- Problems begin when high-energy processes need to be described.
  - The theory predicts linear rise of the neutrino cross-sections with energy, which violates the unitarity principle - the probability for a particular process to occur should be less than or equal to unity. At the energy about 300 GeV the cross-section exceeds the maximal value allowed by the unitarity principle.
  - Very large cross-section at high energies contradict also cosmic-ray data. Neutrinos are the important part of secondary cosmic rays produced in the atmosphere but their interactions with matter were not frequent and put strong limit on their cross-section.
bosons (82 GeV) range 1/1000 fermi

*: Fermi's theory as a low-energy limit of an interaction mediated by the exchange of bosons.
**New ideas**

- Fermi's basic principle: analogy with electromagnetism.
- But electromagnetic processes are described as an exchange of photons.
- There should be vector particles which transmits the weak force.
- Weak interaction is of short range, hence the vector particle should be heavy.
- Since β-decay changes nuclear charge, the vector particles should carry the charge.
- Four-fermion point-like interactions were abandoned and replaced with a particle ($W$) exchange mechanism.

\[ e^- \rightarrow W^- \rightarrow e^- \]
\[ \nu_e \rightarrow W^- \rightarrow \nu_e \]
\[ \mu^+ \rightarrow W^+ \rightarrow \mu^- \]
\[ v_\mu \rightarrow W^+ \rightarrow v_\mu \]

Examples of basic lepton processes with the creation or destruction of W-boson
Problems with W-boson

• Neutron $\beta$-decay can now be shown as an exchange of W-boson.

• W-boson helped to solve some problems of Fermi's theory but produced others.

• W-boson has spin 1 and non-zero mass. As a consequence at high-momentum transfer its propagator is proportional to $p^2 / M^2 c^2$. The diagrams are diverging and the results of the theory are infinite.
Electroweak theory and Higgs mechanism

• First step: 1954 - C. N. Yang and R. Mills developed a theory of interacting massless particles - gauge theory (the Lagrangian describing the interaction of particle wavefunctions remains invariant under certain symmetry transformations). The theory could accommodate particles like the photon and W-boson, but it required them to be massless. The infinities in the model could be reabsorbed (the model was "renormalisable").
•1964 - Peter Higgs found a way to transform the Lagrangian describing two massless scalar particles and one massless vector gauge particle into the Lagrangian with one massive scalar particle and one massive vector gauge particle ("Higgs mechanism" for symmetry breaking - mechanism that can give masses to W-bosons).

•1967 - 1968 - Sheldon Glashow, Steven Weinberg and Abdus Salam - development of the standard model of electroweak interactions, which unifies electromagnetic and weak interactions.
Unification of the interactions

• In the last decades the belief has grown that the strong, electromagnetic, weak and gravitational interactions are different aspects of a single universal interaction, which would be manifested at very high energies inaccessible by existing accelerators.
• In our everyday particle physics experiments at relatively low energies this symmetry is broken.
• The first successful attempt to unify two apparently different interactions was done by Maxwell in 1865. He showed that electricity and magnetism could be unified into a single theory involving a vector field (electromagnetic field) interacting between electric charges and currents.
• Maxwell equations contain one arbitrary constant - the velocity of light - which is not predicted by the theory and has to be determined from experiment.

• Glashow, Weinberg and Salam - unification of the electromagnetic and weak interactions into electroweak interaction with a single coupling described by the elementary electric charge, $e$. 
Electroweak theory and neutral current

• Start - massless Yang-Mills particles: $W^+$, $W^-$, $W^0$ and $B$ (not to be confused with B-meson). $W$-particles form a triplet of the new symmetry, "weak isospin", $B$ is an isosinglet.

• Higgs mechanism gives masses to the $W$-bosons.

• $W^0$ and $B$ mix to produce two physical particles, the photon (represented by the field $A$) and the $Z$-boson. The photon is massless. The $Z$-boson acquires a mass comparable to that of $W$-boson.
• Early 70s - G. 't Hooft et al. have shown that the theory is renormalisable. Introducing a finite number of arbitrary parameters (to be determined from the experiment), it is possible to make predicted amplitudes for physical processes finite at all energies and to all orders in the coupling constant.

• Very important predictions of the new theory - there should be a neutral Z-boson which does not change the charge of particles. Fermi theory predicted only charge W-boson. Z-boson is responsible for the reaction with neutral current, such as $\nu_\mu e^- \rightarrow \nu_\mu e^-$, which were not allowed in the Fermi's theory.
Early searches for neutral currents

- The searches for neutral current events have been started in the early 60s when the electroweak theory of Glashow-Weinberg-Salam was not yet believed to be renormalisable.
- The neutral-current processes were not found in the decays:
  \[ K^+ \rightarrow \pi^+ e^- e^+ \text{ and } K_{L^0} \rightarrow \mu^+ \mu^- \. \]
- The searches had been limited to the neutral currents which changed strangeness. If strangeness is not changed and the reaction goes via a Z-exchange, then in most cases a photon can be exchanged too. Thus the effect of Z is masked by much larger electromagnetic effect.
One way to avoid this was to look for neutrino scattering with the emission of Z. This process could not occur via a photon exchange because neutrino does not couple to a photon. The signature for such a process is the absence of a charged lepton in the final state.
References

4. Lecture notes by Dr. Vitaly Kudryavtsev Sheffield Univ.