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NOTES ON ELECTROSTATIC ION OPTICS AND
BEAM TRANSPORT FOR ION IMPLANTATION

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Introduction. A charged particle moving from ion source to target is influenced by externally applied electric and magnetic fields and by the presence of nearby particles (space charge). External fields used to accelerate, focus, and deflect beams of particles may be subdivided into fields applied transversely to the beam (e.g. dipole deflectors, quadrupole lenses, space charge lenses, etc.) and fields applied longitudinally to the beam (e.g. accelerators, gap lenses, einzel lenses, etc.). Given knowledge of the distribution of applied fields one may in principle predict (classically) the past and future for every particle in the beam from its position and velocity coordinates at any intermediate location. When a focused image must be produced on target (as required for microscopy) this study is properly named "ion optics" whereas if the purpose simply is to move particles efficiently from place to place, this study is more appropriately named "beam transport".

Matrices and first-order beam transport. For many applications first order analysis is sufficient to describe very accurately the motions of charged particles. Processes not amenable to first order analysis are usually called "aberrations" because they distort the final image in some way. However, caution is in order; for example, quadrupole lenses exhibit astigmatism which, despite the insinuation of aberration, is a first order effect. First order ion optics rests on the assumption of a linear relationship between particle position and velocity at one instant in time and the position and velocity some time later. In one plane (see fig. 1), this relationship may be expressed by the linear equations

$$x = a_{11}x_0 + a_{12}x'_0 + a_{13}, \quad (1)$$

$$x' = a_{21}x_0 + a_{22}x'_0 + a_{23}, \quad (2)$$

where x is displacement perpendicular to the beam axis z , divergence "angle" $x' = \dot{x}/\dot{z}$ is the ratio of transverse to longitudinal velocities, (x_0, x'_0) are initial particle coordinates, (x, x') are final particle coordinates, and the a_{ij} are constants that relate initial and final states. A similar set of equations apply to the orthogonal direction y ; usually, motions in x and y are not coupled and may be studied separately. Eqs. 1 and 2 may be written in matrix form as

$$\begin{pmatrix} x \\ x' \\ 1 \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_0 \\ x'_0 \\ 1 \end{pmatrix}. \quad (3)$$

The 2x2 sub-matrix on the diagonal in eq. 3 is a standard form^{1,2}. The third column is not standard, often it is omitted and one or more alternative columns substituted. A useful property of linear equations is that sequences of separate linear operations perpetuate the same linear form. The advantage of matrix methods is that matrices may be derived for individual devices (e.g. accelerators, bending magnets, deflectors, lenses, or empty space) and the effect of using several such devices sequentially is ascertained by multiplying together individual matrices taken in proper order. Matrices do not change the physics of beam transport but they simplify the bookkeeping. Eq. 3 pertains to one particle. Using two vectors representing two particles or rays it is easy to expand this description to include properties of a beam of particles³.

Matrices for a field-free drift space of length l , a thin lens of focal length f , and a general lens having object and image focal lengths (f_1 and f_2) and focal points (F_1 and F_2) are

$$\begin{pmatrix} \text{drift} \\ 1 & l \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} \text{lens} \\ 1 & 0 \\ 1/f & 1 \end{pmatrix} \cdot \begin{pmatrix} \text{general lens} \\ F_2/f_2 & (F_1 F_2 - f_1 f_2)/f_2 \\ -1/f_2 & -F_1/f_2 \end{pmatrix}. \quad (4.5.6)$$

Both lens matrices represent impulses having no insertion length; the finite length of real devices is included by adding drift space to each side of the impulse.

Transverse fields. Electrodes or magnet poles placed on opposite sides of the beam produce fields perpendicular to the beam. Two poles produce dipole fields that deflect or steer the beam sideways. Electric dipoles accelerate particles approaching an attracting electrode and decelerate particles approaching a repelling electrode. The resulting linear differential in particle speeds across the gap causes the deflected beam to converge toward a (usually distant) line focus. The impulse matrix for a deflector is

$$\begin{pmatrix} 1 & 0 & 0 \\ 1/f & 1 & \alpha - \beta \\ 0 & 0 & 1 \end{pmatrix} \cdot \left\{ \begin{array}{l} f = -L/(2\alpha^2 - \alpha\beta), \\ \alpha = \frac{1}{2}(E/U)L, \\ \beta = 2L/(2mU/q)^{1/2}. \\ f = -(L/\alpha^2) [1 - \frac{1}{2}(\Delta m/m_0)] \\ \alpha - \beta = \frac{1}{2}\alpha(\Delta m/m_0). \end{array} \right\} \text{ Wien filter} \quad (7-12)$$

where α is the electric deflection angle, β is the magnetic deflection angle, qU is particle energy, and Δm is the (small) difference in mass from monoenergetic particles of mass m_0 that pass undeflected through the crossed electric and magnetic fields of a Wien velocity filter.

Four regularly spaced poles of alternating polarity surrounding the beam produce quadrupole fields. Fig. 2 shows separate x and y components of the quadrupole field and how these add vectorially in the x,y plane. The magnitude of each field component increases linearly away from the axis. This produces converging (concave) and diverging (convex) cylinder lenses simultaneously in the x and y planes whose matrices are

$$\begin{pmatrix} \cos\theta & (L/\theta)\sin\theta \\ -(L/\theta)\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \cosh\theta & (L/\theta)\sinh\theta \\ (\theta/L)\sinh\theta & \cosh\theta \end{pmatrix} \quad (13,14)$$

$$\theta_x = (L/a)(V_a/U)^{1/2}, \quad \theta_y = L \left[(dB/dy)/(2mE/q) \right]^{1/2} \quad (15,16)$$

where θ is a geometric parameter describing particle trajectories through a lens of length L . V is the electrode potential at radius a in an electric lens, and dB/dy is the field gradient in a magnetic lens. Fig. 3 illustrates how linearly changing transverse fields function as a lens. A quadrupole "singlet" lens necessarily diverges the beam in one plane; however, two singlet lenses of opposite polarity (alternating gradients) form a quadrupole "doublet" lens that provides net convergence in both planes. A quadrupole "triplet" lens consisting of back-to-back doublet lenses has mirror symmetry about its midplane; this preserves more symmetry between incoming and outgoing beams than is possible using a doublet lens. The space charge lens also produces transverse focusing fields that are closely related to quadrupole fields. Whereas the quadrupole singlet lens obeys eq. 13 (converging) in one plane and simultaneously eq. 14 (diverging) in the orthogonal plane, the space charge lens containing an axially-symmetric cloud of trapped charge obeys either eq. 13 or eq. 14 but not both at once.

Longitudinal fields. Longitudinal fields are used to accelerate and, indirectly, to focus beams of charged particles. Matrices for a uniform field accelerator and for lenses that form in fringing fields at the entrance (converging) and exit (diverging) are

$$\begin{pmatrix} \text{exit} & & \text{accelerator} & & \text{entrance} \\ 1 & 0 & \begin{pmatrix} 1 & 2L/(R+1) \\ 0 & 1/R \end{pmatrix} & \begin{pmatrix} 1 & 0 \\ 1/R & 1 \end{pmatrix} & f = 4LR^2/(R^2-1), \\ & & & & F = -4L/(R^2-1). \end{pmatrix} \quad (17-19)$$

where $R = (U/U_0)^{1/2}$ is the ratio of particle speeds at exit and entrance. The entrance lens is the dominant optical element; its focal length is inversely proportional to accelerator gradient. Crossing the beam through a small diameter waist at the accelerator entrance effectively nullifies the entrance lens.

Where longitudinal fields are not uniform (such as at the ends of a graded accelerator tube), changes in longitudinal field produce transverse field components which to first order have the properties of a lens. To understand this, suppose that the electric potential distribution near the axis may be approximated by the power series expansion

$$\phi(r, z) = a + bz + c(z^2 - \frac{1}{2}r^2) + \dots \quad (20)$$

Eq. 20 obeys Laplace's equation (in cylindrical coordinates)

$$\partial^2 \phi / \partial r^2 + (1/r) \partial \phi / \partial r + \partial^2 \phi / \partial z^2 = 0, \quad (21)$$

which describes potential distributions in empty space. Odd powers of r are omitted from eq. 20 in order to preserve rotational symmetry about the axis so that $\phi(r) = \phi(-r)$. The electric field components associated with eq. 20 are

$$E_z = \partial \phi / \partial z = b + 2cz + \dots \quad (22)$$

$$E_r = \partial \phi / \partial r = -cr + O(r^3). \quad (23)$$

Observe that radial forces vary in proportion to r (from eq. 23). Such forces constitute a lens (see fig. 3). Forces proportional to r^3 blur the image and produce aberration.

Lenses created by changing longitudinal fields lack the simplicity of quadrupole lenses in which the transverse field component usually is almost constant except for fringing at the ends. Longitudinal field lenses exist only within the fringing fields and existing analytic procedures are inadequate to provide formal equations of motion. Usually the potential distribution within a lens is determined numerically (first using analog methods but now almost exclusively by digital computer) and then numerical integrations are used to transport sample rays through the potential. Focal properties obtained this way are published in tables and graphs^{4,5}. Unfortunately, one must select from among many possible lens geometries without much guidance.

Two popular designs are the single-gap lens and two-gap einzel lens fabricated from cylinders of equal diameter. Fig. 4 shows that in effect each gap consists of two "semi-lenses", one converging and the other diverging. The beam always spends more time on the converging side producing a net converging lens. In an einzel lens the center electrode may either accelerate or decelerate the incoming beam. For a given applied voltage the decelerating lens is stronger but it suffers the disadvantage that initial deceleration in the diverging semi-lens expands the size of the beam causing substantially more aberration (because of the r^3 nonlinearity indicated in eq. 26).

Beams and phase space area. Fig. 1 illustrates a typical beam transport situation. A small diameter beam (e.g. emerging from an ion source or mass analysis slits) expands to such larger diameter before being focused by a lens to another small spot. These small spots are called "waists" or "crossovers" and correspond closely (but not exactly) to objects and images of light optics. The lens cannot reduce the target spot smaller than some minimum size imposed by the finite area occupied by the beam in x, x' "phase space". Brighter beams occupy smaller areas in phase space and can be focused to smaller size spots.

Fig. 1 also shows typical phase space area "ellipses" associated with the beam. Ellipses or other shapes in phase space shear horizontally (without changing area) as the beam drifts in field-free space (because particle angles do not change but particle position changes linearly with angle as a function of time, as implied by eq. 4). Ellipses shear vertically within a thin lens (because particle positions do not change but particle angles change linearly with position, as implied by eq. 5 and fig. 3). Spot magnification from waist to waist is controlled by geometry. Once the lens has focused the beam to a waist on target it can do nothing more to control the waist size. Two lenses are required to both produce a waist and control the size of the waist. Within limits, the first lens controls the size of the beam entering the second lens; this determines (by inverse proportions) the size of the waist that the second lens will deliver to the target.

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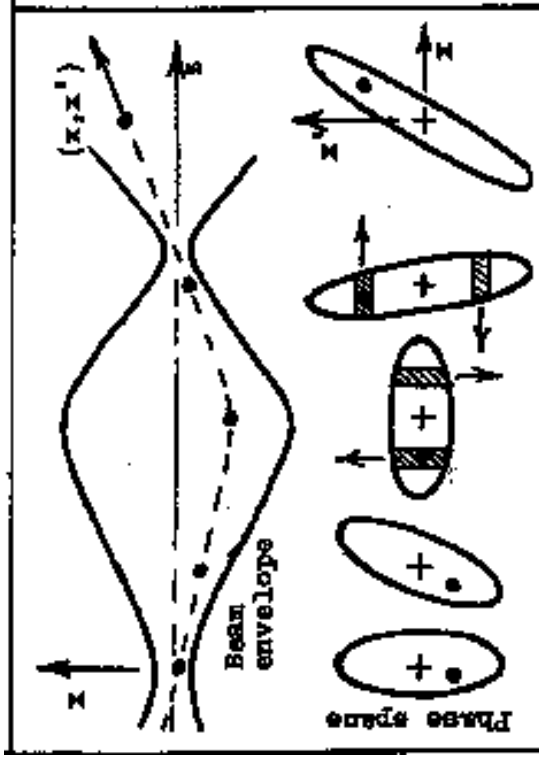


Fig. 1. Beam and phase space ellipses.

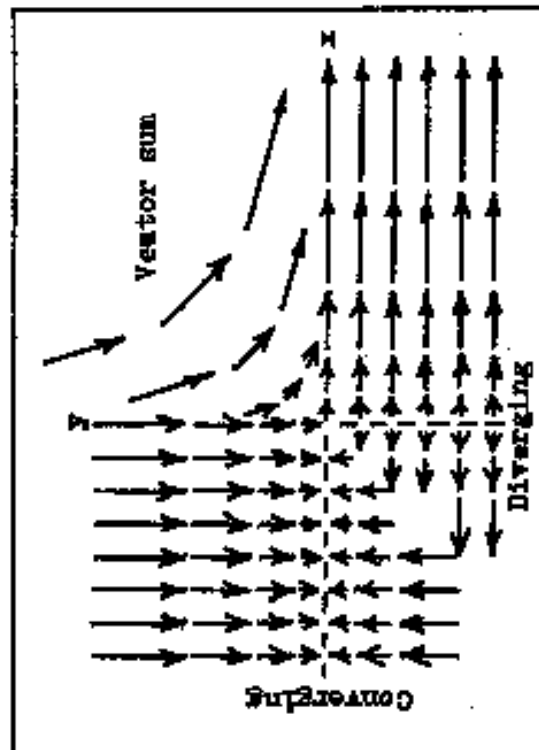


Fig. 2. Quadrupole fields.

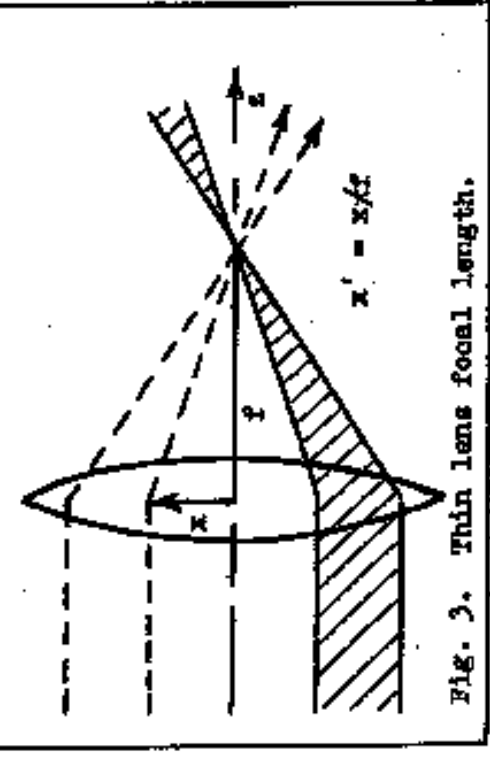


Fig. 3. Thin lens focal length.

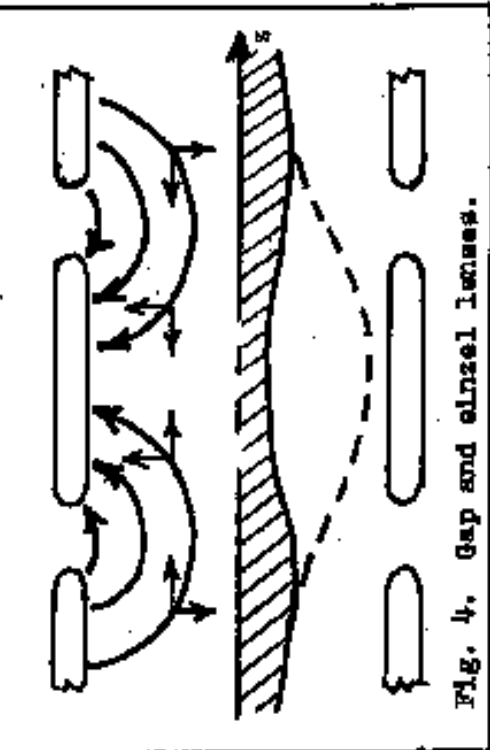


Fig. 4. Gap and einzel lenses.