1 Fundamental Symmetries in the Nucleus

1.1 Parity-Mixing Measurements

1.1.1 Parity Violation with Polarized Epithermal Neutrons (The TRIPLE Collaboration) - General


The nucleon-nucleon force consists of the strong parity conserving (PC) interaction and the weak parity nonconserving (PNC) interaction. The PNC interaction has a strength of order $10^{-7}$ relative to the PC interaction. The weak interaction can be detected by measurement of pseudo-scalar observables of the type $\langle \vec{\sigma} \cdot \vec{k} \rangle \Gamma$ where $\vec{k}$ is the momentum and $\vec{\sigma}$ is the spin of the nucleon. Resonances formed with polarized low-energy neutrons show strong PNC effects. The weak interaction causes the mixing of nuclear levels of the same spin and opposite parity. In heavy nuclei the combination of statistical and kinematic enhancements amplify the PNC effects (by $10^4$ to $10^6$) in the helicity dependence of the neutron cross section.

The TRIPLE collaboration uses the high-flux epithermal neutron beam available at the Manuel Lujan Neutron Scattering Center (MLNSC) at the Los Alamos Neutron Scattering Center (LANSCE) to study the neutron-nucleus weak interaction. The longitudinal asymmetry is measured for neutron energies up to several hundred eV. The analysis treats the PNC matrix elements as random variables - our initial results (for $^{238}$U and $^{232}$Th) yielded root-mean-squared PNC matrix elements with values $M \approx 1$ meV. This value agrees well with the estimate $M_{sp}/N^{1/2}$ where the weak single-particle matrix element is $M_{sp} \approx 0.5$ eV and $N \approx 10^6$ is the approximate number of quasiparticle components in the wave function of a compound-nuclear (CN) state in a heavy nuclide. The $^{232}$Th data showed an unexpected result - all of the longitudinal asymmetries had the same sign. All of the many attempts to explain the ‘sign effect’ as a general feature of the weak neutron-nucleus interaction failed. The early results are summarized in two review articles [Bow93Tfra93].

Following these initial results we developed a new large-area high-polarization proton target for polarizing the neutron beam, a new neutron detector for transmission experiments with large samples and a large solid angle pure CsI detector for capture experiments with

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isotopic samples.

With the new experimental system we performed transmission measurements on $^{238}\text{U}$ and $^{232}\text{Th}$ (repeating our initial measurements) obtained new transmission data on natural In and Ag and performed a capture measurement on $^{113}\text{Cd}$ (with a preliminary version of the capture detector). PNC effects were observed in all of these nuclei. In 1995 the focus was on capture experiments and on targets in the mass $A \approx 100$ region. (The motivation was to determine experimentally the mass dependence of the weak neutron-nucleus interaction.) Capture data were obtained on the separated isotopes $^{106}\text{Pd}$, $^{108}\text{Pd}$, $^{107}\text{Ag}$ and $^{121}\text{Sb}$. Transmission measurements were performed on natural Sb, Xe, I, and Cs.

The most important result is that the sign correlation in the $^{232}\text{Th}$ longitudinal asymmetries is confirmed—eight of eight statistically significant ($> 5\sigma$) effects are positive. Excluding $^{232}\text{Th}$ the new data (combined with a few older measurements) yield longitudinal asymmetries which have approximately 20 plus values and 15 negative values. This suggests that the sign correlation observed in $^{232}\text{Th}$ is specific to that nuclide and is not a general feature of the weak nucleon-nucleus interaction. This result has led to a number of new local doorway-state models.

Subsequent PNC transmission measurements have been performed with natural Pd. A number of PNC effects were observed. Capture measurements are being performed with separated isotopes in order to establish to which isotope these resonances belong.

We have improved the data processing and analysis in several ways. For example there is a $\gamma$-ray component in the neutron beam and these $\gamma$ rays are detected by our neutron detector array. We have developed a procedure that permits the determination of the $\gamma$-ray content in any pulsed neutron beam. Our data have been corrected for this effect.

Initially the PNC longitudinal asymmetries were determined with an empirical single level code. There were several difficulties: (1) a multilevel analysis is often required (2) the shape of the background flux can be appreciably distorted (by strong neighboring s-wave resonances) from the typical smooth energy dependence (3) the observed resonance shape is dominated at higher energies by the asymmetric beam energy resolution function (4) the effects of Doppler broadening and the beam energy resolution cannot be simulated by one symmetric convolution. We have written a multilevel code that addresses all of these (and other) concerns. The data are now being reanalyzed with this new code. Figure 1.1–1 shows a fit of the neutron transmission yield for the $63.5$ eV $p$-wave resonance in $^{238}\text{U}$ for both positive and negative helicity neutrons. The figure demonstrates the ability of the fitting code to precisely determine the difference in the positive and negative helicity cross sections.

The determination of the rms PNC matrix element $M$ from the longitudinal asymmetries has been extended to include targets with spin and to situations where only partial spectroscopic information is available. In cases with all spectroscopic information available the uncertainty is determined by the sample size. However in most cases there is incomplete information available for some or all of the relevant spectroscopic parameters and the
precision with which $M$ can be determined is governed by the status of the relevant nuclear spectroscopic information. We derived probability density functions when there is complete knowledge, partial knowledge, or no information concerning the spectroscopic parameters [Bow96]. Given the probability density functions, the likelihood method can be used to determine the relevant parameters from the experimental results. With the relevant spectroscopic parameters known, one can obtain reliable and reasonably precise values for the rms PNC matrix element $M$ from the longitudinal asymmetry data. With partial information, $M$ can still be obtained, but with increased uncertainty. Our considerations provide a framework for establishing priorities among the various spectroscopic measurements. Much of the recent effort has been to improve the spectroscopic information—spin and orbital angular momentum values, isotopic identification—either by our own measurements and analysis or via a collaboration with a group at the Institute for Reference Materials in Geel, Belgium.

The status of the data and analysis for each target is summarized in the next section.


1.1.2 Parity Violation with Polarized Epithermal Neutrons (The TRIPLE Collaboration) - Status of Experiments


232Th – The thorium target was studied in transmission in 1991 and with the improved system in 1993. For all practical purposes thorium is monoisotopic. Preliminary analysis of 28 $p$-wave resonances yields much improved results but which are qualitatively consistent with the earlier data. There are now eight $5\sigma$ PNC effects and the sign correlation is confirmed. The data are now being reanalyzed with the new more comprehensive code.

238U – The uranium target was studied in transmission in 1990 and with the improved system in 1993. Since the target was depleted to $0.2\%$ $^{235}$U the target was essentially $^{238}$U. Analysis of 20 $p$-wave resonances yields five $4\sigma$ PNC effects. The value of $M$ is very close to that obtained in the earlier measurement.

115In – Natural indium ($95.7\%$ $^{115}$In and $4.3\%$ $^{113}$In) was studied in transmission. An enriched $^{115}$In target was also studied with the capture detector. The combination of these two measurements significantly improved the neutron spectroscopy for $^{115}$In and a paper was published on these spectroscopic results [Fra93]. In addition the group at IRRM Geel plans to study an enriched $^{115}$In target with an emphasis on resonance spin determination. Preliminary analysis of 36 $p$-wave resonances yields six $3\sigma$ PNC effects. These data will be reanalyzed with the new code.

107Ag – Natural silver ($51.8\%$ $^{107}$Ag and $48.2\%$ $^{109}$Ag) was studied in transmission. An enriched $^{107}$Ag target was studied with the capture detector. The combination of these two measurements significantly improved the neutron spectroscopy for $^{107}$Ag. A paper on these spectroscopic results has been accepted for publication [Low97]. In addition the group at IRRM Geel is now studying this same enriched target with an emphasis on the resonance spin determination. Their preliminary results are in excellent agreement with our measurements. Preliminary analysis of 15 $p$-wave resonances yields seven $4\sigma$ PNC effects. These data are being reanalyzed with the new program.

109Ag – The combination of measurements described above identified new resonances in $^{109}$Ag. In addition an enriched $^{109}$Ag target has been studied in neutron capture at IRRM Geel. We plan to combine the LANSCE and IRRM data in order to obtain the best neutron spectroscopy of both silver isotopes. Preliminary analysis of 10 $p$-wave resonances yields four $3\sigma$ PNC effects. These data are being reanalyzed with the new program.

108Pd – An enriched $^{108}$Pd target was studied with the capture detector. These measure-

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ments make a significant improvement in the neutron spectroscopy for $^{106}$Pd. Preliminary analysis indicates one PNC effect at the $3\sigma$ level.

$^{108}$Pd – An enriched $^{108}$Pd target was studied with the capture detector. Preliminary analysis indicates no PNC effects.

Pd – A natural Pd target was studied in transmission. Natural Pd is a mixture of many isotopes. Several PNC effects were observed. In order to identify to which isotope these resonances belong, several enriched isotopic targets ($^{104,105,110}$Pd) have been studied with the capture detector. Five $3\sigma$ PNC effects are observed in $^{106}$Pd.

$^{113}$Cd – An enriched $^{113}$Cd target was studied with the capture detector. Preliminary results indicate three PNC effects at the $3\sigma$ level. Resonance spin measurements have been performed with the same target at IRRM Geel.

$^{117}$Sn – An enriched $^{117}$Sn target was studied with the capture detector. Preliminary analysis indicates six PNC effects.

Sb – Natural (57.25\% $^{121}$Sb and 42.75\% $^{123}$Sb) antimony was studied in transmission. An enriched $^{121}$Sb target was studied with the capture detector. Preliminary analysis indicates two $3\sigma$ PNC effects.

$^{127}$I – A natural iodine target was studied in transmission. Iodine is monoisotopic. Preliminary analysis indicates four PNC effects at the $3\sigma$ level.

Xe – Natural xenon was studied in transmission. (This experiment was in collaboration with an Indiana group. Some xenon isotopes can be highly polarized and the polarization maintained extremely well suggesting that xenon could be a possible polarized target for time reversal studies. The first step was to find suitable resonances displaying PNC effects.) One PNC effect was observed. Subsequent measurements at JINR (Dubna) have identified the resonance as belonging to $^{131}$Xe.

$^{133}$Cs – A natural cesium target was studied in transmission. Cesium is monoisotopic. Preliminary analysis indicates one PNC effect at the $3\sigma$ level.


1.1.3 Parity-Violation Tests with Charged Particles


The neutron resonance parity violation experiments (see preceding sections) have been performed at the $3\pi$ and $4\pi$ neutron strength function maxima near $A = 100$ and $A = 230$. These measurements do not provide a broad dynamic range in $A$ and have large

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uncertainties. Parity-violation tests in light or medium mass nuclei would be of special value in the attempt to determine the mass dependence of the weak spreading width. However, the larger level spacing in light nuclei means that a completely different experimental system is required to perform the neutron experiments in light nuclei. This is not feasible. It appears that parity violation experiments can be performed using charged-particle resonances.

High-resolution \((p\bar{p})\) and \((p\bar{\alpha})\) resonance data from TUNL exist for the targets \(^{27}\text{Al}\), \(^{31}\text{P}\), \(^{35}\text{Cl}\), and \(^{39}\text{K}\). We considered resonance pairs with the same \(J\) and different \(\pi\). For each of these resonance pairs, the values of \(A_Z(\theta)\) and \(A_X(\theta)\) were calculated for both the \((p\bar{p}_0)\) and \((p\bar{\alpha}_0)\) reactions; we also calculated the value of \(A_Z\) for the \((p\bar{\alpha}_0)\) reaction integrated over \(4\pi\).

The Hamiltonian was \(H = H_0 + H_{PV}\) where \(H_0\) is parity-conserving and \(H_{PV}\) is parity-nonconserving. Perturbed reduced-width amplitudes were obtained using experimentally determined resonance parameters and first-order perturbation theory; we assumed internal mixing between two states. Differential cross sections were then calculated for a longitudinally polarized proton beam and convoluted with a 500-eV FWHM Gaussian to simulate finite beam-energy-resolution. Since the longitudinal analyzing power \(A_Z(\theta)\) is to first order proportional to \(V\) (the matrix element of \(H_{PV}\)), the ratio \(\frac{A_Z}{A_X}\) is a suitable measure of the relative enhancement. We also calculated the analyzing power component \(A_X\).

The results depend dramatically on energy, angle, and the specific resonance parameters. A suitable figure of merit for \(A_Z(\theta)\) is \(\beta_P = \frac{(A_Z)^2}{(A_X)^2}\) since maximizing this ratio minimizes the time to reach a given sensitivity in \(V\) (assuming that all other experimental factors remain the same). Results for \(A_Z(\theta)\) and \(A_X(\theta)\) are comparable but fluctuate from resonance to resonance. The \(\frac{A_Z}{A_X}\) values are typically one to two orders of magnitude smaller for the \((p\bar{p}_0)\) reaction than for the \((p\bar{\alpha}_0)\) reaction. Since measuring \(A_Z\) appears much easier experimentally, we have focused on that measurement. The calculations are discussed in two recent papers [JFS94, MIT96].

The proposed experiments require a number of improvements including improvement in the stability of the polarized beam energy, position, and polarization direction. A recently installed stripper bias amplifier has reduced the energy fluctuations due to accelerator voltage fluctuations by a factor of approximately 10. This is equivalent to terminal voltage fluctuations of about 30 V. An improved slit feedback steering system has been constructed to provide the necessary improvements in beam position stability. Since the counting rates are a primary concern, a large solid-angle detector is of particular value. However, since often the sign of the parity violation changes with angle, a segmented detector is essential. There are a number of examples where the difference in the net effect for a single detector and a segmented detector is two orders of magnitude.

We have constructed an array of silicon strip detectors that subtends approximately 80% of \(2\pi\) at backward angles. The detector array is segmented by 16 in polar angle \(\theta\) and 4 in azimuthal angle \(\phi\) for a total of 64 individual detector elements. Sixteen-channel
preamplifiers provide energy and timing outputs, with the energy signals going directly to charge-integrating analog-to-digital converters. Gates for the ADC’s are generated from the timing signals by constant-fraction discriminators. By using short shaping times (300 ns) and gates (75 ns) pulse pile-up is kept to a minimum at high counting rates.

In a recent test 3.4 MeV polarized protons were incident on a 10 μg/cm² ³¹P target with a 2 μg/cm² carbon backing. Beam currents were typically 0.7 μA. In tests of individual detector strips the detector performed very well with good energy resolution and low pile-up. At the resonance peak the background under the alpha peak was less than 10%. Work is presently underway to scale up the data acquisition to 64 simultaneous channels and to optimize the throughput of data.

In order to confront issues of systematic errors one needs to know the value of the experimental quantity $A_2$ not the enhancement factor $A_2$. Although the determination of $V$ is the goal of the measurement an estimate for $\langle V \rangle$ can be obtained by assuming a value for the weak spreading width $\Gamma_W = \frac{2\pi W}{D(J)}$ where $D(J)$ is the local average level spacing for resonances with spin $J$. We assume that $\Gamma_W$ is constant and equal to the average of the values obtained in the TRIPLE measurements for U and Th and use our experimental values of $D(J)$. This yields an estimate for $\langle V \rangle$ for each resonance pair. The estimates for $\langle V \rangle$ are usually between 50 and 150 meV with values for $A_2$ ranging from $10^{-3}$ to $10^{-7}$ (most are between $10^{-4}$ and $10^{-6}$). Measurements at the $10^{-4}$ level would provide significant new information – this level of sensitivity is our initial goal. We have performed a detailed study of the sources of systematic errors as well as methods for monitoring and controlling them. A paper considering these methods for performing precision measurements of parity violating asymmetries was published very recently [Wil97].

From our calculations for the enhancement one can estimate the time required to measure $V$ at a given level for a particular pair of resonances. Incorporating beam intensity $k$ target thickness $\theta$ solid angle $\Delta$ and detector efficiency into a constant $\beta$ the time required to determine $V$ at a level $V_0$ is $t = \frac{k}{[\beta(V_0)^2]}$. One can invert this procedure and ask what limit is set on $V$ if one measures for a time $t_0$ and no parity violation is observed. For ³¹P there are eight resonance pairs with $\beta_0$ larger than $10^{-6}$. For these resonances the limits placed on $V$ by a null result after one day of measurement would average about 25 meV. This limit would constrain $\Gamma_W$ to be much less than the value measured in heavy nuclei. Therefore even a set of null results would be quite important.


1.2 Time-Reversal Invariance

1.2.1 Neutron Resonance Tests of P-even T-violation

E. D. Davis¹, C. R. Gould, D. G. Haase, and P. R. Huffman²

The most precise dynamical bounds on parity conserving T-odd cross sections come from our 1996 work with polarized MeV neutrons and a cryogenically aligned $^{165}$Ho target [Huf96]. In this work we measured asymmetries at the $10^{-6}$ level and compared to traditional detailed balance studies, achieved a factor of four hundred improvement in a relative cross-section measurement.

Bounds on the underlying T-odd $\rho$-meson exchange coupling were still modest however and weaker than those inferred by Haxton [Hax94] from atomic edm measurements and by Simonius [Sim97] from an analysis of the TRIUMF and IUCF charge-symmetry breaking measurements. Resonance tests hold promise for large enhancements and we have been investigating the conditions under which resonance polarized neutron-transmission tests will yield bounds comparable to or better than the CSB and atomic edm work.

We have focused on holmium because it is easily aligned cryogenically and many of the spins of the resonances are known from previous work. A test of time reversal is carried out on a resonance formed by mixed $s$-wave and $d$-wave amplitudes where the $d$-wave amplitude can interfere with a neighboring $s$-wave resonance of the same spin. In this situation there will be sensitivity to the T-odd matrix element $W_{rms}$ coupling the states. The sensitivity scales with the magnitude of the $d$-wave partial-width amplitude which can itself be determined in a separate measurement of the deformation-effect cross section $\sigma_{\text{DEF}}$.

To analyze sensitivity we have considered two level interference between the $d$-wave component of the (presumed weak) resonance showing the deformation effect labelled 2 and having energy $E_2$ and the $s$-wave component of the (presumed strong) neighboring resonance labelled 1 and having energy $E_1$. The ratio of the five-fold correlation (FC) cross section $\sigma_{\text{FC}}$ to the deformation-effect cross section $\sigma_{\text{DEF}}$ on top of resonance 2 is given by

$$\frac{\sigma_{\text{FC}}}{\sigma_{\text{DEF}}} = \frac{-2w z W_{rms}}{(E_2 - E_1)} \left( \frac{\Gamma_{n_{1}}(0)}{\Gamma_{n_{2}}(0)} \right),$$

(1.1)

where

$$z \equiv \frac{1 + \alpha \tan \phi}{1 - \beta \tan \phi},$$

(1.2)

is related to the ratio of partial-width amplitudes $\Gamma$.

$$\tan \phi \equiv \frac{g_{l_{2}}^{J}(2\frac{3}{2})}{g_{l_{2}}^{J}(2\frac{5}{2})}.$$
through constants $\alpha$ and $\beta \Gamma$ and $w$ is a Gaussian distributed random variable with zero mean and unit variance.

The equation tells us that the sensitivity to the TRV matrix element $W_{\text{rms}}$ is determined by the probability distribution density of the product $r = wz$ of two random variables $\Gamma w$ and $z$. In the limit $I \gg 1$,

$$P(r) = \frac{1}{\sqrt{3\pi}} \frac{1}{\pi} e^{r^2/3} E_1 \left( r^2/3 \right),$$

where $E_1$ is the exponential integral. Numerical tests show that the $E_1$ form differs by 10% or less from the exact integral result for $I = 7/2$.

To set a bound on $W_{\text{rms}}$ (at some prescribed level) we consider measurements carried out on $N$ different resonances labelled $i$. Let $C_i$ denote the value of 

$$\frac{-2}{(E_2 - E_1)} \sqrt{\frac{\Gamma^f_{n1}(0)}{\Gamma^f_{n2}(0)}}$$

for the $i$th measurement $R_i$ of the ratio $\sigma_{\text{FC}}/\sigma_{\text{DEF}}$ with experimental error $\delta_i$.

If we take the Bayesian prior for $W_{\text{rms}}$ to be a constant for $W_{\text{rms}} \geq 0$ and zero for $W_{\text{rms}} < 0$ (the least contentious choice) $\Gamma$ then the probability distribution $P(W_{\text{rms}})$ for $W_{\text{rms}}$ is given by

$$P(W_{\text{rms}}) = \frac{1}{N} f(W_{\text{rms}}) \Theta(W_{\text{rms}})$$

where

$$f(w) = \prod_{i=1}^{N} \left\{ \int_{-\infty}^{+\infty} \exp \left[ -\frac{(S_i - wr)^2}{2\delta_i^2} \right] P(r) dr \right\}$$

and $N$ is the normalization constant

$$N = \int_{0}^{\infty} f(w) dw.$$

This equation allows us to set bounds on $W_{\text{rms}}$ based on experimental measurements $R_i \pm \delta_i$. In general each measurement will have different sensitivities depending on the constants $C_i$ and on the statistical accuracy of each measurement $\delta_i$. For holmium values of $C_i$ are typically in the range (1-10)/eV with lower values occurring at higher energies. Experimental accuracies depend on the (unknown) $d$-wave amplitudes but our earlier estimates predict values $\delta_i = 0.01 \Gamma$ i.e. the ratios will be bounded at the 1% level.

Results quantitatively confirm the earlier analysis of Davis [Dav94] that at least three resonance results are needed to sensibly bound $W_{\text{rms}}$ in an FC experiment. With only one result $P(W_{\text{rms}})$ is unbounded. With two results the 95% confidence bound is still 0.2 eV but for three it drops to 0.02 eV and for seven or more results to 0.002 eV or better.
Multiple measurements on many different resonances dramatically improve the confidence limits in FC experiments.

Recent work on microscopic time-reversal violating optical potentials [Eng94] has shown how to relate asymmetries in MeV-neutron FC measurements to the strength of PC TRV meson-nucleon coupling constants. From \( \Gamma_T \sim 2\pi W_{rms}^2/D \sim 2\pi 10^5 a_T^2 \), a bound of 0.02 eV on \( W_{rms} \) implies a bound of \( 2 \times 10^{-5} \) on \( a_T \), the ratio of the time-reversal violating to time-reversal conserving parts of the nucleon-nucleon effective interaction.

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### 1.2.2 Statistics of On-Resonance Deformation-Effect Measurements

**E. D. Davis**\(^1\) and **C. R. Gould**

We have begun investigating the statistics of on-resonance measurements of the deformation-effect cross section \( \sigma_{02} \) in unpolarized neutron transmission through an aligned \(^{165}\)Ho target. We consider an unpolarized neutron beam of momentum \( p \) incident on a aligned target of nuclei with spin \( I \) (below \( \hat{I} \) denotes the unit vector specifying the orientation of the aligned target). Deformation-effect cross sections \( \sigma_{0K} \) are related to the total cross section \( \sigma_{tot} \) observed in neutron transmission with an unpolarized beam by

\[
\sigma_{tot} = \sum_K \sigma_{0K} \tilde{f}_{K0}(I),
\]

and can be identified through their distinctive dependence on \( \hat{f} \cdot \hat{p} [\sigma_{0K} \propto P_k(\hat{f} \cdot \hat{p})] \). For epithermal neutrons the dominant deformation-effect cross section is \( \sigma_{02} \Gamma \) which in this regime dominates by transitions between \( s \) and \( d \) partial waves and \( p \) partial waves (barrier penetrabilities suppress contributions from higher partial waves and the higher-order deformation-effect cross sections \( \sigma_{0K} \Gamma K \geq 4 \)).

In the simplest reaction model for an on-resonance measurement of the deformation-effect cross section \( \sigma_{02} \Gamma \) only the contribution to the \( S \) matrix of the resonance at which the

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measurement is performed is taken into account. If $E_J$ and $\Gamma_J$ denote the energy and total width respectively of this resonance then for $E \approx E_J$,

$$\sigma_{02} = 2\pi^2 I P_2(\hat{\mathbf{T}} \cdot \hat{\mathbf{p}}) \frac{g_J \Gamma_J}{(E - E_J)^2 + \Gamma_J^2/4} C(J),$$

where the statistical weight $g_J = (2J + 1)/(2I + 1)$ and $\Gamma_J$ is the reduced partial width $\Gamma_{n01}^J$ of the resonance $\Gamma$.

$$C(J) = \left\{ \begin{array}{l}
C_{sd} = -2W(J^J_1/2; \Gamma^J_2/2) \gamma_{n01}^J \gamma_{n22}^J + \sqrt{6} W(J^J_1/2; \Gamma^J_2/2) \gamma_{n01}^J \gamma_{n22}^J + 3 \sqrt{6} W(J^J_1/2; \Gamma^J_2/2) \gamma_{n01}^J \gamma_{n22}^J \\
C_{pp} = -2W(J^J_1/2; \Gamma^J_2/2) \gamma_{n11}^J \gamma_{n10}^J - W(J^J_2/2; \Gamma^J_2/2) \left( \gamma_{n14}^J \right)^2
\end{array} \right.$$ 

for measurements at an $s$-wave resonance (with a $d$-wave admixture) and a $p$-wave resonance $\Gamma$ respectively.

For measurements at $s$-wave resonances we anticipate that there will be no spectroscopic data to constrain the partial-width amplitudes $\gamma_{n2j}^J$ [even the $d$-wave neutron partial width $\Gamma_{n2}^J = (\gamma_{n22}^J)^2 + (\gamma_{n21}^J)^2$ of a predominantly $s$-wave resonance will be unobservable] but one can take advantage of the fact that $s$-wave partial widths $\Gamma_{n01}^J$ will be measurable to extract values of

$$D_n \equiv g_J C_{sd} / \sqrt{\Gamma_{n01}^J P_d(E_J)} = -g_J \sum_j \frac{(-1)^{2j+1}/2 \gamma_{n1j}}{\Gamma_{n1j}^J} W(J^J_1/2; I_j) \gamma_{n2j}^J,$$

where $P_d(E_J)$ is the $d$-wave penetrability factor required to convert $d$-wave partial widths $\Gamma_{n2j}^J$ to reduced partial widths $\Gamma_{n01}^J$ (we assume phases are chosen so that $\gamma_{n01}^J = \sqrt{\Gamma_{n01}^J}$).

Elimination of the dependence on $s$-wave partial amplitudes $\gamma_{n01}^J$ guarantees that the fluctuations in $D_n$ are far simpler than those of $C_{sd}$.

The standard (Porter-Thomas) assumption about the reduced partial-width amplitudes $\gamma_{n21}^J$ and $\gamma_{n22}^J$ of a set of $s$-wave resonances of a given spin $J$ is that they are sampled from independent gaussian distributions of zero mean and variances $\langle \Gamma_{n1j}^J \rangle$ and $\langle \Gamma_{n1j}^J \rangle$ respectively where $\langle \Gamma_{n1j}^J \rangle$ denotes the running energy average of the reduced partial widths $\Gamma_{n1j}^J$ of the $s$-wave resonances. Under this assumption the prediction for measurements of $\sigma_{02}$ restricted to resonances of a given spin $J$ is that the values of $D_n$ are drawn from a gaussian distribution of zero mean and variance

$$v_j^2 = g_J^2 \sum_j (2j + 1) \left[ W(J^J_1/2; I_j) \right]^2 \langle \Gamma_{n1j}^J \rangle \simeq g_J^2 \langle \Gamma_{n1j}^J \rangle / (2I + 1),$$

for measurements at an $s$-wave resonance (with a $d$-wave admixture) and a $p$-wave resonance $\Gamma$ respectively.

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$$D_n \equiv g_J C_{sd} / \sqrt{\Gamma_{n01}^J P_d(E_J)} = -g_J \sum_j \frac{(-1)^{2j+1}/2 \gamma_{n1j}}{\Gamma_{n1j}^J} W(J^J_1/2; I_j) \gamma_{n2j}^J,$$

where $P_d(E_J)$ is the $d$-wave penetrability factor required to convert $d$-wave partial widths $\Gamma_{n2j}^J$ to reduced partial widths $\Gamma_{n01}^J$ (we assume phases are chosen so that $\gamma_{n01}^J = \sqrt{\Gamma_{n01}^J}$).

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$$v_j^2 = g_J^2 \sum_j (2j + 1) \left[ W(J^J_1/2; I_j) \right]^2 \langle \Gamma_{n1j}^J \rangle \simeq g_J^2 \langle \Gamma_{n1j}^J \rangle / (2I + 1),$$

for measurements at an $s$-wave resonance (with a $d$-wave admixture) and a $p$-wave resonance $\Gamma$ respectively.
if $\langle \Gamma^{(0)}_{nlj} \rangle$ is approximately independent of $j$. In practice, the spins of the $s$-wave resonances are unlikely to be identified in which case the measured values of $D_s$ have probability density

$$p_s(x) = \sum_{J=I-\frac{1}{2}, I+\frac{1}{2}} \frac{p_J}{\sqrt{2\pi v_J^2}} \exp \left( -\frac{x^2}{2v_J^2} \right),$$

where $p_J$ is the probability a $s$-wave resonance has spin $J$ (the ratio $p_{I+\frac{1}{2}}/p_{I-\frac{1}{2}} \simeq (1 + 1/I) \exp[-(2I + 1)/(4\sigma^2)]\Gamma$ where $\sigma$ is the appropriate level-density spin cut-off factor).
1.3 Quantum Chaos in Nuclei

1.3.1 A Complete Level Scheme for $^{30}$P


In the past year we have continued our measurements on the $^{28}$Si(pP)$^{30}$P reaction. These measurements have been undertaken with the goal of establishing a complete level scheme for $^{30}$P. Once the level scheme is established we will analyze the energy eigenvalue fluctuations and transition-strength distributions to study the role of chaos and isospin breaking in this system.

Measurements with a pair of fixed HPGe detectors determined branching ratios for 47 resonances in this reaction. For a number of resonances those results were sufficient to determine the spin $J$, the parity $\pi$, and the isospin $T$ of the resonance level; these results were published within the last year [P. M. Wallace et al. Phys. Rev. C 54, 2916 (1996); G. A. Vavrina et al. Phys. Rev. C 55, 1119 (1997)]. For other resonances as well as for some lower-lying states additional measurements are needed. We are currently measuring angular distributions for this reaction by moving the unsuppressed HPGe detector through a series of five angles while holding the Compton-suppressed HPGe detector fixed as a monitor.

As an example, we consider the results obtained for the $E_x = 7759$ keV level. The branching ratio measurements allowed spin/parity combinations of $1^+\Gamma 2^+\Gamma 2^-\Gamma$ or $3^+$. The isospin was unknown. Angular distributions were measured for seven primary $\gamma$ rays and for one secondary $\gamma$ ray.

A code has been written to analyze primary and secondary transitions. Only secondary transitions which are fed by a single (primary) transition can be included. The analysis consists of a simultaneous fit to the entrance channel proton-mixing ratio (which is common to all transitions) to a $\gamma$-ray mixing ratio for each primary (if the transition can be mixed) to a $\gamma$-ray mixing ratio for each secondary (if the transition can be mixed) and to an overall normalization constant for each transition. Because the mixing ratios appear quadratically two equally good solutions are sometimes found for the mixing ratios (although not in the present case).

In the case of the $E_x = 7759$ keV level the angular distributions for the seven primaries were incompatible with the $1^+\Gamma 2^+\Gamma 2^-\Gamma$ possibilities. The angular distribution for the secondary $2724 \rightarrow 0$ transition was inconsistent with the primary $r \rightarrow 2724$ transition if $2^+$ was assumed; thus this level is assigned a spin/parity of $3^+$. Graphs of the angular distribution data and fits for these two transitions are shown in Figure 1.3–1.

Comparison of reduced transition probabilities with the recommended upper limits for

\footnote{Tennessee Technological University, Cookeville, TN.}
Table 1.3-1: Mixing ratios for the $E_x = 7739$ keV level.

<table>
<thead>
<tr>
<th>Mixing Ratio</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta_p$</td>
<td>0.2±0.2</td>
</tr>
<tr>
<td>$\delta_{r\rightarrow4422.8}$</td>
<td>0.07±0.08</td>
</tr>
<tr>
<td>$\delta_{r\rightarrow3928.6}$</td>
<td>0.2±0.2</td>
</tr>
<tr>
<td>$\delta_{r\rightarrow2839.3}$</td>
<td>-0.04±0.09</td>
</tr>
<tr>
<td>$\delta_{r\rightarrow2723.7}$</td>
<td>0.01±0.01</td>
</tr>
<tr>
<td>$\delta_{r\rightarrow2539.0}$</td>
<td>0.4±0.2</td>
</tr>
<tr>
<td>$\delta_{r\rightarrow1973.3}$</td>
<td>0.26±0.06</td>
</tr>
<tr>
<td>$\delta_{r\rightarrow1451.2}$</td>
<td>0.00±0.02</td>
</tr>
<tr>
<td>$\delta_{2723.7\rightarrow0}$</td>
<td>-4.8±0.4</td>
</tr>
</tbody>
</table>

those quantities [End93] shows that the isospin for this level can be assigned with knowledge of the mixing ratio for the $r\rightarrow2724$ keV transition. One finds that for a $3^+$ assignment the $\gamma$-ray mixing ratio for this transition must be greater than 0.80 to allow $T = 0$ or less than 0.71 for $T = 1$. The angular distributions yield a mixing ratio of 0.01 ± 0.01; thus the state has isospin $T = 1$.

The mixing ratios measured for this resonance are listed in Table 1.3–1.

Figure 1.3–1: Angular distributions and fits for two transitions in the $^{28}\text{Si}(p,\gamma)$ reaction.

1.3.2 Reduced Transition-Probability Distributions

A. A. Adams, G. E. Mitchell, W. E. Ormand\textsuperscript{1}, and J. F. Shriner, Jr.\textsuperscript{2}

Although eigenvalue analyses have been the most commonly used method of searching for chaos in quantum systems the stringent data requirements make alternative signatures most desirable. Several theoretical studies [Allh92,Mer93] have studied statistical distributions of transition strengths within various models. They suggest that the strengths follow a $\chi^2(\nu = 1)$ distribution if the system is chaotic and a $\chi^2(\nu < 1)$ distribution if the system is regular.

A study of $B(M1)$ and $B(E2)$ values in $^{22}\text{Na}$ calculated with the shell model showed $\chi^2$ distributions with $\nu = 1$. Results from this aspect of our study have been published [A. A. Adams et al., Phys. Lett. B 392 1 (1997)].

We then applied these same techniques to the large body of experimental data available for the nuclide $^{20}\text{Al}$ [End88a,End88b]. Here an interesting result is observed: the data are no longer consistent with a $\chi^2$ distribution at all! We show in Figure 1.3–2 the normalized distribution of reduced transition strengths in $^{20}\text{Al}$ and a $\chi^2(\nu = 1)$ distribution for comparison. Since all other $\chi^2$ distributions also peak at values of $\log_{10} y = 0$ it is clear that the experimental distribution shown will not be well characterized by any $\chi^2$ distribution. The most reasonable explanation for this behavior is that we are observing the effects of broken isospin symmetry on the transition-strength distribution. However there is no theoretical study of this particular aspect of transition distributions to guide expectations. Further study is underway.

\begin{itemize}
\end{itemize}

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\textsuperscript{2}Tennessee Technological University, Cookeville, TN.
Figure 1.3–2: Experimental distribution of reduced transition probabilities in $^{26}\text{Al}$. The variable $y$ represents a normalized transition probability. The solid curve shows a $\chi^2(\nu = 1)$ distribution for comparison.